

# CLASSICAL ELECTRON MODEL WITH NEGATIVE ENERGY DENSITY IN EINSTEIN-CARTAN THEORY OF GRAVITATION

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## Abstract

Experimental result regarding the maximum limit of the radius of the electron  $\sim 10^{-16}$  cm and a few of the theoretical works suggest that there might have some negative energy density regions within the particle in general theory of relativity. It is argued in the present investigation that such a negative energy density also can be obtained with a better physical interpretation in the framework of Einstein-Cartan theory.

## 1. INTRODUCTION

Recently, Cooperstock and Rosen [1989], Bonnor and Cooperstock [1989], and Herrera and Varela [1994] have shown that within the experimentally obtained upper limit of the size of the electron (  $\sim 10^{-16}$  cm ) [Quigg, 1983], when it is modeled as a charged sphere obeying Einstein-Maxwell theory, must contain some negative gravitational mass density regions within the particle . According to Cooperstock, Rosen and Bonnor (CRB) [1989], the rest mass or active gravitational mass within this sphere, by virtue of the relation

$$M = m - \frac{q^2}{2a}, \quad (1)$$

is negative and about  $10^{-52}$  cm (when the inertial mass or effective gravitational mass, charge and radius, respectively, of the electron, are  $m = 6.76 \times 10^{-56}$  cm,  $q = 1.38 \times 10^{-34}$  cm, and  $a = 10^{-16}$  cm in relativistic units ). Further, Herrera and Varela (HV) [1994] have shown, in one of the cases of their paper, that the matter-energy density

$$\rho = (\alpha q^2 + \frac{2}{3}\pi\sigma_0^2)(a^2 - r^2), \quad (2)$$

for the constant  $\alpha = -4.77 \times 10^{95}\text{cm}^{-6}$  (when radius  $a \sim 10^{-16}$  cm) is also negative,  $\sigma_0$  being the constant charge density at the centre of the spherical distribution. These models, however,

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lack spin and magnetic moment and hence do not possess the actual physical characteristics required for an electron.

As an alternative way both the groups suggest the stationary Kerr-Newman (KN) metric [Newman et al., 1965] related to the solution of Einstein-Maxwell equations [Misner et al., 1973] to be more appropriate than those described earlier. However, in this context it is also to be mentioned here that the KN metric cannot be valid for distance scales of the radius of a subatomic particle [Mann and Morris, 1993; Herrera and Varela, 1994].

We, therefore, feel that the problem can be tackled in the framework of Einstein-Cartan (EC) theory, where torsion and spin are inherently present in the formulation of the theory itself.

## 2. AN OVERVIEW: THE NEGATIVE DENSITY MODELS

Before going into the Einstein-Cartan theory let us have a birds' eye view of the negative matter-energy density models already we have mentioned in the introduction.

### (i) THE COOPERSTOCK-ROSEN-BONNOR (CRB) MODEL

Cooperstock and Rosen [1989] and Bonnor and Cooperstock [1989] in their papers have shown that any spherically symmetric distribution of charged fluid, irrespective of its equation of state, whose total mass, radius and charge correspond to the observed values of the electron, must have a negative energy distribution (at least for some values of the radial coordinate). Considering a static spherically symmetric charge distribution with the line-element

$$ds^2 = e^{2\nu(r)} dt^2 - e^{2\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (3)$$

they have argued that when the Einstein-Maxwell equation

$$R^0_0 - \frac{1}{2}\delta^0_0 R = 8\pi(T^0_0^{(m)} + T^0_0^{(em)}) \quad (4)$$

is written in the form

$$e^{-2\lambda} = e^{2\nu} = 1 - \frac{1}{r} \int_0^r (8\pi\rho + e^{-(\nu+\lambda)} E^2) r^2 dr \quad (5)$$

and hence is equated with the Reissner-Nordström exterior metric on the boundary  $r = a$ , which as usual gives

$$1 - \frac{2m}{a} + \frac{q^2}{a^2} = 1 - \frac{1}{a} \int_0^a (8\pi\rho + e^{-(\nu+\lambda)} E^2) r^2 dr. \quad (6)$$

Then for the previous specifications of mass, charge and radius of the electron it can be shown that

$$\frac{q^2}{a^2} - \frac{2m}{a} \sim 2 \times 10^{-36} > 0. \quad (7)$$

So, the left hand side of the above equation (6) must be greater than unity and hence on the right hand side  $\rho < 0$  for some values of  $r$  implying that the electron must contain some negative rest mass density though the net mass is as usual a positive quantity.

(ii) THE HERRERA-VARELA (HV) MODEL

Following the CRB model [1989] Herrera and Varela [1994] have discussed the fact that the electron, when modeled as a relativistic spherically symmetric charged distribution of matter, must contain some negative rest mass if its radius is not larger than  $\sim 10^{-16}$  cm. In this regard they have analyzed some extended electron models and have shown that negative energy density distributions result from the requirement that the total mass of these models remains constant in the limit of a point particle. Among all these extended electron models the model of Tiwari et al. [1984] demands special attention to us which will be seen very much relevant to our present work. Herrera and Varela [1994] generalize this model of Tiwari et al. [1984] by introducing a condition of anisotropy in the form

$$p_{\perp} - p_r = \alpha q^2 r^2 \quad (8)$$

where  $\alpha$  is a constant.

Thus the solution obtained by Herrera and Varela [1994] is as follows:

$$e^{-2\lambda} = e^{2\nu} = 1 - \frac{16}{45}\pi^2\sigma_0^2 r^2(5a^2 - 2r^2) - \frac{8}{15}\pi\alpha q^2 r^2(5a^2 - 3r^2), \quad (9)$$

$$p_r = -(\alpha q^2 + \frac{2}{3}\pi\sigma_0^2)(a^2 - r^2), \quad (10)$$

$$p_{\perp} = \alpha q^2 r^2 - (\alpha q^2 + \frac{2}{3}\pi\sigma_0^2)(a^2 - r^2), \quad (11)$$

$$m = \frac{64}{45}\pi^2\sigma_0^2 a^5 + \frac{8}{15}\pi\alpha q^2 a^5, \quad (12)$$

$$\rho = (\alpha q^2 + \frac{2}{3}\pi\sigma_0^2)(a^2 - r^2). \quad (13)$$

$$q = \frac{4}{3}\pi\sigma_0 a^3 \quad (14)$$

The value of  $\alpha$  can be obtained from the equation (12) as  $\alpha = -4.77 \times 10^{95} \text{cm}^{-6}$  and hence the energy density, as given by the equation (13) is negative for the radius of the electron  $a = 10^{-16} \text{cm}$ .

Now, from the equation (12) it can be seen that the effective gravitational mass,  $m$ , is of purely electromagnetic origin and corresponds to the TRK model [1984] with  $\alpha = 0$  case. This type

of models where mass, including all the other physical parameters, originates from the electromagnetic field alone are known as the electromagnetic mass models [EMMM] in the literature [Feynman et al., 1964] and have been investigated by several authors [Florides, 1962, 1983; Cooperstock and de la Cruz, 1978; Tiwari et al., 1984, 1986, 1991, 2000; Gautreau, 1985; Grøn, 1985, 1986a, 1986b; de Leon, 1987a, 1987b, 1988; Tiwari and Ray, 1991a, 1991b, 1997; Ray et al., 1993; Ray and Ray, 1993]. In the present paper we shall construct such a model within the framework of Einstein-Cartan theory with negative matter-energy density for some values of the radial coordinate.

### 3. THE FIELD EQUATIONS OF EINSTEIN-CARTAN THEORY

The EC field equations are given by

$$R^i_j - \frac{1}{2}\delta^i_j R = -\kappa t^i_j, \quad (15)$$

$$Q^i_{jk} - \delta^i_j Q^l_{lk} - \delta^i_k Q^l_{jl} = -\kappa S^i_{jk}, \quad (16)$$

where  $t^i_j$  is the canonical energy-momentum tensor (asymmetric),  $Q^i_{jk}$  is the torsion tensor and  $S^i_{jk}$  is the spin tensor (with  $\kappa = -8\pi$ ,  $G$  and  $c$  being unity in relativistic units ).

The asymmetric energy-momentum tensor here is given by

$$t^i_j = T^i_j + \frac{1}{2}g^{ik}\nabla_m(S^m_{jk}), \quad (17)$$

$\nabla_m$  being covariant derivative with respect to the torsionless, symmetric Levi-Civita connection  $\Gamma^i_{jk}$  and the symmetric energy-momentum tensor  $T^i_j$  will consist of two parts, viz., matter and electromagnetic tensors and which, respectively, are

$$T^i_j{}^{(m)} = (\rho + p)u^i u_j - pg^i_j, \quad (18)$$

$$T^i_j{}^{(em)} = \frac{1}{4\pi}(-F_{jk}F^{ik} + \frac{1}{4}\delta^i_j F_{kl}F^{kl}), \quad (19)$$

where  $\rho$  is the matter-energy density,  $p$  is the fluid pressure,  $u^i$  is the velocity four-vector (with  $u^i u_i = 1$ ) and  $F_{ij}$  is the electromagnetic field tensor.

The conservation equations for the EC theory can be given through the Bianchi identities as

$$\nabla_k[(\rho + p)u^k - g^{ki}u^l\nabla_m(u^m S_{li})] = u^j\nabla_j p, \quad (20)$$

$$[(\rho + p)u^k - g^{ki}u^l\nabla_m(u^m S^{li})]\nabla_k u_j = -\nabla_l(u^l u_j) + u^k S_{jm}R^m_k - \frac{1}{2}u^k S_{lm}R^{lm}_{jk} \quad (21)$$

Now, electromagnetic fields not being coupled with torsion [Novello, 1976; Raychaudhuri, 1979] the Maxwell equations as usual take the form

$$\nabla_j F^{ij} = J^i, \quad (22)$$

$$(J^i \sqrt{-g})_{,i} = 0. \quad (23)$$

The electromagnetic field tensor,  $F_{ij}$ , in the above equation (22) is related to the electromagnetic potentials as  $F_{ij} = A_{i,j} - A_{j,i}$  which is equivalent to  $F_{[i,j,k]} = 0$ ,  $A_i$  being the electrostatic potentials. Here and in what follows a comma denotes the partial derivative with respect to the coordinate indices involving the index.

Again, the spin tensor and the intrinsic angular momentum density tensor are related as

$$S^i{}_{jk} = u^i S_{jk}, \quad (24)$$

with

$$u^i S_{ik} = 0. \quad (25)$$

Now, assuming that the spins of the individual charged particles composing the fluid distribution are all aligned in the radial directions [Prasanna, 1975; Raychaudhuri, 1979; Tiwari and Ray, 1997] and the matter is at rest with respect to the observer, the non-vanishing components of the spin tensor can be obtained, from equations (24) and (25), as

$$S^0{}_{23} = -S^0{}_{32} = s(g_{00})^{-1/2}, \quad (26)$$

whereas, from equation (16), we have the torsion tensor as

$$Q^0{}_{23} = -Q^0{}_{32} = -\kappa s(g_{00})^{-1/2}, \quad (27)$$

$s = S_{23}$  being the only non vanishing component of the intrinsic angular momentum density tensor. Here, we have followed the convention  $(t, r, \theta, \phi) = (0, 1, 2, 3)$ .

The Einstein-Cartan-Maxwell equations with source then can be written as [Tiwari and Ray, 1997]

$$e^{-2\lambda} \left( \frac{2\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi\tilde{\rho} + E^2, \quad (28)$$

$$e^{-2\lambda} \left( \frac{2\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = 8\pi\tilde{p}_r - E^2, \quad (29)$$

$$e^{-2\lambda} \left[ \nu'' + \nu'^2 - \nu'\lambda' + \frac{(\nu' - \lambda')}{r} \right] = 8\pi\tilde{p}_\perp + E^2, \quad (30)$$

$$(r^2 E)' = 4\pi r^2 \sigma e^\lambda, \quad (31)$$

where  $\tilde{\rho}$ ,  $\tilde{p}_r$ ,  $\tilde{p}_\perp$  and  $E$  are the effective matter-energy density, effective pressures (radial and tangential) and electric field respectively, and are defined as

$$\tilde{\rho} = \rho - 2\pi s^2, \quad (32)$$

$$\tilde{p}_r = p_r - 2\pi s^2, \quad (33)$$

$$\tilde{p}_\perp = p_\perp - 2\pi s^2, \quad (34)$$

$$E = -exp[-(\nu + \lambda)]\phi' = \frac{q}{r^2}, \quad (35)$$

$\rho$ ,  $p_r$ ,  $p_\perp$ ,  $s$ ,  $\phi$  and  $q$  being the ordinary matter-energy density, ordinary pressures (radial and tangential), spin density, electrostatic potential and electric charge respectively. Here,  $\sigma$  denotes the electric charge density and prime is used for the derivative with respect to the radial coordinate  $r$ .

Also, the conservation equations (20) and (21) here reduce to

$$\frac{d\tilde{p}_r}{dr} = -(\tilde{\rho} + \tilde{p}_r)\nu' + \frac{1}{8\pi r^4} \frac{d}{dr}(q^2) + \frac{2(\tilde{p}_\perp - \tilde{p}_r)}{r}. \quad (36)$$

This is the key equation which is to be solved for constructing EMM.

#### 4. THE SOLUTIONS

Addition of (28) and (29), under the assumption  $g_{00}g_{11} = -1$  ( or equivalently, in terms of energy-momentum tensors  $T^0_0 = T^1_1$ ), provides the pure charge condition

$$\tilde{\rho} + \tilde{p}_r = 0, \quad (37)$$

where, in general,  $\tilde{\rho}$  is assumed to be positive and hence  $\tilde{p}_r$  is negative. However, as is evident from equation (32),  $\tilde{\rho}$ , being the effective energy-density, can even be negative due to the positive second term related to the spin on the right hand side. Thus, the possibility of equation (33) being satisfied with  $\tilde{p}_r$  being positive is not ruled out.

To make (31) and (36) solvable, we further assume that

$$\sigma e^\lambda = \sigma_0, \quad (38)$$

$$p_\perp - p_r = \tilde{p}_\perp - \tilde{p}_r = \alpha q^2 r^2 \quad (39)$$

following TRK Model [Tiwari et al., 1984] and HV model [Herrera and Varela, 1994] respectively, where  $\sigma_0$  and  $\alpha$  are two constants as mentioned earlier.

By substituting (37) – (39) in (31) and (36), we get

$$E = \frac{q}{r^2} = \frac{4}{3}\pi\sigma_0 r, \quad (40)$$

$$p_r = 2\pi s^2 - (\alpha q^2 + \frac{2}{3}\pi\sigma_0^2)(a^2 - r^2), \quad (41)$$

$$p_\perp = 2\pi s^2 - \alpha q^2(a^2 - 2r^2) - \frac{2}{3}\pi\sigma_0^2(a^2 - r^2), \quad (42)$$

$$\rho = 2\pi s^2 + (\alpha q^2 + \frac{2}{3}\pi\sigma_0^2)(a^2 - r^2). \quad (43)$$

The active gravitational mass

$$M(r) = 4\pi \int_0^r (\rho - 2\pi s^2 + \frac{E^2}{8\pi})r^2 dr \quad (44)$$

takes the form, by virtue of (40) and (43), as

$$M(r) = \frac{8}{135}\pi^2\sigma_0^2 r^3 [8\pi\alpha a^6(5a^2 - 3r^2) + 3(5a^2 - 2r^2)]. \quad (45)$$

Thus, the metric potentials  $\lambda$  and  $\nu$  are given by

$$e^{-2\lambda} = e^{2\nu} = 1 - \frac{2M(r)}{r}, \quad (46)$$

whereas, the effective gravitational mass in (1), can be obtained as

$$m = \frac{64}{45}\pi^2\sigma_0^2 a^5 (1 + \frac{2}{3}\pi\alpha a^6), \quad (47)$$

which corresponds to the second case (B) of HV model [1994] and of purely electromagnetic origin. This again corresponds to the TRK model [1984] with  $\alpha = 0$  case. It can be noted here that unlike the matter-energy density the effective gravitational mass is independent of spin.

In this context it is to be mentioned here that the junction conditions in the EC theory are different from that of general theory of relativity and indeed read like this [Arkuszewski et al., 1975]

$$n_i u^i |_- = 0 \quad (48)$$

$$p |_- = 2\pi G(n_i S^i) |_- \quad (49)$$

where  $S^i$  is the spin density pseudo-vector. Here condition (48) is the same as in classical relativistic hydrodynamics and has already been incorporated by matching the interior solution

to the exterior Reissner-Nordström field at the boundary of the spherical distribution. The condition (49), however, is the additional condition to be satisfied in EC theory. In the present case, it is only the effective pressure (radial) that vanishes on the boundary and not the ordinary radial pressure which, by virtue of equation (41), equals  $2\pi s^2$ . The spin is aligned in the radial direction and hence the spin density pseudo-vector is hypersurface orthogonal. Thus, the boundary condition (49) will become

$$p_r|_{r=a-0} = 2\pi s^2|_{r=a-0}, \quad (50)$$

which, depending on whether  $s$  is a constant or function of coordinates, will automatically be satisfied.

In this connection it is to be mentioned here that the spin density,  $s$ , in the final solutions (41) – (43) remains arbitrary (function of  $r$ ). An explicit functional form of this spin density can also be obtained by assuming some additional physically viable possibility, such as the one used by Prasanna [1975] by splitting the conservation equation into two parts, the second part relating to conservation of spin only, giving the functional form of spin density as  $s = s_0 e^{-\nu}$  (where  $s_0$  is the value of  $s$  at  $r = 0$  i.e. the central spin density). This can, using equations (38) and (46) and the condition  $g_{00}g_{11} = 1$ , (that is,  $\nu + \lambda = 0$ ) equivalently be written as  $s\sigma = s_0\sigma_0 = \text{constant}$ . The functional form of spin density is, however, not relevant in our discussion as our problem is concerned with the properties related to the ‘electron’, an elementary particle whose radius is of the order of  $10^{-16}$  cm. Indeed, as the spin function is arbitrary, there is no loss in generality, even if we assume it to be almost a constant (that is, the quantized value of the spin of the electron).

## 6. THE NEGATIVE ENERGY DENSITY MODEL

Let us have a closer observation of the results of the previous section 4. The equation (43) related to matter-energy density has the spin density part in the first term where spin density is defined as  $s = 3S/4\pi a^3$ , where  $S$  is the spin of electron the quantized value of which is  $S = \hbar/2$ . Then substituting the standard values for different parameters in the relativistic units, as mentioned in the introductory part, the numerical value for the matter-energy density (43) can be shown as

$$\rho = 6.14 \times 10^{-37} - 6.81 \times 10^{27}(10^{-32} - r^2). \quad (51)$$

The first term related to spin, being of the order of  $10^{-37}$ , is too small compared to the last term and hence the spin contribution is negligible. Now, the equation (51) indicates that the central

density at  $r = 0$  is negative and its magnitude is about  $10^{-5}$ . On the other hand, the total density at the boundary,  $r = a$ , is positive as usual with the numerical value about  $10^{-37}$ . This change in the sign of the energy density is because of the presence of the spin term in equation (52) which, indeed, is the contribution of the EC theory. In the absence of spin, however, we could have negative and zero densities at the centre and boundary of the electron respectively. This change in the sign again indicates that the central negative value gradually increases along the radius and somewhere, in the region  $0 < r_c < a$ , it becomes zero, where  $r_c$  is the critical radius. Obviously, the amount of negative energy density is less than its positive counterpart the balance of which ultimately provides the net density as the positive one.

It is already mentioned that though, in general, for any spherical fluid distribution the density on the surface should be zero we are getting here some non-zero value for it. This finite value is solely coming from the spin contributed part  $2\pi s^2$ . Thus for the vanishing spin the situation corresponds to the general behaviour (vide equation (17) of Herrera and Varela, 1994). In this context it is also to be noted here that up to the critical radius behaviour of our model is similar to that of Herrera and Varela [1994]. Beyond this cut off radius the energy density is regulated by spin which makes the overall density of the model as positive. This particular aspect lack in the model of Herrera and Varela [1994] where the total energy density is a negative quantity. This increase of matter-energy density due to spin density can probably be accounted for the kinetic energy through the angular motion of the electron here.

Similar kind of examination is also possible for the pressures, radial and tangential, both. The radial pressure, in this case, takes the form as

$$p = 6.14 \times 10^{-37} + 6.81 \times 10^{27}(10^{-32} - r^2). \quad (52)$$

However, pressure is throughout positive here from the centre to boundary. These results, i.e. negative energy density and positive pressure, are in accordance with the pure charge condition (37) which reads as  $\rho = -p_r + 4\pi s^2$ .

## 7. CONCLUSIONS

(i) The possible origin of the intriguing negative matter-energy density in the work of Cooperstock and Rosen [1989], Bonnor and Cooperstock [1989], Herrera and Varela [1994] and present paper may be due to the finiteness of the total mass of the Reissner-Nordström solution [Papapetrou, 1974; Visser, 1989]. Since the electrostatic energy of a point charge is infinite, the only way to produce a finite total mass is the presence of an infinite amount of negative energy at the

center of symmetry. According to Bonnor and Cooperstock [1989] the negativity of the energy density and hence the active gravitational mass is consistent with the phenomenon, known as the Reissner-Nordström repulsion [de la Cruz and Israel, 1967; Cohen and Gautreau, 1979; Tiwari et al., 1984; Cooperstock and Rosen, 1989]. In this regards Bonnor and Cooperstock [1989] also have discussed about the singularity theorems of general relativity [Hawking and Ellis, 1973]. They have shown that the negative regions are liable to exit over distances of order  $10^{-13}$  cm and as the proof of the singularity theorems depends on the manifold structure of space-time valid down to lengths of order  $10^{-15}$  cm so might break down below this. On the other hand, in the context of Einstein-Cartan theory of gravitation the idea of negative mass is not a new one as stated by Sabbata and Sivaram: "...torsion provides a natural framework for the description the negative mass under extreme conditions of such as in the early universe, when a transition from positive to negative mass can take place."

(ii) We have considered in the present paper an extended static spherically symmetric distribution of an elementary particle like electron having the radius of the order of  $10^{-16}$  cm, and even if for the finite size of the physical system the spin in Einstein-Cartan theory can be related to orbital rotation (which indeed is not the case), for systems of dimensions of subatomic particle the orbital rotation loses its meaning. In this case, the only way is to take the spin to be the 'intrinsic angular momentum', that is, the spin of quantum mechanical origin (in our problem since  $s$  is arbitrary, we can consider its quantized value or an average value). In this respect we would like to quote here from Hehl et al. [1974], "It is crucial to note that *spin* in  $U_4$  theory is canonical spin, that is, the *intrinsic* spin of elementary particles, not the so-called spin of galaxies or planets."

(iii) Following other authors [Prasanna, 1975; Raychaudhuri, 1979; Tiwari and Ray, 1997], in the present work the spins of all the individual particles are assumed to be oriented along the radial axis of the spherical systems. As to how this alignment is brought about is not very much clear. We have discussed here only a few possible ways of realizing this situation. According to Raychaudhuri [1979], in general, there will be an interaction between the spins of the particles and the magnetic field. The overall effect is the alignment of the spins. In this context, Prasanna [1975] mentions that such an alignment may be meaningful either in the case of spherical symmetry when magnetic field is present or else one has to consider axially symmetric field. As stated above, our view point is that in the case of physical systems of the size of the electron, the radial alignment of the spin is not ruled out. The solution obtained supports this view.

(iv) Though our present approach via Einstein-Cartan theory to inject spin may be interesting, we feel even that there should have some room to discuss the relationship of our work with an alternative means to provide spin and magnetic moment. This is, we think, possible through Dirac-Maxwell theory where spin and magnetic moment are naturally incorporated through the Dirac spin [Bohun and Cooperstock, 1999; Lisi, 1995] and would like to persue the problem in future investigations.

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