

Mach's Principle and the Creation of Matter [and Discussion]
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You must realize that these exact solutions are very special solutions, and we cannot in this way get anything which would represent a gravitational wave falling on a body and diffracting. In that sense diffraction is excluded.

J. L. Synge

Mr Chairman, you assume axial symmetry in your work. Now axial symmetry includes spherical symmetry as a special case. Therefore all you say must hold in the case of spherical symmetry. I was just wondering what happens in that case, since in view of the so-called Birkhoff's theorem, there isn't any radiation.

H. Bondi

Yes. This comes out quite clearly from the boundary conditions on the axis. In the case of spherical symmetry the news functions would have to be spherically symmetrical. Since the news function must vanish to the second order on the axis of symmetry, it must vanish everywhere in the spherical symmetrical case. Therefore there is no news in the case of spherical symmetry, and no loss of mass.

A. Lichnerowicz

One of my students Dr Lee, has shown recently that there exists some type of permanence of the type of solution in the radiative types of solution if Ω vanishes. If on a space-like hypersurface we have a solution with a fixed type, and if Ω vanishes on the hypersurface, then the type is the same outside the hypersurface.

W. Kundt

In a pure radiation field, the conform tensor type cannot change along the rays. (For twist-free fields this can be read from the metric form; it also follows from propagation equations derived from the vacuum Bianchi identities.) Consequently, if such a field of fixed type is given on a (two-dimensional) wave surface, it will be of this type along the total wave hypersurface containing the wave surface. On the other hand, the type can change arbitrarily as a function of retarded time.

Mach's principle and the creation of matter

By F. Hoyle, F.R.S. and J. V. Narlikar
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This is a short report of a small contribution to a very old problem, the problem expressed by Newton in the following well-known words:

'The effects which distinguish absolute motion from relative motion are the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion they are greater, or less, according to the quantity of the motion.'

After this, Newton went on to discuss his experiments of a rotating water-filled bucket suspended from a twisted thread. The crucial point was that whenever rotation occurred relative to some particular reference frame the surface of the water became depressed; an absolute effect not a relative one.

It was also clear that the reference frame, relative to which inertial forces were observed, coincided within experimental error with the frame in which distant objects in the universe were non-rotating. More accurate later experiments have confirmed this coincidence. Since the coincidence can scarcely be accidental it is necessary to attempt an explanation of it.

A suggestion, not an explanation, was made by Mach to the effect that inertial forces are related to the distant parts of the universe, for the very good reason that they are generated by the distant parts of the universe. Nowadays this rather vague concept is dignified by the name Mach's principle. Mach's contribution to the subject lay in his clear perception of the existence of a crucial problem, rather than in the development of a quantitative theory. This is not to say that Newton himself had not been fully aware of the problem, but intervening generations had been so bemused by the great success of Newton's theory that they had forgotten the difficulties contained in the basic postulates.

Einstein was much impressed by the ideas of Mach, and it was his hope that the general theory of relativity might provide the necessary mathematical foundation of Mach's principle. In the event this hope was not realized. Yet the coincidence of the inertial frame of Newton's theory with that in which the distant parts of the universe are non-rotating can be deduced from cosmology. The apparent paradox that the coincidence in question can be deduced from normal cosmology but not from the normal relativity theory is resolved by the circumstance that normal cosmology contains postulates that lie outside the relativity theory. These postulates are:

(i) The world lines of matter form a geodesic congruence normal to a space-like surface. Taking the congruence as defining the co-ordinate system x^μ, t the line element can be written as
$$ds^2 = dt^2 + g_{\mu\nu}\,dx^\mu\,dx^\nu \tag{1}$$
the six functions $g_{\mu\nu}$ ($\mu, \nu = 1, 2, 3$) being dependent on x^μ, t. This is known as Weyl's postulate.

(ii) The subspaces t = constant are geometrically homogeneous and isotropic. This postulate, known as the cosmological principle, makes it possible to choose co-ordinates r, θ, ϕ that are constants along the world lines of every particle (by (i) all such world lines belong to the geodesic congruence) and such that
$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right], \tag{2}$$
where $k = 0, \pm 1$ and where r can be chosen zero for an observer attached to any particular particle. The form (2) is the well-known Robertson–Walker line element.

Now a transformation $r' = r'(r,t)$, $t' = t'(r,t)$, $r' \to 0$ as $r \to 0$ can be found that for sufficiently small r' expresses (2) in the form
$$ds^2 = dt'^2\{1 - r'^2(\dot{S}/S)\} - \frac{dr'^2}{\{1-(kr'^2/S^2) - r'^2(\dot{S}^2/S^2)\}} - r'^2(d\theta^2 + \sin^2\theta\,d\phi^2) \tag{3}$$

the coefficients being correct to order r'^2. In the case of the de Sitter line element $k = 0$, $S = \exp(Ht)$, H a constant, the transformation in question is a familiar one.

$$r' = r\exp(Ht), \quad t' = t - (1/2H)\ln(1 - r'^2H^2). \tag{4}$$

Indeed in this case the form (3) is both accurate and it applies whatever r' may be.

Returning to (3), in any local experiment we have the case $r' \to 0$ and (3) becomes the metric of special relativity. Rotation relative to this inertial frame produces inertial forces in the usual kinematical way. Moreover, if we wish to study the gravitational field of a local mass point then (3) provides the boundary condition at large distance from the mass. In practice the terms in r'^2 are very small, even for distances that can be considered large. For example, at 1 pc from the sun the terms in r'^2 turn out to be $\sim 10^{-19}$. Hence for practical purposes, in considering the application of the Schwarszchild's metric within the solar system, we can take the boundary condition 'at infinity' as that of special relativity. Now in this metric θ, ϕ are constant for the distant objects—just because θ, ϕ were unaffected by the transformation to r', t'. Hence we have arrived at the required situation.

If one now turns to Einstein's field equations

$$R^{ik} - \tfrac{1}{2}g^{ik}R + \lambda g^{ik} = -\kappa T^{ik} \tag{5}$$

and inserts a metric tensor of the form (2), then reading the equation from left to right one can show that T^{ik} is of the form

$$T^{ik} = (\rho + p + \tfrac{4}{3}u)\frac{dx^i}{ds}\frac{dx^k}{ds} - (p + \tfrac{1}{3}u)g^{ik} \tag{6}$$

with $dx^i/ds = \delta^i_4$ where the precise properties of u, p, ρ depend on the function $S(t)$ in (2). (These are respectively the radiation density, gas pressure and matter density.)

Plainly the present procedure is the direct contrary of all that is implied by Mach's principle. Mach's principle requires us to read equation (5) from right to left. Suppose we simply try to reverse the above situation. Given (6) in some co-ordinate system can the line element (2) be deduced? The answer is, it cannot. This is known from a solution of (5) with T^{ik} of the form (6) due to Gödel (1949), which does not give (2). Gödel's solution, which does not satisfy the condition of isotropy, is fundamentally different from the form given by (2), and cannot be obtained from it by a co-ordinate transformation.

The most that can be achieved within the framework of the relativity theory is the following. Take any suitable space-like surface and use this surface to define co-ordinates x^μ, t leading to a line element of the form (1). This simply removes the arbitrariness of co-ordinates. On this surface specify the matter and the kinematical situation and specify also

$$g_{\mu\nu}, \quad \frac{\partial g_{\mu\nu}}{\partial x^i}, \quad \frac{\partial^2 g_{\mu\nu}}{\partial x^\lambda \partial t}, \quad \frac{\partial^2 g_{\mu\nu}}{\partial x^\lambda \partial x^\sigma} \quad (i = 1, 2, 3, 4;\ \lambda, \mu, \sigma, \nu = 1, 2, 3)$$

the specifications being made consistently with (5). Then (5) allows one in principle to calculate both the dynamical situation and the form of the metric tensor at points off the initial surface.

Now while it is the case that these specifications can be made in such a way that the line element turns out everywhere to be of the form (2), in order to achieve this result—and hence the correct boundary condition for Newton's bucket, and for the solar Schwarzschild solution—it is essential that conditions on $g_{\mu\nu}$ be laid down, as well as conditions on the matter. The upshot is that Newton's absolute space has been replaced in the relativity theory by initial boundary conditions on the metric tensor.

In normal forms of cosmology the situation is dealt with by supposing that the required boundary conditions were imposed at the origin of the universe—the universe might have been created with any specified degree of anisotropy, but it just did not happen to be that way. If this view is correct then the coincidence implicit in Newton's bucket experiment must simply be accepted. It can never be explained.

One of the main aims of the so-called steady-state cosmology is to dispense with the need for special initial conditions. The theory requires the introduction of a new field, which one may take to be of the simplest form, a scalar C. The usual equations (5) can be obtained from an action principle. The effect of C can also be deduced from an action principle, as was first pointed out by M. H. L. Pryce (private communication). The action is

$$A = \frac{1}{16\pi G}\int R\sqrt{(-g)}\,\mathrm{d}^4x - \Sigma m\int \mathrm{d}s + \tfrac{1}{2}f\int C_i C^i \sqrt{(-g)}\,\mathrm{d}^4x - \Sigma m\int C_i \frac{\mathrm{d}x^i}{\mathrm{d}s}\,\mathrm{d}s. \quad (7)$$

The quantity f is a coupling constant, which turns out to determine the rate of expansion of the universe. The conditions $\delta A = 0$ with respect to variations of the metric tensor and of the C field give

$$R^{ik} - \tfrac{1}{2}g^{ik}R = -\kappa[T^{ik} - f\{C^iC^k - \tfrac{1}{2}g^{ik}C^lC_l\}], \quad (8)$$

$$C^i{}_{;i} = \frac{1}{f}j^i{}_{;i}, \quad T^{ik}{}_{;\,k} = fC^i\,C^k{}_{;k}, \quad (9)$$

where $j^i = \rho(\mathrm{d}x^i/\mathrm{d}s)$ is the mass current. It is possible to study the homogeneous isotropic case by using the form of T^{ik} given by (6). If, as an example, one puts $u = 0$, $p = 0$, (8) and (9) lead to

$$3\frac{\dot{S}^2 + k}{S^2} = \kappa(\rho - \tfrac{1}{2}f\dot{C}^2), \quad (10)$$

$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2 + k}{S^2} = \tfrac{1}{2}\kappa f\dot{C}^2, \quad (11)$$

$$\ddot{C} + 3\frac{\dot{S}}{S}\dot{C} = \frac{1}{f}\left(\dot{\rho} + 3\frac{\dot{S}}{S}\rho\right), \quad (12)$$

where C must be a function of t only in the homogeneous-isotropic case.

These equations have a remarkable property. In the case $C = 0$ they become the usual cosmological equations. For example, when $k = 0$ the general solution tends to the well-known Einstein de Sitter universe. This means that, if one so considers

perturbations of the Einstein de Sitter solutions, the perturbations die away—this being true for all perturbations in which $C^i{}_{;i}$ remains zero. But if one admits a perturbation in which $C^i{}_{;i} \neq 0$, i.e. one in which matter is created, then the S function is moved on to a new class of solution that always tends to $\exp(Ht)$, $H = \sqrt{(\kappa f/6)}$. This is the steady-state solution. As $S \to \exp(Ht)$ the density $\rho \to 3H^2/4\pi G_1$ and the creation rate per unit volume $\to 9H^3/4\pi G$. It is to be emphasized that although in the asymptotic situation creation and expansion are in exact balance, no such balance is assumed—the balance is proved.

In the steady solution (and even during the unsteady phase in the case $C^i{}_{;i} \neq 0$) C is just the time t used in setting up the line element (2). This suggests that if we attempt to read equation (8) from right to left, returning to the project of deducing the geometry from the physics, the C field may turn out to make a crucial difference. In particular one should read the equation $C = t$ from right to left—when we choose the surface of Weyl's postulate we choose a surface $C =$ constant. The matter is normal to this surface because it is created so,

$$T^{ik}{}_{;k} = fC^iC^k{}_{;k}.$$

We can still set up a system of surfaces $C =$ constant, even if it be supposed that the metric is not of the form (2) and we can define $t = C$. Suppose we are given the distribution of matter and its velocity on a particular member of the set of C-surfaces. To permit a calculation of the situation off the particular surface it is still necessary to specify the metric tensor and the derivatives on the surface. But could it be the case that as the time proceeds, i.e. as C increases—it does not matter how the free choices on the metric tensor were made, that over any constant physical (not co-ordinate) distance the line element tends to the form (2)? If so, the Newtonian bucket experiment would be explained as an asymptotic condition to which all initial conditions tend.

This problem has been studied taking as our initial conditions perturbations from the form (2). The perturbed $g_{\mu\nu}$ are of the form

$$g_{\mu\nu} = -(\eta_{\mu\nu} + h_{\mu\nu})\,e^{2Ht}, \tag{13}$$

where $\eta_{\mu\nu} = 0$ if $\mu \neq \nu$ and $\eta_{\mu\nu} = 1$ if $\mu = \nu$. In considering small perturbations the squares and higher powers of $h_{\mu\nu}$ and its derivatives can be neglected. Substitution in the field equations leads to linear equations for $h_{\mu\nu}$. The quantities $h_{\mu\nu}$ turn out to have the form

$$h_{\mu\nu} = \alpha_{\mu\nu} + \beta_{\mu\nu}\,e^{-2Ht} + \gamma_{\mu\nu}\,e^{-3Ht}, \tag{14}$$

where $\alpha_{\mu\nu}$, $\beta_{\mu\nu}$, $\gamma_{\mu\nu}$ are functions of x^μ only, satisfying certain partial differential equations. Clearly, the last two terms in (14) die away in times of order H^{-1}. The variation of the first term over any given proper volume becomes smaller and smaller because of expansion. This is precisely the effect required.

The above calculation does not take account of the effects of rotation. This is brought about by the $g_{4\alpha}$ terms in the line element. The following simple demonstration shows, however, that though rotation cannot be altogether removed by expansion, it can be made as small as we please over any given proper volume.

Following the notation of Raychaudhuri (1955) we define rotation for the line element

$$ds^2 = dt^2 + 2g_{\mu 4}\,dt\,dx^\mu + g_{\mu\nu}\,dx^\mu\,dx^\nu \tag{15}$$

by means of an anti-symmetrical tensor

$$\omega_{ik} = \tfrac{1}{2}(u_{i;k} - u_{k;i}) = \frac{1}{2}\left(\frac{\partial g_{i4}}{\partial x^k} - \frac{\partial g_{k4}}{\partial x^i}\right), \tag{16}$$

where $u^i = dx^i/ds = \delta^i_4$ is the matter tensor. Since matter still follows geodesics $g_{\mu 4}$ are independent of t. The spin ω is given by

$$2\omega^2 = g^{ik}g^{lm}\omega_{il}\omega_{km}. \tag{17}$$

The only non-zero components of ω_{ik} are $\omega_{23} = -\omega_{32} = \omega_1$, $\omega_{31} = -\omega_{13} = \omega_2$, $\omega_{12} = -\omega_{21} = \omega_3$ (say). If it is assumed that universe continues to expand indefinitely then $g_{\mu 4}$ terms can be neglected in comparison with $g_{\mu\nu}$. Equation (17) then reduces to

$$\omega^2 \cong (1/2g)\,g_{\mu\nu}\omega_\mu\omega_\nu \quad (\text{sum on } \mu, \nu = 1, 2, 3). \tag{18}$$

If the expansion is not basically different in various directions, $g_{\mu\nu}/g \to 0$. Since ω_μ are independent of t we get $\omega \to 0$. For an almost isotropic expansion—such as the one observed now—one recovers the familiar classical law $S^2\omega \cong$ constant (in time).

The above work includes only gravitational effects and therefore is not inconsistent with the fact that irregularities actually develop in the universe—galaxies form—provided the irregularities owe their origin to local non-gravitational forces—e.g. to pressure and cooling effects. Although mathematical difficulties have so far prevented cases other than perturbations to be studied the results have the interesting feature that they would already be applicable to a situation in which there was a greater measure of anisotropy than observation suggests the universe to have.

The essential point is that creation of matter keeps the universe going, it keeps ρ at a more or less constant level and the initial non-homogeneity and non-isotropy is expanded away, over specified physical distances. Rotation, such as occurs in the Gödel solution is not destroyed, but is rendered more and more ineffective.

The situation has some analogy to the case of an electrical network containing a periodic generator. The currents in the network do not necessarily oscillate at any given moment with the period of the generator, because of transients; but given enough time the transients will decay and the remaining currents will indeed be periodic. Einstein's equations may be said to possess transients as solutions. If the universe itself is transient then there is no reason why the geometry of space-time should have any particular regularity (such as we find it to have). But if the universe persists, due to a field such as the above C field, then the transients may be expected to die away, leaving the required homogeneous, isotropic regularity in any finite portion of space-time, however large.

References (Hoyle & Narlikar)

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DISCUSSION

P. A. M. Dirac

Professor Hoyle spoke about Newton's bucket experiment. He said that the frame of absolute non-rotation provided by Newton's bucket experiment is in agreement with the frame provided by the distant stars. I should like to ask the experimental people how accurate this agreement is. We heard a good deal this morning about how the accuracy of fundamental experiments has been pushed up in recent years, and I think that it is important that the accuracy of this particular experiment should be pushed up still further. It might very well show a discrepancy which would be fundamental to all theories of cosmology. Can any experimental person tell me how accurate this result is, and whether there are chances of improving the accuracy?

H. Bondi

Without pretending to be an experimentalist, I should like to point out that the best bucket we have is the solar system. Clemence told me some years ago that in order to account properly for the motions in the solar system, one needed a rest-frame a great deal better than that given by the stars of our galaxy, because of the galactic rotation, and he was in fact trying to tie the rest-frame—the non-rotating frame defined by the motions of the solar system—to the external galaxies; he felt this was a long-term task because of the fact that the galaxies are rather fuzzy objects. It is a difficult task to perform, but in principle he felt that our knowledge of the dynamical frame arising from the study of the solar system was now so good that it could be tied to the external galaxies. In a preliminary way the agreement is very good, but it would need a lot of work to see just how good it is.

R. Schlegel, University of Michigan

I wish to ask Professor Hoyle: in introducing a perturbation in the C_i in some initial state, do you not lose the philosophical appeal of the steady-state theory of Professor Bondi's perfect cosmological principle?

F. Hoyle

It is a question of from what point of view you feel the aesthetic appeal. I feel that more in the equations. I feel that if I can see a set of equations where a small perturbation will make one go into a steady-state solution, I like that better. That to me is more aesthetic.

H. Bondi

We do not all agree.

W. Davidson, Battersea College of Technology

Does the stability of the steady state depend on taking C_i^i = constant, rather than $C_i^i \neq 0$? Surely if it is other than a constant, you do not find $R(t)$ tending to exponential form.

F. Hoyle

Yes, you do. For the equation I wrote down this tendency is quite stable.

W. Davidson

In connexion with the lack of isotropy, which Professor Hoyle says the steady state can get rid of, would not any cosmological model which is uniform in the large have this property?

F. Hoyle

I think it is probably true that in any modern cosmology one does finally expand away any initial irregularities only by the time the density has tended to zero—only by the time that nothing ever happens any more. The essential thing here is that there is still a full complement of matter left, in which there are creatures which can live and see that things have smoothed out, whereas by the time things have smoothed out in other cosmologies, no people would be alive.

J. L. Synge

There is one point I am not quite clear about in connexion with the theory. You have Einstein's field equations on the left; on the right you have T_{ik} minus a certain expression in C. Now suppose I write '$= \psi_{ik}$' and rub out the middle part, am I not back with the original Einstein equations that I am familiar with?

F. Hoyle

Yes, in so far as Einstein intended them to be a relation between the geometry and the physical energy-momentum tensor, this is correct. All that one is really saying here is that one is adding an extra piece on to the physical energy-momentum tensor in the same way as one has had to do since Einstein's day in order to add the nuclear fields—the meson fields and the neutrino field.

J. L. Synge

Is it not merely a question of the interpretation of the symbols?

F. Hoyle

I think you must take the C-field as part of the energy tensor, and it must not be thought of as a geometrical quantity, but as a physical field.

I think I might just add that one could say that throughout physics people have had the idea of conservation, but the quantity which is physically conserved has changed as the years have gone on, and all that is being suggested here is that a further change is necessary.