Cosmic Mimicry: Is LCDM a Braneworld in Disguise?

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Abstract. For a broad range of parameter values, braneworld models display a remarkable property which we call *cosmic mimicry*. Cosmic mimicry is characterized by the fact that, at low redshifts, the Hubble parameter in the braneworld model is virtually indistinguishable from that in the LCDM (Λ + Cold Dark Matter) cosmology. An important point to note is that the $\Omega_{\rm m}$ parameters in the braneworld model and in the LCDM cosmology can nevertheless be quite different. Thus, at high redshifts (early times), the braneworld asymptotically expands like a matter-dominated universe with the value of $\Omega_{\rm m}$ inferred from the observations of the local matter density. At low redshifts (late times), the braneworld model behaves almost exactly like the LCDM model but with a *renormalized* value of the cosmological density parameter $\Omega_{\rm m}^{\rm LCDM}$. The value of $\Omega_{\rm m}^{\rm LCDM}$ is smaller (larger) than $\Omega_{\rm m}$ in the braneworld model with positive (negative) brane tension. The redshift which characterizes cosmic mimicry is related to the parameters in the higher-dimensional braneworld Lagrangian. Cosmic mimicry is a natural consequence of the scale-dependence of gravity in braneworld models. The change in the value of the cosmological density parameter (from $\Omega_{\rm m}$ at high z to $\Omega_{\rm m}^{\rm LCDM}$ at low z) is shown to be related to the spatial dependence of the effective gravitational constant $G_{\rm eff}$ in braneworld theory. A subclass of mimicry models lead to an older age of the universe and also predict a redshift of reionization which is lower than $z_{\rm reion} \simeq 17$ in the LCDM cosmology. These models might therefore provide a background cosmology which is in better agreement both with the observed quasar abundance at $z \gtrsim 4$ and with the large optical depth to reionization measured by the Wilkinson Microwave Anisotropy Probe.

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1. Introduction

Braneworld models have been extensively applied to cosmology, where they demonstrate qualitatively new and very interesting properties (see [1, 2] for recent reviews). Theories with the simplest generic action involving scalar-curvature terms both in the bulk and on the brane are not only good in modeling cosmological dark energy but, in doing so, they also exhibit some interesting specific features, for example, the possibility of superacceleration (supernegative effective equation of state of dark energy $w_{\rm eff} \leq -1$) [3, 4] and the possibility of cosmological loitering even in a spatially flat universe [5].

It was noted some time ago [6] that the cosmological evolution in braneworld theory, from the viewpoint of the Friedmann universe, proceeds with a time-dependent gravitational constant. In this letter, we further study this property and report another interesting feature of braneworld cosmology, which we call "cosmic mimicry." It turns out that, for a broad range of parameter values, the braneworld model behaves *exactly as a LCDM (Λ + Cold Dark Matter) universe* with different values of the effective cosmological density parameter $\Omega_{\rm m}$ at different epochs. This allows the model to share many of the attractive features of LCDM [7, 8] but with the important new property that the cosmological density parameter inferred from the observations of the large-sale structure and cosmic microwave background (CMB) and that determined from neoclassical cosmological tests such as observations of supernovae (SN) can potentially have different values.

An important feature of this model is that, although it is very similar to LCDM at the present epoch, its departure from "concordance cosmology" can be significant at intermediate redshifts, leading to new possibilities for the universe at the end of the "dark ages" which may be worth exploring.

We relate the "mimicry" properties of the braneworld cosmology with the properties of gravity in braneworld theories. In particular, we show that the change in the cosmological density parameter $\Omega_{\rm m}$ as the universe evolves can be related to the spatial scale dependence of the effective gravitational constant in braneworld theory [9]. This can have important consequences for cosmological models based on the braneworld theory and calls for more extensive analysis of their cosmological history.

2. Cosmic mimicry

We consider the simplest generic braneworld model with an action of the form

$$S = M^3 \left[\int_{\rm bulk} \left(\mathcal{R} - 2\Lambda_{\rm b} \right) - 2 \int_{\rm brane} K \right] + \int_{\rm brane} \left(m^2 R - 2\sigma \right) + \int_{\rm brane} L \left(h_{ab}, \phi \right) . \qquad (1)$$

Here, $\mathcal{R}$ is the scalar curvature of the metric g_{ab} in the five-dimensional bulk, and R is the scalar curvature of the induced metric $h_{ab} = g_{ab} - n_a n_b$ on the brane, where n^a is the vector field of the inner unit normal to the brane, which is assumed to be a boundary of the bulk space, and the notation and conventions of [10] are used. The quantity $K = h^{ab} K_{ab}$ is the trace of the symmetric tensor of extrinsic curvature

$K_{ab} = h^c{}_a \nabla_c n_b$ of the brane. The symbol $L(h_{ab}, \phi)$ denotes the Lagrangian density of the four-dimensional matter fields ϕ whose dynamics is restricted to the brane so that they interact only with the induced metric h_{ab}. All integrations over the bulk and brane are taken with the corresponding natural volume elements. The symbols M and m denote the five-dimensional and four-dimensional Planck masses, respectively, $\Lambda_{\rm b}$ is the bulk cosmological constant, and σ is the brane tension.

Action (1) leads to the Einstein equation with cosmological constant in the bulk:

$$\mathcal{G}_{ab} + \Lambda_{\rm b} g_{ab} = 0 \,, \tag{2}$$

while the field equation on the brane is

$$m^2 G_{ab} + \sigma h_{ab} = \tau_{ab} + M^3 \left(K_{ab} - h_{ab} K \right) \,, \tag{3}$$

where τ_{ab} is the stress–energy tensor on the brane stemming from the last term in action (1).

The cosmological evolution on the brane that follows from (2) and (3) is described by the main equation [3, 11, 12, 13]

$$H^2 + \frac{\kappa}{a^2} = \frac{\rho + \sigma}{3m^2} + \frac{2}{\ell^2} \left[1 \pm \sqrt{1 + \ell^2 \left(\frac{\rho + \sigma}{3m^2} - \frac{\Lambda_{\rm b}}{6} - \frac{C}{a^4} \right)} \right] , \tag{4}$$

where C is the integration constant describing the so-called "dark radiation" and corresponding to the black-hole mass of the Schwarzschild–(anti)-de Sitter solution in the bulk, $H \equiv \dot{a}/a$ is the Hubble parameter on the brane, and the term κ/a^2 corresponds to the spatial curvature on the brane. The length scale ℓ is defined as

$$\ell = \frac{2m^2}{M^3} \,. \tag{5}$$

The "$\pm$" signs in (4) correspond to two different branches of the braneworld solutions [3]. Models with the lower ("$-$") sign were called Brane 1, and models with the upper ("$+$") sign were called Brane 2 in [3], and we refer to them in this way throughout this paper.

In what follows, we consider a spatially flat universe ($\kappa = 0$) without dark radiation ($C = 0$). It is convenient to introduce the dimensionless cosmological parameters

$$\Omega_{\rm m} = \frac{\rho_0}{3m^2 H_0^2} \,, \quad \Omega_\sigma = \frac{\sigma}{3m^2 H_0^2} \,, \quad \Omega_\ell = \frac{1}{\ell^2 H_0^2} \,, \quad \Omega_{\Lambda_{\rm b}} = -\frac{\Lambda_{\rm b}}{6 H_0^2} \,, \tag{6}$$

where the subscript "0" refers to the current values of cosmological quantities. The cosmological equation with the energy density ρ dominated by dust-like matter can now be written in a transparent form:

$$\frac{H^2(z)}{H_0^2} = \Omega_{\rm m}(1{+}z)^3 + \Omega_\sigma + 2\Omega_\ell \pm 2\sqrt{\Omega_\ell}\,\sqrt{\Omega_{\rm m}(1{+}z)^3 + \Omega_\sigma + \Omega_\ell + \Omega_{\Lambda_{\rm b}}} \,. \tag{7}$$

The model satisfies the constraint equation

$$\Omega_{\rm m} + \Omega_\sigma \pm 2\sqrt{\Omega_\ell}\,\sqrt{1 + \Omega_{\Lambda_{\rm b}}} = 1 \tag{8}$$

reducing the number of independent Ω parameters. The sign choices in Eqs. (7) and (8) always correspond to each other if $1 + \Omega_{\Lambda_{\rm b}} > \Omega_\ell$. In the opposite case, $1 + \Omega_{\Lambda_{\rm b}} < \Omega_\ell$, both signs in (8) correspond to the lower sign in (7), and the option with both upper signs in Eqs. (7) and (8) does not exist (see the appendix of the first paper in [3] for details). The signs in (8) correspond to the two possible ways of bounding the Schwarzschild–(anti)-de Sitter bulk space by the brane [11, 13].

In the "normal" case $1 + \Omega_{\Lambda_{\rm b}} > \Omega_\ell$, substituting Ω_σ from (8) into (7), we get

$$\frac{H^2(z)}{H_0^2} = \Omega_{\rm m}(1{+}z)^3 + 1 - \Omega_{\rm m} + 2\Omega_\ell \mp 2\sqrt{\Omega_\ell}\,\sqrt{1+\Omega_{\Lambda_{\rm b}}} \pm 2\sqrt{\Omega_\ell}\,\sqrt{\Omega_{\rm m}(1{+}z)^3 - \Omega_{\rm m} + \left(\sqrt{1+\Omega_{\Lambda_{\rm b}}} \mp \sqrt{\Omega_\ell}\right)^2}\,. \tag{9}$$

In the "special" case $1 + \Omega_{\Lambda_{\rm b}} < \Omega_\ell$, the Brane 1 model [the lower sign in (7)] corresponds to both signs of (8), and the cosmological equation reads as follows:

$$\frac{H^2(z)}{H_0^2} = \Omega_{\rm m}(1{+}z)^3 + 1 - \Omega_{\rm m} + 2\Omega_\ell \mp 2\sqrt{\Omega_\ell}\,\sqrt{1+\Omega_{\Lambda_{\rm b}}} - 2\sqrt{\Omega_\ell}\,\sqrt{\Omega_{\rm m}(1{+}z)^3 - \Omega_{\rm m} + \left(\sqrt{\Omega_\ell} \mp \sqrt{1+\Omega_{\Lambda_{\rm b}}}\right)^2}\,. \tag{10}$$

For sufficiently high redshifts, the first term on the right-hand side of (9) or (10) dominates, and the model reproduces the matter-dominated Friedmann universe with the density parameter $\Omega_{\rm m}$. Now we note that, for the values of z and parameters $\Omega_{\Lambda_{\rm b}}$ and Ω_ℓ which satisfy

$$\Omega_{\rm m}(1{+}z)^3 \ll \left(\sqrt{1+\Omega_{\Lambda_{\rm b}}} \mp \sqrt{\Omega_\ell}\right)^2 , \tag{11}$$

both Eqs. (9) and (10) can be well approximated as

$$\frac{H^2(z)}{H_0^2} \simeq \Omega_{\rm m}(1{+}z)^3 + 1 - \Omega_{\rm m} - \frac{\sqrt{\Omega_\ell}}{\sqrt{\Omega_\ell} \mp \sqrt{1+\Omega_{\Lambda_{\rm b}}}}\left[\Omega_{\rm m}(1{+}z)^3 - \Omega_{\rm m}\right]. \tag{12}$$

We introduce the positive parameter α by the equation

$$\alpha = \frac{\sqrt{1+\Omega_{\Lambda_{\rm b}}}}{\sqrt{\Omega_\ell}}\,. \tag{13}$$

Then, defining a new density parameter by the relation

$$\Omega_{\rm m}^{\rm LCDM} = \frac{\alpha}{\alpha \mp 1}\,\Omega_{\rm m}\,, \tag{14}$$

we get

$$\frac{H^2(z)}{H_0^2} \simeq \Omega_{\rm m}^{\rm LCDM}(1{+}z)^3 + 1 - \Omega_{\rm m}^{\rm LCDM}\,, \tag{15}$$

which is precisely the Hubble parameter describing a LCDM universe. [Note that the braneworld parameters Ω_ℓ and $\Omega_{\Lambda_{\rm b}}$ have been effectively absorbed into a "renormalization" of the matter density $\Omega_{\rm m} \to \Omega_{\rm m}^{\rm LCDM}$, defined by (14). Since the value of the parameter $\Omega_{\rm m}^{\rm LCDM}$ should be positive by virtue of the cosmological considerations, in the case of the upper sign in (12) and (14), we restrict ourselves to the parameter region $\alpha > 1$, i.e., we exclude the special case of Brane 1 model with the upper sign in (8) from consideration.]

Thus, our braneworld displays the following remarkable behaviour which we refer to as *"cosmic mimicry"*:

- A Brane 1 model, which at high redshifts expands with density parameter $\Omega_{\rm m}$, at lower redshifts *masquerades as a LCDM universe* with a *smaller value* of the density parameter. In other words, at low redshifts, the Brane 1 universe expands as the LCDM model (15) with $\Omega_{\rm m}^{\rm LCDM} < \Omega_{\rm m}$ [where $\Omega_{\rm m}^{\rm LCDM}$ is determined by (14) with the lower ("+") sign].
- A Brane 2 model at low redshifts also masquerades as LCDM but with a *larger value* of the density parameter. In this case, $\Omega_{\rm m}^{\rm LCDM} > \Omega_{\rm m}$ with $\Omega_{\rm m}^{\rm LCDM}$ being determined by (14) with the upper ("−") sign.

The range of redshifts over which this cosmic mimicry occurs is given by $0 \leq z \ll z_{\rm m}$, with $z_{\rm m}$ determined by (11). Specifically,

$$z_{\rm m} = \frac{\left(\sqrt{1+\Omega_{\Lambda_{\rm b}}} \mp \sqrt{\Omega_\ell}\right)^{2/3}}{\Omega_{\rm m}^{1/3}} - 1\,, \tag{16}$$

which can also be written as

$$(1+z_{\rm m})^3 = \frac{\Omega_{\rm m}\,(1+\Omega_{\Lambda_{\rm b}})}{(\Omega_{\rm m}^{\rm LCDM})^2} \tag{17}$$

for both braneworld models.

Examples of cosmic mimicry are shown in Fig. 1 for the Brane 1 model, and in Fig. 2 for the Brane 2 model. One striking consequence of Fig. 2 is that a low-density ($\Omega_{\rm m} = 0.04$) universe consisting *entirely* of baryons mimics a higher-density LCDM model ($\Omega_{\rm m}^{\rm LCDM} = 0.2$) and can therefore be in excellent agreement with the SN data.

In view of relation (17), it is interesting to note that we can use the equations derived in this paper to relate the three free parameters in the braneworld model: $\{\Omega_\ell\,, \Omega_{\Lambda_{\rm b}}, \Omega_{\rm m}\}$ to $\left\{\Omega_{\rm m}, z_{\rm m}, \Omega_{\rm m}^{\rm LCDM}\right\}$. These relations (which turn out to be the same for Brane 1 and Brane 2 models) are:

$$\frac{1+\Omega_{\Lambda_{\rm b}}}{\Omega_{\rm m}^{\rm LCDM}} = \frac{\Omega_{\rm m}^{\rm LCDM}}{\Omega_{\rm m}}(1+z_{\rm m})^3\,, \tag{18}$$

$$\frac{\Omega_\ell}{\Omega_{\rm m}^{\rm LCDM}} = \left[\sqrt{\frac{\Omega_{\rm m}^{\rm LCDM}}{\Omega_{\rm m}}} - \sqrt{\frac{\Omega_{\rm m}}{\Omega_{\rm m}^{\rm LCDM}}}\right]^2 (1+z_{\rm m})^3\,. \tag{19}$$

Furthermore, if we assume that the value of $\Omega_{\rm m}^{\rm LCDM}$ is known (say, from the analysis of SN data), then the two braneworld parameters Ω_ℓ and $\Omega_{\Lambda_{\rm b}}$ can be related to the two parameters $\Omega_{\rm m}$ and $z_{\rm m}$ using (18), so it might be more convenient to analyze the model in terms of $\Omega_{\rm m}$ and $z_{\rm m}$ (instead of Ω_ℓ and $\Omega_{\Lambda_{\rm b}}$).

We also note that, under condition (11), the brane tension σ, determined by (8), is positive for Brane 1 model, and negative for Brane 2 model.

Since the Hubble parameter in braneworld models departs from that in LCDM at *intermediate* redshifts ($z > z_{\rm m}$), this could leave behind an important cosmological signature especially since several key cosmological observables depend upon the Hubble parameter either differentially or integrally. Examples include:

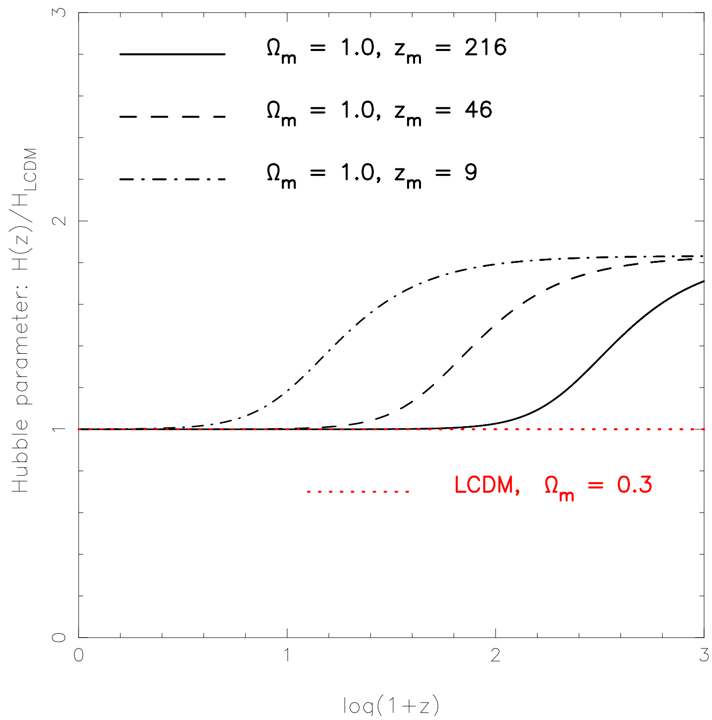

Figure 1. An illustration of cosmic mimicry for the Brane 1 model. The Hubble parameter in three high-density Brane 1 models with $\Omega_{\rm m} = 1$ is shown. Also shown is the Hubble parameter in the LCDM model (dotted line) which closely mimics this braneworld but has a lower mass density $\Omega_{\rm m}^{\rm LCDM} = 0.3$ ($\Omega_\Lambda = 0.7$). The brane matter density ($\Omega_{\rm m}$) and the matter density in LCDM are related through $\Omega_{\rm m} = \Omega_{\rm m}^{\rm LCDM} \times \left[1 + \sqrt{\Omega_\ell / (1+\Omega_{\Lambda_b})}\right]$, so that $\Omega_{\rm m} \gtrsim \Omega_{\rm m}^{\rm LCDM}$. The redshift interval during which the braneworld masquerades as LCDM, so that $H_{\rm BRANE1} = H_{\rm LCDM}$ for $z \ll z_{\rm m}$, is $z_{\rm m} = 9$, 46, 216 (left to right) for the three braneworld models.

- the luminosity distance $d_L(z)$:

$$\frac{d_L(z)}{1+z} = c \int_0^z \frac{dz'}{H(z')}\,, \tag{20}$$

- the angular-size distance

$$d_A(z) = \frac{c}{1+z} \int_0^z \frac{dz'}{H(z')}\,, \tag{21}$$

- the product $d_A(z)H(z)$, which plays a key role in the Alcock–Paczynski anisotropy test [14],
- the product $d_A^2(z)H^{-1}(z)$, which is used in the volume-redshift test [15],
- the deceleration parameter:

$$q(z) = \frac{H'(z)}{H(z)}(1+z) - 1\,, \tag{22}$$

- the effective equation of state of dark energy:

$$w(z) = \frac{2q(z)-1}{3\left[1-\Omega_{\rm m}(z)\right]}\,, \quad \Omega_{\rm m}(z) = \Omega_{\rm m}\left[\frac{H_0}{H(z)}\right]^2 (1+z)^3\,, \tag{23}$$

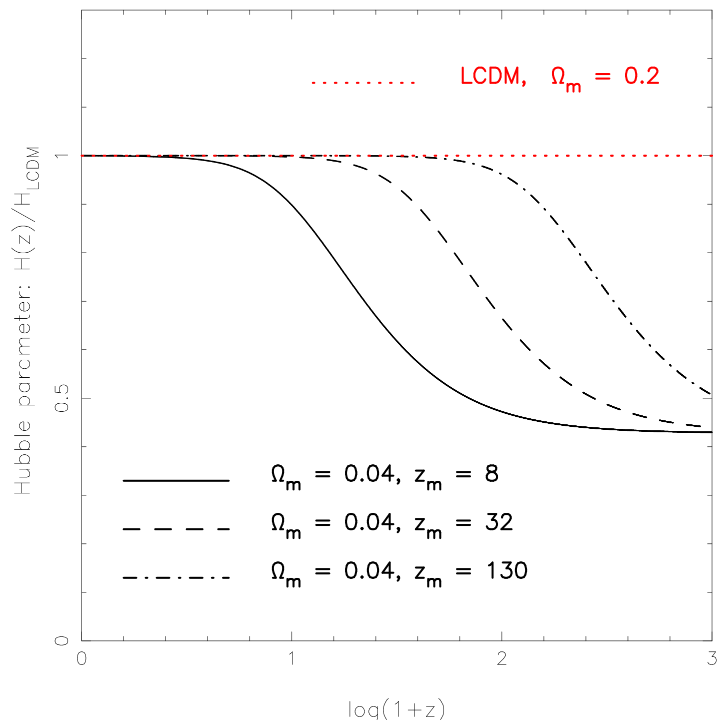

Figure 2. An illustration of cosmic mimicry for the Brane 2 model. The Hubble parameter in three low-density Brane 2 models with $\Omega_{\rm m} = 0.04$ is shown. Also shown is the Hubble parameter in a LCDM model (dotted line) which mimics this braneworld but has a higher mass density $\Omega_{\rm m}^{\rm LCDM} = 0.2$ ($\Omega_\Lambda = 0.8$). The brane matter-density parameter ($\Omega_{\rm m}$) and the corresponding parameter of the masquerading LCDM model are related as $\Omega_{\rm m} = \Omega_{\rm m}^{\rm LCDM} \times \left[1 - \sqrt{\Omega_\ell/\left(1+\Omega_{\Lambda_b}\right)}\right]$, so that $\Omega_{\rm m} \lesssim \Omega_{\rm m}^{\rm LCDM}$. The redshift interval during which the braneworld masquerades as LCDM, so that $H_{\rm BRANE2} = H_{\rm LCDM}$ for $z \ll z_{\rm m}$, is $z_{\rm m} = 8,\ 32,\ 130$ (left to right) for the three braneworld models.

- the age of the universe:

$$t(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}\,, \tag{24}$$

- the "statefinder pair" [16]:

$$r = \frac{\dddot{a}}{aH^3} \equiv 1 + \left[\frac{H''}{H} + \left(\frac{H'}{H}\right)^2\right](1+z)^2 - 2\frac{H'}{H}(1+z)\,,$$
$$s = \frac{r-1}{3(q-1/2)}\,, \tag{25}$$

- the electron-scattering optical depth to a redshift $z_{\rm reion}$

$$\tau(z_{\rm reion}) = c\int_0^{z_{\rm reion}} \frac{n_e(z)\sigma_T\,dz}{(1+z)H(z)}\,, \tag{26}$$

where n_e is the electron density and σ_T is the Thomson cross-section describing scattering between electrons and CMB photons.

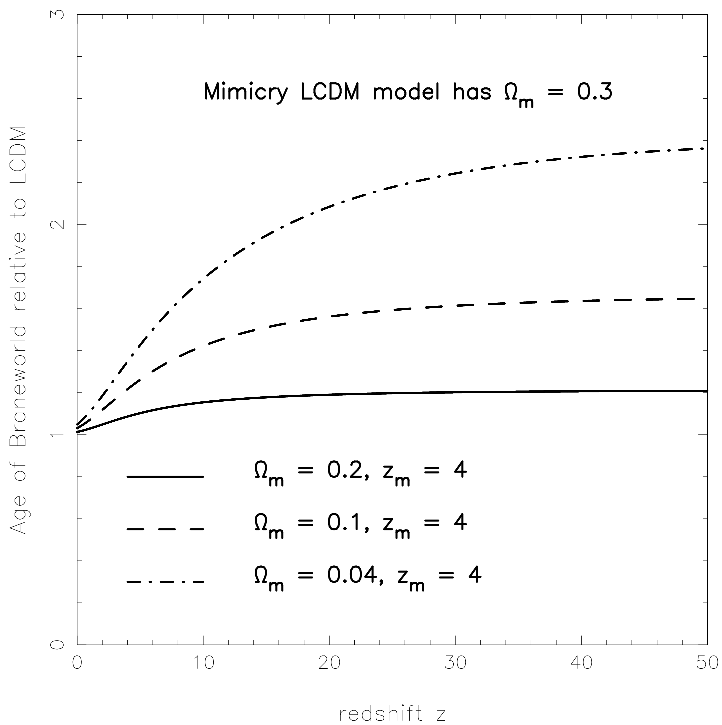

Figure 3. The age of the universe in the Brane 2 model is shown with respect to the LCDM value. The mimicry redshift (17) is $z_{\rm m} = 4$ so that $H_{\rm brane}(z) \simeq H_{\rm LCDM}(z)$ at $z \ll 4$. The braneworld models have $\Omega_{\rm m} = 0.2,\ 0.1,\ 0.04$ (bottom to top) whereas $\Omega_{\rm m}^{\rm LCDM} = 0.3$. Note that the braneworld models are older than LCDM.

A degree of caution should be exercised when comparing the late-time LCDM behaviour (15) of our model with different sets of observations, since the parameter $\Omega_{\rm m}^{\rm LCDM}$, residing in (15), which is effectively used in determinations of the luminosity distance (20) and angular-size distance (21), may very well be different from the value of $\Omega_{\rm m}$ inferred from observations of gravitational clustering. These issues should be kept in mind when performing a maximum-likelihood analysis using data belonging to different observational streams.

Cosmological tests based on the luminosity distance and angular-size distance typically probe lower redshifts $z \lesssim 2$. Therefore, if the mimicry redshift is $z_{\rm m} \geq 2$, the braneworld model will, for all practical purposes, be indistinguishable from the LCDM cosmology on the basis of these tests alone. However, tests which probe higher redshifts should be able to distinguish between these models. For instance, since $H(z) < H_{\rm LCDM}(z)$ in Brane 2 at redshifts larger than the mimicry redshift, it follows that the age of the universe will be greater in this model than in the LCDM cosmology. This is illustrated in Fig. 3 for three distinct values of the cosmological density parameter: $\Omega_{\rm m} = 0.2,\ 0.1,\ 0.04$, all of which are lower than $\Omega_{\rm m}^{\rm LCDM} = 0.3$.

Since the late-time evolution of the universe is

$$t(z) \simeq \frac{2}{3H_0\sqrt{\Omega_{\rm m}}}(1+z)^{-3/2}\,, \tag{27}$$

one finds, for $z \gg 1$,

$$\frac{t_{\rm brane}}{t_{\rm LCDM}}(z) \simeq \sqrt{\frac{\Omega_{\rm m}^{\rm LCDM}}{\Omega_{\rm m}}}\,. \tag{28}$$

Since $\Omega_{\rm m} < \Omega_{\rm m}^{\rm LCDM}$ in the Brane 2 model, we find that the age of a Brane 2 universe is greater than that of a LCDM universe. (The reverse is true for the Brane 1 model, for which $\Omega_{\rm m} > \Omega_{\rm m}^{\rm LCDM}$.)

The altered rate of expansion in the braneworld model at late times ($z > z_{\rm m}$) also affects other cosmological quantities including the redshift of reionization which, for the Brane 2 model, becomes smaller than that in the LCDM cosmology. This is because the lower value of $H(z)$ in the Brane 2 model (relative to the LCDM model), when substituted to (26), gives a correspondingly lower value for $z_{\rm reion}$ for an identical value of the optical depth τ in both models. (In fact, it is easy to see that, for the Brane 2 model, the value of $z_{\rm reion}$ decreases with decreasing $z_{\rm m}$ and $\Omega_{\rm m}$.) Hence, braneworld cosmology makes it possible to have a value for the reionization redshift which is lower than $z \simeq 17$ inferred for the LCDM model from the data of the Wilkinson Microwave Anisotropy Probe (WMAP) [17].

Both an increased age of the universe and a lower redshift of reionization are attractive properties of the braneworld model which, as we have seen, mimics the LCDM cosmology at lower redshifts $z < z_{\rm m}$.§ It is important to note that the presence of high-redshift quasi-stellar objects (QSO's) and galaxies at redshifts $z \gtrsim 6$ indicates that the process of structure formation was already in full swing at that early epoch when the LCDM universe was less than a billion years old. Most models of QSO's rely on a central supermassive black hole ($M_{\rm BH} \gtrsim 10^9 M_\odot$) to power the quasar luminosity via accretion. Since structure forms hierarchically in the cold dark matter scenario, the presence of such supermassive black holes at high redshift suggest that they formed through an assembly mechanism involving either accretion or mergers or both. It is not clear whether either of these processes is efficient enough to assemble a large number of high-redshift QSO's in a LCDM cosmology [18, 19]. (At the time of writing, over 400 quasi-stellar objects with redshifts ≥ 4 have been reported in the literature.) In addition, it is interesting to note the discovery of an "old" quasar APM 08279+5255 at redshift $z = 3.91$, which appears to have an older age (2.1 Gyr) than the LCDM model (at that redshift) and may, therefore, be problematic for concordance cosmology as well as several other dark-energy models [20]. Since Brane 2 can be considerably older than LCDM, this makes it easier to account for the existence of APM 08279+5255 within the braneworld context than in LCDM. (We shall return to this issue in a companion paper.)

Another "surprise" for the concordance cosmology emerges from the WMAP data which suggest that the optical depth to reionization is $\tau = 0.17 \pm 0.06$. For the LCDM model, this translates into a rather high redshift of reionization $z = 17 \pm 5$ [17], suggesting

§ Note that the decreased redshift of reionization and the increased age of the universe are properties that the Brane 2 model shares with the loitering universe discussed in [5].

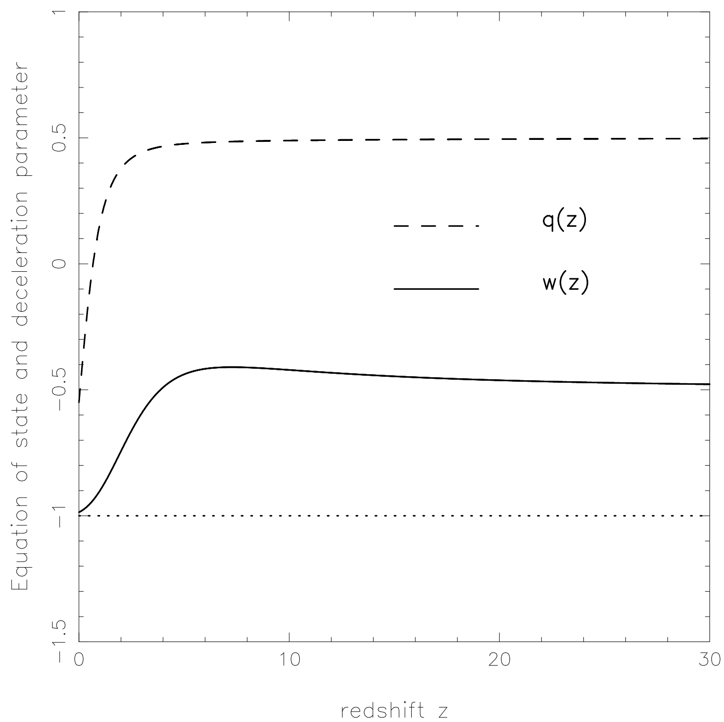

Figure 4. The effective equation of state (solid line) and the deceleration parameter (dashed line) of the Brane 2 model are shown. (The dotted line shows $w = -1$ which describes the LCDM model.) The mimicry redshift (17) is $z_{\rm m} = 4$ so that $H_{\rm brane}(z) \simeq H_{\rm LCDM}(z)$ at $z \lesssim 4$. The braneworld has $\Omega_{\rm m} = 0.2$ whereas $\Omega_{\rm m}^{\rm LCDM} = 0.3$.

that population III stars and/or mini-QSO's were already in place at $z > 17$ in order to have efficiently reionized the universe. Whether both the structure-assembly issue and the high reionization redshift can be successfully accommodated within the LCDM paradigm remains to be seen [21]. However, both issues are important enough to compel the serious theorist to look for alternative models which, while preserving the manifold strengths and successes of the LCDM model at low z, will also be able to flexibly accommodate the striking properties of the universe at higher redshifts. As we have demonstrated in this paper, braneworld cosmology may successfully alleviate some of the tension currently existing between theory and observations at moderate redshifts, while allowing the universe to be "LCDM-like" at the present epoch.

The effective equation of state and the deceleration parameter of the Brane 2 model are shown in Fig. 4. The braneworld has $\Omega_{\rm m} = 0.2$ and, at $z \lesssim 4$, masquerades as a higher-density LCDM model with $\Omega_{\rm m}^{\rm LCDM} = 0.3$. Note that the *effective* equation of state (23) is a *model-dependent* quantity, involving the model-dependent cosmological parameter $\Omega_{\rm m}$ in its definition. In our case, we use the braneworld theory as our model with $\Omega_{\rm m}$ defined in (6), and the effective equation of state (23) is then *redshift-dependent* even during the mimicry period when $H_{\rm brane}(\Omega_{\rm m}, z) \simeq H_{\rm LCDM}(\Omega_{\rm m}^{\rm LCDM}, z)$. A theorist who is unaware of the possibility of cosmic mimicry, when reconstructing the cosmic equation of state from (23) with $\Omega_{\rm m}^{\rm LCDM} = 0.3$ in the place of $\Omega_{\rm m}$, will arrive at a different conclusion that $w = -1$. This example demonstrates some of the pitfalls associated with the cosmological reconstruction of the equation of state, which depends on the underlying theoretical model and for which an accurate knowledge of $\Omega_{\rm m}$ is essential; see [16, 22, 23, 24] for a discussion of related issues.

In concluding this section, we would like to mention an additional important consequence of braneworld cosmology, namely that the departure of the brane Hubble parameter from its LCDM counterpart is likely to affect the growth of density perturbations. From the linearized perturbation equation

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\,\delta = 0\,, \tag{29}$$

where $\delta = (\rho - \bar{\rho})/\bar{\rho}$ is the relative density perturbation, it follows that a large value of H will, via the second term on the left-hand side, dampen the growth of perturbations, while a small value of H should hasten their growth. Although this argument is also likely to carry over to the braneworld, the direct application of (29) to the study of gravitational instability (as well as CMB anisotropy) should be treated with some caution since (29) was derived within the general-relativistic framework, and it is likely to require some modification before it can be applied to the study of cosmological perturbations on the brane. Indeed, according to the analysis of [4, 25], the linearized equation describing the evolution of inhomogeneities in the braneworld models of Dvali–Gabadadze–Porrati (DGP) type [26], corresponding to $\Lambda_{\rm b} = 0$, on spatial scales much shorter than ℓ has the form

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\left(1 + \frac{1}{3\beta_*}\right)\delta = 0\,, \tag{30}$$

where the time-dependent parameter β_* is given by [4]

$$\beta_*(t) = 1 \mp \ell H\left(1 + \frac{\dot{H}}{3H^2}\right)\,. \tag{31}$$

Another equation which might possibly describe the behaviour of density perturbations in the braneworld models under consideration in this paper will be discussed in the next section.

3. Properties of braneworld gravity

3.1. Multiscale gravity

Here, we would like to discuss the results of the previous section in the light of some generic properties of braneworld gravity.

Cosmology determined by action (1) has two important scales, namely, the length scale (5)

$$\ell = \frac{2m^2}{M^3}\,, \tag{32}$$

which describes the interplay between the bulk and brane gravity, and the momentum scale

$$k = \frac{\sigma}{3M^3}\,, \tag{33}$$

which determines the role of the brane tension in the dynamics of the brane. In a model characterized by the Randall–Sundrum constraint [27]

$$\Lambda_{\rm RS} \equiv \frac{\Lambda_{\rm b}}{2} + \frac{\sigma^2}{3M^6} = 0\,, \tag{34}$$

the absolute value of k is equal to the inverse warping length $\ell_{\rm warp} = \sqrt{-6/\Lambda_{\rm b}}$ of the Randall–Sundrum solution. Note that k is negative when $\sigma < 0$. We take M to be positive in our paper, since, in the opposite case, the massive Kaluza–Klein gravitons become ghosts [28, 29].

Following the procedure first employed in [30] for the Randall–Sundrum (RS) model [27] and subsequently applied in [6] to the model under consideration, we contract once the Gauss identity

$$R_{abc}{}^{d} = h_a{}^{f} h_b{}^{g} h_c{}^{k} h^{d}{}_{j} \mathcal{R}_{fgk}{}^{j} + K_{ac} K_b{}^{d} - K_{bc} K_a{}^{d} \tag{35}$$

on the brane and, using Eq. (2), obtain the equation

$$G_{ab} + \Lambda_{\rm eff} h_{ab} = 8\pi G_{\rm eff} \tau_{ab} + \frac{1}{\beta + 1} \left(\frac{1}{M^6} Q_{ab} - W_{ab} \right)\,, \tag{36}$$

where

$$\beta = \frac{2\sigma m^2}{3M^6} = k\ell \tag{37}$$

is an important dimensionless parameter,

$$\Lambda_{\rm eff} = \frac{\Lambda_{\rm RS}}{\beta + 1} \tag{38}$$

is the effective cosmological constant on the brane,

$$8\pi G_{\rm eff} = \frac{\beta}{\beta + 1} \cdot \frac{1}{m^2} \tag{39}$$

is the effective gravitational constant,

$$Q_{ab} = \frac{1}{3} E E_{ab} - E_{ac} E^{c}{}_{b} + \frac{1}{2} \left(E_{cd} E^{cd} - \frac{1}{3} E^2 \right) h_{ab} \tag{40}$$

is a quadratic expression constructed from the "bare" Einstein equation $E_{ab} \equiv m^2 G_{ab} - \tau_{ab}$ on the brane, and $E = h^{ab} E_{ab}$. The symmetric traceless tensor $W_{ab} \equiv n^c n^d W_{acbd}$ in (36) is a projection of the bulk Weyl tensor W_{abcd}. It is related to the tensor Q_{ab} through the conservation equation

$$D^a \left(Q_{ab} - M^6 W_{ab} \right) = 0\,, \tag{41}$$

where D^a denotes the covariant derivative on the brane associated with the induced metric h_{ab}.

It is important to note that all couplings in Eq. (36), including the effective cosmological and gravitational constants, are inversely proportional to $\beta + 1$, which indicates that the theory becomes singular for the special case $\beta = -1$ (see [29, 31]).

In the absence of the curvature term on the brane ($m = 0$), we obtain Eq. (36) in which $8\pi G_{\rm eff} = 2\sigma/3M^6$ is the gravitational constant in the RS model [27], and $\beta = 0$;

in this form, Eq. (36) was first derived in [30]. The conditions $\sigma = 0$ and $\Lambda_{\rm b} = 0$ are characteristic of the DGP model [26], which also has $\beta = 0$. In this model, the effective gravitational constant (39) turns to zero, i.e., the term linear in the stress–energy tensor on the brane vanishes in Eq. (36).

Let us now turn to the "cosmic mimicry" exhibited by braneworld cosmology and show how it can be related to the gravitational properties of braneworld theory. In this context, it is remarkable that the parameter β introduced in (37) is very close (in absolute terms) to the parameter α introduced in (13) in our discussion of mimicry models. Specifically,

$$\beta = \frac{1 - \Omega_{\rm m}}{2\Omega_\ell} \mp \alpha \, , \tag{42}$$

so that

$$|\beta| \approx \alpha \tag{43}$$

when $|1 - \Omega_{\rm m}| \ll \Omega_\ell$. This last inequality follows naturally from condition (11) for values of α of order unity, which are of interest to us in this paper. As a consequence, the term which appears in the "renormalization" of the cosmological mass density $\Omega_{\rm m}$ in (14) is almost identical to the term which redefines the gravitational constant in (39). This coincidence can be explained by inspecting the brane equation (36). First, we note that cosmological solutions without dark radiation are embeddable in the anti-de Sitter bulk spacetime, so that $W_{ab} = 0$ for these solutions. For large cosmological matter densities, the quadratic expression (40) dominates in Eq. (36), and the universe is described by the "bare" Einstein equation $m^2 G_{ab} - \tau_{ab} = 0$, with the effective gravitational coupling being equal to $1/m^2$. As the matter density decreases, the role of this quadratic term becomes less and less important, and the effective gravitational coupling eventually is determined by the linear part of Eq. (36), i.e., by the gravitational constant (39). Thus, comparing (39) and (14), one has the following natural relation, valid to a high precision in view of (43):

$$\Omega_{\rm m}^{\rm LCDM} = \frac{8\pi G_{\rm eff} \rho_0}{3H_0^2} \, . \tag{44}$$

Now we consider some relevant properties of braneworld gravity. The expression for Q_{ab} in Eq. (40) is quadratic in the curvature as well as in the stress–energy tensor. On the other hand, the tensor W_{ab} is related to Q_{ab} through the conservation equation (41). One might, therefore, expect that the term in the parentheses on the right-hand side of Eq. (36), namely $Q_{ab}/M^6 - W_{ab}$, will be insignificant on sufficiently large length scales, and that the braneworld theory on those scales should reduce to Einstein gravity with the effective constants given by (38) and (39). This expectation is borne out by a detailed analysis [29] carried out for a positive-tension brane ($\sigma > 0$) in the specific case when the braneworld satisfies the RS constraint (34). In this case, the gravitational potential of a unit mass on large scales (on the positive-tension brane) has the Newtonian form with a small RS correction [29]‖:

$$V(r) = -\frac{G_{\rm eff}}{r}\left[1 + \frac{2}{3(\beta+1)(kr)^2}\right] , \quad kr \gg 1 , \tag{45}$$

where $G_{\rm eff}$ is given by (39).

On smaller spatial scales, $kr \ll 1, \beta$, the potential in linear theory again has the Newtonian form, this time with a small logarithmic correction:

$$V(r) = -\frac{\widetilde{G}_{\rm eff}}{r} - \left(\frac{15}{8} + \frac{2}{\beta}\right)\frac{k}{3\pi^2 m^2}\log\left[\left(\frac{15}{8} + \frac{2}{\beta}\right)kr\right] , \quad kr \ll 1, \beta , \tag{46}$$

and with a different expression for the effective gravitational constant [29]

$$\widetilde{G}_{\rm eff} = \left[1 + \frac{1}{3(1+\beta)}\right]\frac{1}{8\pi m^2} = \left(1 + \frac{4}{3\beta}\right) G_{\rm eff} . \tag{47}$$

For $k \to 0$ (hence, also $\beta \to 0$), this reproduces the result obtained for the DGP model in [9] on scales $r \ll \ell$.

It is worth noting that gravity on these smaller scales $kr \ll 1, \beta$, in general, involves the massless scalar radion, i.e., it is of scalar–tensor type. As a consequence, for the spherically symmetric solution, it violates the property $h_{00}(r) = -h_{rr}^{-1}(r)$, or, in the linear approximation, $\gamma_{00}(r) = \gamma_{rr}(r)$, where $\gamma_{\alpha\beta}(r)$ are the components of metric perturbation in the spherically symmetric coordinate system. Specifically, in the model with the RS constraint one can obtain the relation:

$$\frac{\Delta\gamma}{\gamma_{00}} \equiv \frac{\gamma_{00} - \gamma_{rr}}{\gamma_{00}} = \frac{1}{1 + 3\beta/4} . \tag{48}$$

Since there are stringent experimental upper bounds [32] on the left-hand side of (48) in the neighbourhood of the solar system (it should not exceed 10^{-5} by order of magnitude), if solution (46) were applicable in this domain, it would imply that only very large values of β are permissible in the braneworld theory under consideration [namely, the braneworld model (4) with the RS constraint (34)]. It is important to note, however, that the applicability region of the linear approximation (46) is bounded from below by a length scale which depends upon the mass of the central source, as has been demonstrated for the DGP model in [9]. Specifically, the dynamics of the radion develops strong nonlinear corrections on sufficiently small scales, leading to the breakdown of linearized theory. (This also creates the so-called strong-coupling problem in the DGP model [33].) In this case, in order to study gravity at small distances from the source, one must turn to the fully nonlinear theory.

Next, let us determine the distances at which the linearized theory, leading to (46), breaks down and to establish the correct behaviour of the potential on such scales. In order to do this, we turn to the effective equation (36). Taking the trace of Eq. (36), we get the following closed scalar equation on the brane:

$$-R + 4\Lambda_{\rm eff} - 8\pi G_{\rm eff}\tau = \frac{Q}{(\beta+1)M^6} , \tag{49}$$

‖ Here, we present the results for the two-brane model as the second (negative-tension) brane is taken to spatial infinity.

where the left-hand side contains terms which are linear in the curvature and in the stress–energy tensor while the right-hand side contains the quadratic term $Q = h^{ab}Q_{ab}$.

Suppose that we are interested in the behaviour of gravity in the neighbourhood of a spherically symmetric source with density ρ_s, total mass $\mathcal{M}_s$, and radius r_s. First of all, we assume that one can neglect the tensor W_{ab} and the effective cosmological constant in the neighbourhood of the source. As regards the effective cosmological constant, this assumption is natural. Concerning the tensor W_{ab}, its smallness in the neighbourhood of the source represents some additional condition on the spherically symmetric solution. We believe that a condition of this sort is likely to arise in any consistent and viable braneworld theory since, without it, one has a large number of spherically symmetric solutions on the brane, many of them non-physical (see [34] for a comprehensive treatment in the framework of the RS model).

Within the source itself, we have two qualitatively different options: an approximate solution can be sought either neglecting the quadratic part or linear part of Eqs. (36) and (49). We should choose the option that gives the smaller error of approximation in Eq. (49). In the first case, neglecting the quadratic part and the effective cosmological constant, we have

$$G_{ab} - 8\pi G_{\rm eff}\tau_{ab} \approx 0 \quad \Rightarrow \quad \frac{Q}{(\beta+1)M^6} \sim \frac{\rho_s^2}{(\beta+1)^3 M^6} \,. \tag{50}$$

In the second case, we neglect the linear part, so that

$$Q_{ab} \approx 0 \quad \Rightarrow \quad E_{ab} \approx 0 \quad \Rightarrow \quad R + 8\pi G_{\rm eff}\tau \sim \frac{\rho_s}{(\beta+1)m^2} \,. \tag{51}$$

The final expression on the right-hand side of (51) is smaller than the corresponding expression in (50) if

$$\rho_s > (\beta+1)^2 \frac{M^6}{m^2} \quad \Rightarrow \quad r_s^3 < r_*^3 \sim \frac{\mathcal{M}_s \ell^2}{(\beta+1)^2 m^2} \,, \tag{52}$$

where we used the relation $\mathcal{M}_s \sim \rho_s r_s^3$. Thus, we can expect that, in the neighbourhood of the source, on distances smaller than r_* given by (52), the solution is determined mainly by the quadratic part Q_{ab} in Eq. (36), which means that it respects the "bare" Einstein equation $m^2 G_{ab} = \tau_{ab}$ to a high precision. This effect is sometimes described as the "gravity filter" of the DGP model [26], which screens the scalar graviton in the neighbourhood of the source making the gravity effectively Einsteinian. Some aspects of this interesting phenomenon are discussed in recent papers [35].

Expression (52) generalizes the length scale [9] of the DGP model, below which nonlinear effects become important, to the case of nonzero brane tension (nonzero β) and bulk cosmological constant satisfying the RS constraint (34). The observable gravitational constant on scales much smaller than r_* will be given by

$$8\pi G_{\rm obs} = \frac{1}{m^2} \,. \tag{53}$$

For the Sun, the scale r_* is estimated as

$$r_* \equiv r_\odot \sim \frac{10^{16}\,{\rm km}}{(\beta+1)^{2/3}\,\Omega_\ell^{1/3}} \,, \tag{54}$$

which, for interesting values of β and Ω_ℓ, will be very large. The corresponding radius for the Earth is smaller only by two orders of magnitude.

This, however, is not the full story. As argued in [4, 25], the gravitational potential of a spherically symmetric body on scales $r_* \lesssim r \ll \ell$ is corrected by the cosmological expansion. Moreover, the critical scale r_* becomes dependent on the value of the Hubble parameter, and can be different from (52) for values of ℓ of the order of the Hubble length. Another effective gravitational constant appears in the equation for the growth of cosmological perturbations on the above-mentioned scales, as was mentioned in the previous section [see Eqs. (30) and (31)]. However, we note that, in this paper, interesting values of the parameter ℓ are smaller than the Hubble length since we would like relation (11) to be satisfied for reasonably high redshifts, and this condition implies that the parameter Ω_ℓ, defined in Eq. (6), is significantly larger than unity. Consequently, the critical scale r_* is given by Eq. (52). Note that gravity on scales $r \ll r_*$, although close to Einstein gravity, is not exactly Einsteinian, and these deviations can be used to test braneworld theory on solar-system scales [4, 9, 25, 36].

The appearance of the distance scale r_* given by (52) can also be justified by using an argument from cosmology. Imagine the central body to be formed of pressureless matter and to represent a part of the homogeneous and isotropic universe. Then the solution inside this body is uniquely described by the cosmological equations of Sec. 2. As the density parameter of the body exceeds the right-hand side of (11), its gravitational evolution is effectively governed by the "bare" Einstein equation $m^2 G_{ab} = \tau_{ab}$. The relevant condition describing this case is the inequality opposite to (11); it can be written in terms of the density ρ_s as

$$\rho_s > (\alpha \mp 1)^2 \frac{M^6}{m^2}\,, \tag{55}$$

which, in view of relations (42) and (43), essentially coincides with condition (52).

Although the preceding reasoning is applicable to both the positive-tension and the negative-tension brane, the current understanding of the braneworld gravitational physics supports only the positive-tension case (which corresponds to Brane 1 model in this paper), while the situation with the negative-tension brane remains unclear (at least, to the authors of this paper). From Eq. (36), one might expect that a negative-tension brane will show reasonable physical behaviour in the case $|\beta| > 1$ (note that $\beta < 0$ for a negative-tension brane), in which the gravitational constant (39) is positive. However, direct calculation (along the lines of [29]) *in the two-brane model* with the RS constraint (34) shows that, in this case, the gravitational interaction between material bodies on large scales is dominated by the ghost-like radion, with the effective gravitational coupling

$$G_{\rm radion} = -\frac{1}{3} G_{\rm eff}\,, \quad kr \gg 1\,, \tag{56}$$

where $G_{\rm eff}$ is given by the same expression (39). The radion-dominated gravity on these scales is formally attractive in the case $G_{\rm eff} < 0$, and is repulsive for $G_{\rm eff} > 0$. However, on smaller spatial scales $kr_* \lesssim kr \ll 1, |\beta|$, Newton's law similar to (46) is reproduced with the gravitational constant given by (47), which is positive if $|\beta| > 4/3$. The gravity on these scales is of scalar–tensor character. On still smaller distances from the central source, $r < r_*$, the theory may approach Einstein gravity with the effective gravitational constant (53). However, all these expectations remain to be verified by more refined calculations which generalize to the braneworld models of interest to us in this paper, including those with negative tension and without the RS constraint (34). In the following subsection, we propose a different approach to this problem, based on alternative boundary conditions in the brane–bulk system.

3.2. The role of boundary conditions in braneworld models

Many papers (see, e.g., the recent review [37]) have stressed that equation (36) is not closed on the brane in the sense that it contains the symmetric traceless tensor W_{ab} whose behaviour on the brane is not determined by the dynamics of matter alone. As of now, this "nonclosure" of the equations on the brane constitutes one of the major problems confronting braneworld theory. Some additional information from the bulk is needed to determine the observable braneworld dynamics completely; one can say that some *boundary conditions* are to be specified to restrict the class of possible braneworld solutions. Usually, one tends to specify the "physical" boundary conditions somewhere in the bulk, by requiring the bulk to be singularity-free (see [37]) and/or by introducing one or more (regulatory) branes in the bulk. However, it seems to be difficult to implement this proposal in practice in a fully nonlinear theory, and it thus remains unclear whether such proposals will lead to a reasonable theory at all. A more practical approach to this problem, adopted in many papers (see, e.g., [38]), consists in making some reasonable assumptions about the tensor W_{ab} on the brane. Here, we make a proposal to take this approach as a way of specifying the boundary conditions in the brane-bulk system. The proposal consists in specifying the boundary conditions on the brane, rather than in the bulk, in such a way as to close equation (36).

Of all possible versions of boundary conditions of this kind, the simplest one appears to consist in setting

$$W_{ab} = 0 \tag{57}$$

on the brane. This condition is very often imposed in practical calculations by neglecting the contribution of this term in various situations. Here, we do not discuss the physical meaning of this proposal in all detail but only consider its consequences very briefly.

With the specification of the boundary condition in the form (57), equation (36) becomes a closed four-dimensional equation on the brane completely determining its dynamics, and solution in the bulk becomes of no crucial importance for an observer on the brane. Note that the specification (57) preserves all homogeneous and isotropic cosmological solutions with the only restriction that the dark radiation is zero, i.e., $C = 0$ in Eq. (4). Thus we are not losing the cosmological properties of the braneworld models (except those for which the nonzero value of the dark radiation is crucial [5]); in particular, the cosmic mimicry phenomenon discussed in this paper remains intact. The boundary condition under consideration affects only the inhomogeneous gravitational physics.

It was shown in [39] that a spherically symmetric metric in the case (57) is unique and is the Schwarzschild metric in a broad region of parameters of the theory. Thus, in particular, there are no higher-dimensional corrections to the Newtonian potential, as is the case in the two-brane model [see Eqs. (45) and (46)], and the theory is not "spoiled" by the presence of the scalar gravitational degree of freedom. The small-scale gravitational physics in this case has the effective gravitational constant equal to $1/m^2$, as in the previous subsection. (From this viewpoint, the two-brane model discussed in the preceding subsection represents a braneworld model with a different type of boundary condition.)

The linear equations for the cosmological scalar density perturbations in the braneworld model under consideration with the boundary condition (57) were, in fact, recently derived in [40]. Condition (57) in [40] was assumed as a reasonable approximation on sufficiently small spatial scales while, in this paper, we propose to take it as an exact boundary condition. The equation governing the evolution of the relativistic potential Φ was shown in [40] to have the form

$$\frac{1}{a^2}\nabla^2\Phi - 3H^2\Phi - 3H\dot{\Phi} = -8\pi G^*_{\rm eff}\delta\rho\,, \tag{58}$$

where

$$8\pi G^*_{\rm eff} = \frac{1}{m^2}\left(1 \pm \frac{1}{\sqrt{1+\ell^2\left(\frac{\rho+\sigma}{3m^2} - \frac{\Lambda_{\rm b}}{6}\right)}}\right) = -\frac{2\dot{H}}{\rho}\,, \tag{59}$$

and the last equality is valid for the case of dust. As in the discussion in the end of the previous section, again the only change is the appearance of a time-dependent effective gravitational constant. Equation (58) was used to test the theory against CMB observations in [40].

We note that the proposal of the boundary conditions (57) works here for branes with positive as well as negative tensions and, hopefully, represents one of the ways of combining the attractive features of the mimicry cosmology of the Brane 2 model with the observable small-scale gravity. More analysis is required to test this proposal, and we are going to return to this issue in the future publications.

4. Conclusion

We have shown that braneworld cosmology, for a large region of parameter space, exhibits a property which can be called "cosmic mimicry." During early cosmological epochs, the braneworld behaves like a matter-dominated Friedmann universe with the *usual* value of the cosmological parameter $\Omega_{\rm m}$ that would be inferred from observations of the local matter density. At late times, however, the universe evolves almost exactly like in the LCDM scenario but with a *renormalized* value of the cosmological density parameter $\Omega_{\rm m}^{\rm LCDM}$. Specifically, a positive-tension Brane 1 model, which at high redshifts expands with density parameter $\Omega_{\rm m}$, at lower redshifts mimics the LCDM cosmology with a *smaller value* of the density parameter $\Omega_{\rm m}^{\rm LCDM} < \Omega_{\rm m}$. A negative-tension Brane 2 model at low redshifts also mimics LCDM but with a *larger value* of the density parameter $\Omega_{\rm m}^{\rm LCDM} > \Omega_{\rm m}$.¶ The transition redshift between the early epoch and the late (mimicry) epoch is, in principle, a free parameter which depends upon constants entering the braneworld Lagrangian; see (16) or (17).

The braneworld models discussed in this paper have interesting cosmological properties. For instance, in the case of Brane 1 (Brane 2), the universe expands faster (slower) than in the LCDM scenario at redshifts greater than the mimicry redshift $z_{\rm m}$, whereas, for $z < z_{\rm m}$, $H_{\rm brane}(z) \equiv H_{\rm LCDM}(z)$ in both models. The smaller value of the Hubble parameter at intermediate redshifts ($z >$ few) in the case of Brane 2 leads to an older universe and also to a redshift of reionization which can be significantly lower than $z \simeq 17$ inferred for the LCDM model from the WMAP data [17]. These features, together with the mimicry property, result in an interesting new cosmology which can successfully ameliorate the current tension between the concordance (LCDM) cosmology and the high-redshift universe.

The effect of cosmic mimicry and the existence of two asymptotic density parameters $\Omega_{\rm m}$ and $\Omega_{\rm m}^{\rm LCDM}$ is a consequence of the time-dependence of the effective gravitational constant in braneworld theory [6], which can be related to the well known property of the scale-dependence of the effective gravitational constant in braneworld models [9]. On large spatial scales, $kr \gg 1$, the braneworld model with positive brane tension (Brane 1) exhibits gravity with the renormalized effective gravitational constant (39), and we showed that this renormalization corresponds to the renormalization of the cosmological density parameter (44). The behaviour of the braneworld model with negative brane tension (Brane 2) on these scales is drastically different: it is dominated by the scalar radion which can even make the gravitational interaction repulsive; see Eq. (56).+ The problem of this kind may not arise in models with stabilized radion [41] or in models with different boundary conditions for the brane–bulk system — but this issue has to be studied separately.

Only two effective gravitational constants appear in the cosmology under consideration, given by (39) and (53), respectively, for low and high energy densities. However, in the local gravitational physics, there also appears the spatial distance (52) depending upon the mass of the central source, so that gravity in the range

$$r_* \lesssim r \lesssim \ell \tag{60}$$

of distances r from the source has a different value of the gravitational constant, given by (47), and, moreover, typically has a scalar–tensor character manifested, in particular, in (48). This may be important for the estimates of masses from the dynamics of clusters of galaxies and from gravitational lensing on these scales in the braneworld theory [9, 25].

¶ The value of $\Omega_{\rm m}^{\rm LCDM}$ is determined by (14) with the upper ("−") sign for Brane 2 and the lower ("+") sign for Brane 1.

+ This result was obtained for the brane with the RS constraint, but the presence of a small effective cosmological constant (38) is unlikely to remedy the situation.

On small distances from the central source, $r \ll r_*$, both positive-tension and negative-tension branes apparently behave similarly reproducing the Einstein gravity to a high precision with the gravitational constant $1/m^2$, which is the bare gravitational coupling in the braneworld action (1). However, this expectation is to be verified by refined calculations in braneworld models with arbitrary sign of brane tension and without the RS constraint (34). In this respect, we should note that the solution for a spherically symmetric source (the analog of the Schwarzschild and interior solution in general relativity) largely remains an open problem in braneworld theory (for recent progress in the DGP model, see [42]).

Specification of boundary conditions is of crucial importance for the construction of a specific braneworld model since they determine the inhomogeneous gravitational physics on the brane. From the general viewpoint, we may regard boundary conditions as any conditions restricting the space of solutions of the brane–bulk system of equations (2), (3).* One of the possible proposals in braneworld theory that we put forward in this paper is to specify the boundary conditions on the visible brane, and we discussed one of the simplest such specifications, given by Eq. (57). In this case, the equations of the theory become completely closed on the brane and allow for both positive and negative brane tensions preserving all cosmological properties. Moreover, this proposal might be extended to the braneworld theory with timelike extra dimension preserving its attractive features [44] but without inheriting its nonphysical properties (the tachyonic character of the Kaluza–Klein gravitational modes). It should be stressed, however, that the homogeneous and isotropic braneworld cosmology is uniquely described by Eq. (4) and, apart from possible restriction on the dark-radiation constant C, is not sensitive to the specification of boundary conditions for system (2), (3).

The cosmological model under consideration appears to safely pass the existing constraints on the variation of the gravitational constant from primordial abundances of light elements synthesized in the big-bang nucleosynthesis (BBN) and from cosmic microwave background (CMB) anisotropy [45]. The value of the gravitational constant at the BBN epoch in our model coincides with the value measured on small scales (53), and the effective gravitational constant (47) or (39) that might affect the large-scale dynamics of the universe responsible for the CMB fluctuations is within the uncertainties estimated in [45].

Cosmic mimicry in braneworld models is most efficient in the case of parameter $\alpha \sim 1$, which, according to (13), implies that the two spatial scales, namely, the brane length scale given by (5) and the curvature scale of the bulk are of the same order: $\ell \sim \ell_{\rm warp} = \sqrt{-6/\Lambda_{\rm b}}$. This coincidence of the orders of magnitude of completely independent scales can be regarded as some tuning of parameters, although it is obviously a mild one.

* This can be compared to the view on boundary conditions in quantum cosmology, which are regarded there as any conditions suitably restricting the space of the wave functions of the universe. In particular, the boundary condition may consist in the universe having no boundary [43].

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References

[1] Maartens R, 2004 *Living Rev. Rel.* **7** 7 [gr-qc/0312059]
[2] Sahni V, *Cosmological surprises from Braneworld models of dark energy*, 2005 *Preprint* astro-ph/0502032
[3] Sahni V and Shtanov Yu V, 2003 *J. Cosmol. Astropart. Phys.* JCAP11(2003)014 [astro-ph/0202346]
Alam U and Sahni V, *Supernova Constraints on Braneworld Dark Energy*, 2002 *Preprint* astro-ph/0209443
[4] Lue A and Starkman G D, 2004 *Phys. Rev.* D **70** 101501 [astro-ph/0408246]
[5] Sahni V and Shtanov Yu, 2005 *Phys. Rev.* D **71** 084018 [astro-ph/0410221]
[6] Maeda K, Mizuno S and Torii T, 2003 *Phys. Rev.* D **68** 024033 [gr-qc/0303039]
[7] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000), [astro-ph/9904398].
[8] V. Sahni, *Dark Matter and Dark Energy*, Lectures given at the 2nd Aegean Summer School on the Early Universe, Ermoupoli, Island of Syros, Greece, astro-ph/0403324.
[9] Gruzinov A, 2005 *New Astron.* **10** 311 [astro-ph/0112246]
Porrati M, 2002 *Phys. Lett.* B **534** 209 [hep-th/0203014]
Dvali G, Gruzinov A and Zaldarriaga M, 2003 *Phys. Rev.* D **68** 024012 [hep-ph/0212069]
[10] Wald R M, 1984 *General Relativity*, (Chicago: The University of Chicago Press)
[11] Collins H and Holdom B, *Phys. Rev.* D **62** 105009 [hep-ph/0003173]
[12] Shtanov Yu V, *On brane-world cosmology*, 2000 *Preprint* hep-th/0005193
[13] Deffayet C, 2001 *Phys. Lett.* B **502** 199 [hep-th/0010186]
[14] Alcock C and Paczynski B, 1979 *Nature* **281** 358
Kujat J, Linn A M, Scherrer R J and Weinberg D H, 2001 *ApJ* **S72** 1 [astro-ph/0112221]
[15] Newman J A and Davis M, 2000 *ApJ* **534** L11 (2000) [astro-ph/0109131]
[16] Sahni V, Saini T D, Starobinsky A A and Alam U, 2003 *JETP Lett.* **77** 201 [astro-ph/0201498]
Alam U, Sahni V, Saini T D and Starobinsky A A, 2003 *MNRAS* **344** 1057 [astro-ph/0303009]
[17] Spergel D N *et al*, 2003 *Astrophys. J. Suppl.* **148** 175 (2003) [astro-ph/0302209]
Tegmark M *et al*, 2004 *Phys. Rev.* D **69** 103501 (2004) [astro-ph/0310723]
[18] Richards G T *et al*, 2004 *Astron. J.* **127** 1305 [astro-ph/0309274]
[19] Haiman Z and Quataert E, *The Formation and Evolution of the First Massive Black Holes*, 2004 *Preprint* astro-ph/0403225
[20] Hasinger G, Schartel N and Komossa S 2002 *Astrophys. J.* **573** L77
Friaça A C S, Alcaniz J S and Lima J A S 2005 *Mon. Not. Roy. Astron. Soc.* **362** 1295 [astro-ph/0504031]
Jain D and Dev A 2005 Age of High Redshift Objects – a Litmus Test for the Dark Energy Models *Preprint* astro-ph/0509212
[21] Barkana R and Loeb A, *In the Beginning: The First Sources of Light and the Reionization of the universe*, 2000 *Preprint* astro-ph/0010468
Ciardi B and Ferrara A, *The First Cosmic Structures and their Effects*, 2004 *Preprint* astro-ph/0409018
[22] Maor I *et al*, 2002 *Phys. Rev.* D **65** 123003 [astro-ph/0112526]
[23] Alam U, Sahni V, Saini T D and Starobinsky A A, 2004 *MNRAS* **354** 275 [astro-ph/0311364]

Alam U, Sahni V and Starobinsky A A, 2004 *J. Cosmol. Astropart. Phys.* JCAP06(2004)008 [astro-ph/0403687]

[24] Bassett B S, Corasaniti P S and Copeland E J, 2004 *ApJ* **617** L1 [astro-ph/0407364]

[25] Lue A and Starkman G, 2003 *Phys. Rev.* D **67** 064002 [astro-ph/0212083]
Lue A, Scoccimarro R and Starkman G, 2004 *Phys. Rev.* D **69** 044005 [astro-ph/0307034]
Lue A, Scoccimarro R and Starkman G D, 2004 *Phys. Rev.* D **69** 124015 [astro-ph/0401515]

[26] Dvali G, Gabadadze G and Porrati M, 2000 *Phys. Lett.* B **485** 208 [hep-th/0005016]
Dvali G and Gabadadze G, 2001 *Phys. Rev.* D **63** 065007 [hep-th/0008054]

[27] Randall L and Sundrum R, 1999 *Phys. Rev. Lett.* **83** 3370 [hep-ph/9905221]
Randall L and Sundrum R, 1999 *Phys. Rev. Lett.* **83** 4690 [hep-th/9906064]

[28] Padilla A, 2004 *Class. Quantum Grav.* **21** 2899 [hep-th/0402079]

[29] Shtanov Yu and Viznyuk A, 2005 *Class. Quantum Grav.* **22** 987 [hep-th/0312261]

[30] Shiromizu T, Maeda K and Sasaki M, 2001 *Phys. Rev.* D **62** 024012 [hep-th/9910076]

[31] Smolyakov M N, 2004 *Nucl. Phys.* B **695** 301 [hep-th/0403034]

[32] Bertotti B, Iess L and Tortora P, 2003 *Nature* **425** 374
Will C M, *Was Einstein Right? Testing Relativity at the Centenary*, 2005 *Preprint* gr-qc/0504086

[33] Luty M A, Porrati M and Rattazzi R, 2003 *J. High Energy Phys.* JHEP09(2003)029 [hep-th/0303116]
Rubakov V A, *Strong coupling in brane-induced gravity in five dimensions*, 2003 *Preprint* hep-th/0303125

[34] Visser M and Wiltshire D L, 2003 *Phys. Rev.* D **67** 104004 [hep-th/0212333]

[35] Kaloper N, 2005 *Phys. Rev. Lett.* **94** 181601; Erratum—*ibid.* **95** 059901 [hep-th/0501028]
Kaloper N, 2005 *Phys. Rev.* D **71** 086003 [hep-th/0502035]

[36] Iorio L, *On the effects of the Dvali–Gabadadze–Porrati braneworld gravity on the orbital motion of a test particle*, 2005 *Preprint* gr-qc/0504053

[37] Lue A, *The Phenomenology of Dvali–Gabadadze–Porrati Cosmologies*, 2005 *Preprint* astro-ph/0510068

[38] Maartens R, 2003 *Progr. Theor. Phys. Suppl.* **148** 213 [gr-qc/0304089]

[39] Kofinas G, Papantonopoulos E and Pappa I, 2002 *Phys. Rev.* D **66** 104014 [hep-th/0112019]
Kofinas G, Papantonopoulos E and Zamarias V, 2002, *Phys. Rev.* D **66** 104028 [hep-th/0208207]

[40] Sawicki I and Carroll S M, *Cosmological Structure Evolution and CMB Anisotropies in DGP Braneworlds*, 2005 *Preprint* astro-ph/0510364

[41] Goldberger W D and Wise M B, 1999 *Phys. Rev. Lett.* **83** 4922 [hep-ph/9907447]

[42] Gabadadze G and Iglesias A *Schwarzschild solution in brane induced gravity*, 2004 *Preprint* hep-th/0407049

[43] Hartle J B and Hawking S W, 1983 *Phys. Rev.* D **28** 2960

[44] Shtanov Yu and Sahni V, 2003 *Phys. Lett.* B **557** 1 [gr-qc/0208047].

[45] Umezu K, Ichiki K and Yahiro M, 2005 *Phys. Rev.* D **72** 044010 [astro-ph/0503578]