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# The photometric study of light scattering from the surface of alumina powder and interpretations by Hapke formula

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## Abstract

A laboratory experiment helps to understand the light scattering property of regolith like samples with known compositions and other physical parameters. The laboratory data so obtained can be compared with the existing in situ data on celestial objects like asteroids. Further, it may be analyzed with the help of various theoretical models to understand the light scattering processes from regolith more clearly. In this work we have performed laboratory based photometry of the light scattered from the surfaces of powdered alumina ($Al_2O_3$) at various tilt angles of the sample and at large phase angles, with the particles having diameter 0.3 μm. The wavelength of observation was 632.8 nm. These data have been fitted by a surface scattering model originally suggested by Hapke. Instead of using empirical Henyey–Greenstein phase function to fix the values of albedo and phase function to be used within Hapke formula, we have used Mie theory for the same. This approach helped us to determine the single particle properties such as particle diameter and complex refractive index from surface scattering phase curve alone. Mie theory depends only on the size parameter $X(=2\pi(\text{radius/wavelength}))$ and complex refractive index $(n, k)$ of the material. Since the absorption coefficient ($k$) for alumina is known to be very low but not exactly zero, the best fit to the experimental data was obtained by least square technique with $k$ as a free parameter, as the other parameters are known. Finally, we compare our results with other published results and discuss the scope of application of the method we adopted.

*Keywords:* Bidirectional reflection; Mie theory; Absorption coefficient; Phase function; Hapke formula

## 1. Introduction

The study of the light scattering properties of powdered samples is known to be an important tool to characterize the physical and compositional properties of asteroids, since the asteroids are known to be covered with fine grained materials known as regolith layers (Hapke, 1993). Therefore, laboratory based experiments on the asteroid surface analogues are important, so that the in situ data can be compared with the laboratory results and the theoretical models can be also tested.

The process of regolith development in small asteroids like Nereus and 1989ML (diameters ⩽ 1 km.) with little surface gravity, may be different from that of other objects like Moon (Kamei et al., 1999). Such asteroids may contain much finer grains in the regolith as compared to others. The laboratory based photometric data at a constant phase angle of 1° for various alumina samples as a regolith analogue with particle diameters 500, 45 and 0.1 μm have been reported by Kamei et al. (1999). The opposition effect on alumina has been also studied by Nelson et al. (2000), (2002), Shkuratov et al. (2002), Ovcharenko et al. (2006). Large phase angle scattering from various alumina samples has been studied by Piatek et al. (2004). Other recent experimental studies on regolith scattering include those by Shkuratov et al. (2006), Munoz et al. (2006), Shkuratov et al. (2007).

In this work, we generate in the laboratory, photometric data at large phase angles for the plane surfaces containing powdered alumina ($Al_2O_3$) particles with 0.3 μm diameter

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and at a laser wavelength of 632.8 nm (Red). The particle size and wavelength are such that, the Mie theory (Van de Hulst 1957) can be used to describe the particle phase function and albedo (also discussed in more detail in Section 4). The studies on regolith surfaces with spherical particles, have been already made by Bondarenko et al. (2006), Ovcharenko et al. (2006), Shkuratov et al. (2006), Zhang and Voss (2005), Zhang and Voss (2008). However, in this work we have tested whether values predicted from the Hapke formula (Hapke, 1993) in combination with Mie theory, can match the phase curve data generated through our experiment on a regolith surface composed of small but irregular grains? Least square technique has been applied for fitting the experimental data to the model, by varying the unknown absorption coefficient *k* of alumina.

## 2. Instrument and sample

The experiment was carried out with the help of a goniometric device (Fig. 1) at the Department of Physics, Assam University, Silchar, India. It consisted of two metal arms having a common horizontal axis of rotation. The sample surface was placed lying along the axis of rotation of the arms with the help of three translation stages (for *X–Y–Z* movement of the sample tray). A miniature goniometer was used for tilting the sample tray. The two arms can be rotated through ±90° from the zenith direction. Also any tilt amount between +20° to −20 ° can be given to the sample tray, with respect to the plane containing the incident ray and emergent ray. A linear polarized He–Ne laser at 632.8 nm was used as the source of light and a CCD (make: SBIG, ST-6) camera was used as the detector. These two were mounted on the two arms, such that their axes were perpendicular to the horizontal sections of the respective arms (Fig. 1). The sample was placed at the common intersection of the axis of rotation and axes of the source and detector. Unpolarized (i. e. polarization randomised) lasers produce fluctuations in linear polarization (Kaasalainen, 2003). Since we used a linear polarized laser, such fluctuations were expected to be much less or absent in our case.

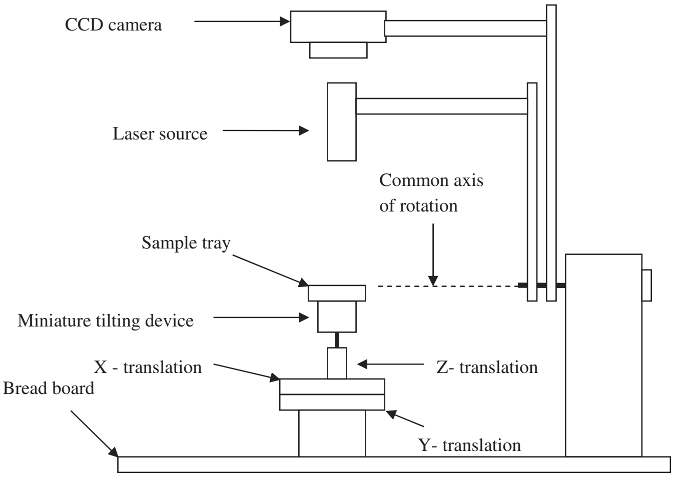

Fig. 1. The schematic diagram of the Goniometric device used in the experimental part (side view) is shown.

The sample used in the present experiment was powdered alumina ($Al_2O_3$) with 0.3 μm as particle diameter. The powdered sample was taken on a sample tray and the bottom of the tray was struck gently on a table to make the sample surface nearly plane. At this stage, the surface roughness of the sample was quite high. To prepare a smooth surface, the sample surface was pressed with a smooth metal spatula, so that the sample surface obtained its smoothness. The thickness of the prepared sample was about 6.2 mm and the porosity was found to be 0.88 which is consistent with previously published results for sample with small alumina particles (Kamei et al., 1999). The porosity *P* was measured with the help of the relation Sakai and Nakamura (2005):

$$P = 1 - (m/\rho V) \tag{1}$$

where, *m* denotes the mass of the powdered sample, *V* denotes its volume (obtained after pressing) and *ρ* is the bulk density of the material, which is 3.9 g/c.c for alumina.

## 3. Measurement and data collection

The tilt angle of the sample was kept fixed at 0° for simplicity to begin with. The detector angle or emergent angle (*e*) was also kept fixed at –45° (anticlockwise when viewed from the front) from the zenith. The angle of incidence (*i*) was varied from 0° to 63° in steps of 9° (which was equivalent to rotation by five divisions of the attached circular scale). Outside the said range of incidence angle, the laser spot did not exactly pass through the axis of rotation, due to the limitation on our mechanical assembly. Thus the phase angle (*g*) was varied from 45° to 108°. A diffuser was placed in front of the CCD to kill the laser speckles produced by the coherent laser beam on scattering from a rough surface (McKechnie, 1976). The images of sample surface were recorded at every angle of incidence in the form of FITS image. The fluctuation of the laser source restricted accurate measurement of the reflectance, but, total experimental errors were not more than 10%.

As the field of view of the detector (0.57° × 0.76°) was larger than the laser spot (linear size 0.52 mm), the geometrical correction (cos *i*/cos *e*) was applied to calculate the intensity values from the detector counts. The background and dark correction were also applied for each observation. The reflectance values were calibrated by using pressed $BaSO_4$ surface (a standard Lambert surface), at the condition of incidence angle 0° and detector angle −45°.

## 4. Results and analysis

### 4.1. Phase curve

The bidirectional reflectance $r(i,e,g)$, is the ratio between the reflected light intensity $I(i,e,g)$ and the incident irradiance $J$ (i.e. flux/sec. per unit normal area). Thus, $r(i,e,g) = \mu_0.$

BRDF, is the ratio of the radiance scattered by a surface into a given direction to the collimated power per unit normal area (Hapke, 1993), normalized to $\pi$ (where BRDF is the bidirectional reflectance distribution function).

Bidirectional reflectance, measured for the alumina sample are shown in Figs. 2 and 3 as a function of phase angle. The data shown in Fig. 2 at zero tilt angle were theoretically fitted with the help of Hapke formula (Hapke, 1993) given by Eq. (2). Hapke formula requires two unknown parameters – opposition surge amplitude $B_0$ and opposition angular width $h$, which are mainly important at phase angles smaller than 15°. At larger phase angles the influence of these two parameters can be neglected. Since, in our case phase angle values were always greater than 45°, we kept these two parameters fixed at their typical values: 1 and 0.65 respectively (Hapke, 1993). The formula also required a single particle phase function $p(\boldsymbol{g})$ and single particle scattering albedo ω. Generally, empirical phase functions like Henyey–Greenstein phase function (Henyey and Greenstein, 1941) are used in Hapke formula and this in turn introduces unknown quantities like *asymmetry parameter* $\xi$. Thus no information about the particle size and/or composition can be introduced into the analysis directly. But in our case, rather than using any empirical phase function, we took the phase function and albedo from the single particle Mie theory and put them into Hapke formula. As a result, the particle diameter and optical constants were directly introduced into the analysis. When the phase angle is not very small or very large, the phase function of a single isolated particle and that of a particle in a regolith do not differ very significantly (e.g. Fig. 1 of Hapke et al., 2009). Therefore, a theoretical phase function (like Mie) can be reliably used in Hapke formula for an approximate analysis. Thus, we tested the applicability of Mie theory phase function and albedo, in Hapke formula in case of a regolith containing grains as small as 0.3 μm and incident radiation with wavelength 632.8 nm. The Hapke formula is reproduced below (Hapke 1993):

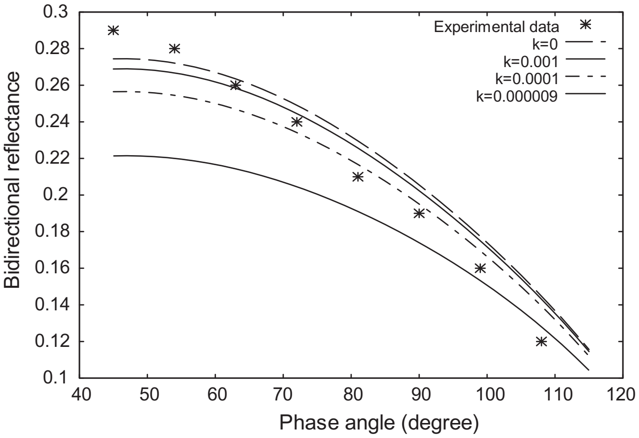

Fig. 2. The points indicate experimental data for alumina sample ($D$ = 0.3 μm) at $\lambda = 0.633$ μm. The X coordinates of the points represent the phase angle $g$ in degrees at fixed $e = 45°$ and at zero tilt. The theoretical curves are obtained by using Hapke formula with Mie theory at same wavelength. The best fit was obtained for $k = 0.000009$.

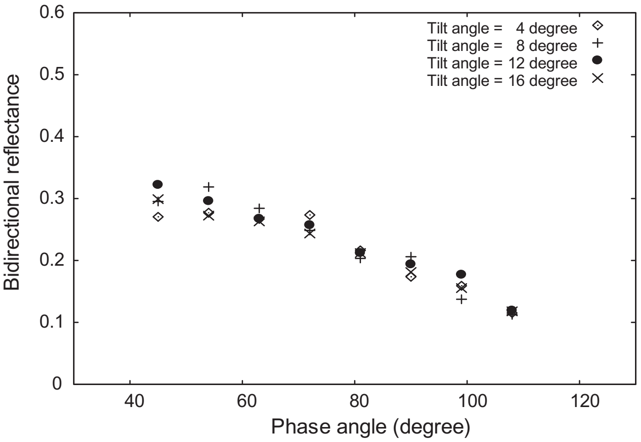

Fig. 3. The points indicate experimental data for alumina sample ($D$ = 0.3 μm) at $\lambda = 0.633$ μm at different tilt angles. The $X$ coordinates of the points represent the phase angle $g$ in degrees at fixed $e = 45°$.

$$r(i,e,g) = (\omega/4\pi)(\mu_0/\mu + \mu_0)\{[1 + B(g)]p(g) + H(\mu)H(\mu_0) - 1\} \tag{2}$$

where,

$$\mu = \cos e \text{ and } \mu_0 = \cos i$$

$$B(g) = \frac{B_0}{1 + (\frac{1}{h})\tan(\frac{g}{2})}$$

$$H(x) = \left[1 - (1 - \sqrt{1-\omega})x\left\{r_0 + (1 - \frac{1}{2}r_0 - r_0 x)\ln\frac{1+x}{x}\right\}\right]^{-1}$$

$$r_0 = \frac{2}{1 + \sqrt{(1-\omega)}} - 1$$

Though we have used Mie theory for generating phase function and albedo values, but it is known that the Mie theory assumes the particles to be smooth and homogeneous spheres, which our alumina particles were not. But, Pollack and Cuzzi (1980) suggested that the Mie theory may be used to calculate the scattering properties of *equant* irregular particles also, if the size parameter $X \leqslant 5$. In our case $X$ value was 1.49 for particle diameter $D = 0.3$ μm and wavelength of the incident light $\lambda = 632.8$ nm. Therefore, in our case, the Mie theory was applied with Hapke formula to calculate the bidirectional reflectance values up to a considerable level of accuracy.

The refractive index of alumina at 632.8 nm is $n = 1.766$ (Gervais 1991), and absorption coefficient '$k$' is known to be very small, but not known exactly. We have varied the unknown parameter '$k$' to fit the laboratory data. The data was fitted by least square technique and the best fit to the data was obtained for $k = 0.000009$. This result was in close agreement with previously obtained value of $k$ for alumina at 635 nm by other authors (Piatek et al., 2004).

Bidirectional reflectance data at other tilt angles have been also shown in Fig. 3. From this figure it can be seen that, the influence of tilt angle on the nature of the phase curve is not very significant for the fine grained alumina sample we used. However, the influence of tilt angle on other samples with different compositions and grain sizes should be tested for a more definite conclusion, so that it contributes further in our knowledge of the disc resolved in situ observations of celestial objects.

### *4.2. I–D curve*

A detailed laboratory study on the opposition effect by powdered alumina surface was performed by Nelson et al. (2000) using alumina samples at 13 different particle diameters ranging from 0.1 to 30.09 μm. Since, the study was confined within opposition region only, the maximum phase angle under consideration was as low as 5°. But, it can be clearly realized from the plots of their data that, except at very small phase angles, the phase curves are parallel to each other. As a result the ratio of reflectances (between two parallel phase curves) at a given phase angle, will practically remain unchanged for phase angle values much larger than 5° .

Now, it was shown in our another work (Deb and Sen 2011) that, the dependence of reflectance or intensity ($I$) on particle diameter ($D$) at a constant phase angle (within range 25°–30° of the phase curve) obtained by using Mie theory and Hapke formula, agreed well with the experimental results for the weakly absorbing materials. It was also shown that, the nature of the curve depended strongly on the absorption coefficient $k$.

As mentioned earlier, for any such work the parameters $B_0$ and $h$ should be exactly known, if the phase angle values are small (less than about 15°). Since, only the typical vales of these two parameters were used in our present analysis (exact values being unknown), we have plotted the theoretical *I–D* curve for alumina at a larger phase angle value $g = 20°$. The theoretical curve had been plotted for $k = 0.000009$, which is shown in Fig. 4. A size distribution of the particles, represented by gamma distribution function $N(r) = k' \cdot r^{(1-3b)/b} e^{(-r/ab)}$ was considered by Mishchenko, 1992 and this was also found to be same as the size distribution function for particles (in the sample) in the experiment reported by Nelson et al. (2000) (with $a = r_0$ (effective radius), $b = 0.04$ and $k' =$ constant). Here we had used the same size distribution function for our light scattering calculations. It can be clearly seen that, their experimental data at $g = 5°$ fitted quite closely to our theoretical curve for small particles. This confirmed the result we found in Section 4.1. Also, it further confirmed the applicability of the analysis done in our other work (Deb and Sen, 2011) in case of low absorbing materials.

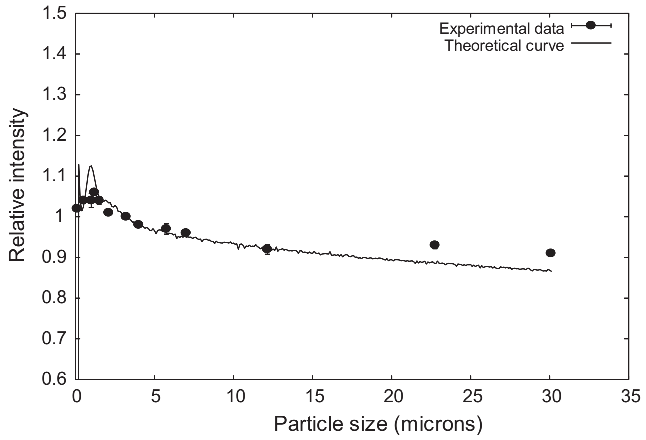

Fig. 4. The points indicate experimental data for alumina sample at $\lambda = 0.633$ μm taken from Nelson et al. (2000). The $X$ coordinates of the points represent the average particle diameters. The theoretical curves were obtained by using Hapke formula with Mie theory at same wavelength and at $i$=0°, $g$=20° (see text). The intensity values are relative to the respective values at $D = 3.2$ μm.

## 5. Conclusions

(1) We found that, the theoretical fit to the large phase angle experimental data was quite satisfactory (Fig. 2) for reflectance studies from a regolith surface using Hapke's model with Mie theory. The features like porosity and roughness were neglected in this treatment, and therefore accurate fitting was not expected. Moreover, particles were hardly smooth spheres, which might have introduced additional uncertainties in the results. But, it was evident from the analysis that, the phase curves of the regolith like samples with grain sizes comparable to the wavelength of incident radiation could be very efficiently studied at large phase angles using directly the theoretical phase functions like that of Mie with the Hapke model, to retrieve the single particle features like grain size and/or composition. However, under the condition the particle diameter $D >> \lambda$, the Mie theory was unable to describe the phase function for the irregular particles (Hapke, (1993)). Therefore, in such cases applicability of other numerical or theoretical phase functions for irregular particles with Hapke formula may be tested.

(2) From the results shown in Fig. 3, we hardly find any significant influence of tilt angle on bidirectional reflectance for 0.3 μm sized alumina particles, within the error limits. This result should be tested for other grain sizes and other samples.

(3) This study again showed that, in case of low absorbing materials, the intensity versus particle size behavior can be very efficiently simulated with the help of Hapke formula and Mie theory, and the *I–D* curve is hardly affected by the irregular shape of the individual grains.

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