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On the electrogravity-dual solution to stringy charged black holes

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Abstract

By resolving the Riemann curvature tensor relative to a timelike unit vector into electric and magnetic parts, electrogravity-duality transformation is defined. Such a transformation interchanges the "active" and "passive" electric parts of this tensor and can be used to obtain spacetime metrics dual to known solutions of any geometric theory of gravity, such as Einstein gravity. The vacuum field equations of general relativity are invariant under such a transformation. It is possible to break this symmetry by introducing matter terms in such a way that the characteristic vacuum solution is still obtained as a special case of that matter distribution. Such a possibility exists for all stationary black hole solutions. Interestingly, solutions exist even for the dual equations with the matter terms. Here, we extend this formalism to study the static, charged black hole solutions of a four-dimensional low-energy effective action of heterotic string theory. We show that analogous to general relativity, the dual solution is itself a similar black hole spacetime, endowed with a global monopole charge.

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I. INTRODUCTION

It is well known that in general relativity, in analogy with Maxwell's theory of electromagnetism, one can extract the "electric" and "magnetic" parts of the gravitational field from the Riemann curvature tensor [1–3]. These parts are defined relative to a timelike unit vector field and are represented by second rank tensors in three-dimensional space. There are two types of electric parts, "active" and "passive". The former is obtained by projecting two of the indices of the Riemann tensor along the timelike unit vector, while the latter is obtained by a similar operation on the double-dual of the Riemann tensor (see section II below). Between them, the active and passive parts comprise of 6 independent components each since they correspond to symmetric tensors. An analogous projection of the left dual of the Riemann tensor yields a traceless second rank tensor in three space dimensions, which is called the magnetic part. It has 8 independent components. Therefore, the two electric parts and the magnetic part completely determine the Riemann tensor.

In general relativity (GR), a field transformation analogous to the electromagnetic-duality transformation in Maxwell's theory can be defined that keeps the Einstein-Hilbert action invariant. Such a transformation simultaneously maps the active electric part and the magnetic part into the magnetic part and minus the passive electric part, respectively. In fact, such a set of transformation equations implies the Einstein vacuum field-equations with a vanishing cosmological constant [4,5]. The electromagnetic-duality transformation in Maxwell's theory does not exhibit such a property since it involves only the fields, which are defined in terms of only the first derivative of the gauge potential. By contrast, the electric and magnetic parts of the gravitational field are obtained from projections of the Riemann tensor or its duals, and hence, contain second-order derivatives of the metric. These parts, therefore, can be appropriately combined to lay down the equations governing the dynamics of the gravitational field.

Another kind of duality transformation can be defined by interchange of the active and passive electric parts of the gravitational field [6]. Such a transformation is a symmetry of the Einstein-Hilbert action: It maps this action into itself, provided one simultaneously transforms the gravitational constant as, $G \rightarrow -G$. The duality transformation, therefore, leaves the vacuum field equations invariant [6]. The need to implement the change in sign of G in the dual solutions can be understood as follows. The source term for the active electric part stems from the matter stress tensor and can be argued to represent matter energy, while the passive part is associated with the gravitational-field energy. Since the vacuum field equations require that the sum of the active and passive parts vanish, the matter energy and the gravitational-field energy contributions should have opposite signs. Thus, G has to change sign under the interchange of active and passive electric parts. The field equations coupled to matter, however, are not duality invariant.

Importantly, the duality transformation can be used to infer new solutions from known ones. In this context, note that the duality equations transform the Ricci tensor into the Einstein tensor, and vice versa. This is because contraction over a pair of Riemann tensor indices yields the Ricci tensor while a similar contraction on its double dual yields the Einstein tensor. For some interesting vacuum solutions, it is possible to introduce matter terms in the field equations such that the original vacuum solution is still retained as a special case. However, since the field equations coupled to matter are not duality invariant, the solution to the dual equations will, in general, be different from the original vacuum solution. This is what happens for the static, spherically symmetric family of black hole solutions in GR. In fact, it has been shown that in such a case, a typical dual solution represents a black hole endowed with a global monopole charge. Here, we extend this formalism to study the static, charged black hole solutions of a four-dimensional (4D) low-energy effective action of heterotic string theory, namely, dilaton gravity coupled to a $U(1)$ gauge field. We show that analogous to general relativity, the dual solution is itself a similar black hole, endowed with a global monopole charge.

The layout of the paper is as follows. In section II, we define the electrogravity-duality transformation and show how the vacuum Einstein field equations remain unchanged under it. We briefly recapitulate how the solution dual to Schwarzschild can be obtained by implementing this transformation, in section III. In section V, we discuss the charged black hole solutions of dilaton gravity and recover its dual by effecting the duality transformation. A few thoughts on these solutions and scope for future work are presented in section VI. We work with the metric signature $(-,+,+,+)$ and employ geometrized units $G = 1 = c$.

II. ELECTROGRAVITY DUALITY

The electric and magnetic parts of the gravitational field in general relativity are defined as follows. Consider a timelike unit vector field u^a, with $u^a u_a = -1$. Then the active and passive parts of the Riemann tensor relative to u^a are

$$E_{ac} = R_{abcd} u^b u^d, \quad \tilde{E}_{ac} = *R*_{abcd}\, u^b u^d \quad , \tag{2.1}$$

respectively. Above, $*R*_{abcd}$ is the double-dual of the Riemann tensor given by:

$$*R*_{abcd} = \frac{1}{4} \epsilon_{abef} \epsilon_{cdgh} R^{efgh} \quad , \tag{2.2}$$

where ϵ_{abcd} is the canonical four-volume element of the spacetime. The magnetic part is the projection of left or right dual of the Riemann tensor and is given by

$$H_{ac} = -*R_{abcd} u^b u^d = H_{(ac)} - H_{[ac]} \quad , \tag{2.3}$$

where we have used the left-dual,

$$*R_{abcd} = \frac{1}{2} \epsilon_{abef} R^{ef}{}_{cd} \,. \tag{2.4}$$

Also, the symmetric and antisymmetric parts of H_{ac} can be expressed as:

$$H_{(ac)} = -*C_{abcd} u^b u^d \quad \text{and} \quad H_{[ac]} = -\frac{1}{2} \epsilon_{abce} R^e_d u^b u^d \quad , \tag{2.5}$$

where C_{abcd} is the Weyl tensor. Thus, the symmetric part is equal to the Weyl magnetic part, whereas the anti-symmetric part represents energy flux. Note that E_{ab} and $\tilde{E}_{ab}$ are both symmetric while H_{ac} is trace-free and they are all purely spacelike, i.e., $(E_{ab}, \tilde{E}_{ab}, H_{ab}) u^b = 0$. The Ricci tensor can then be expressed in terms of the electric and magnetic parts as

$$R_a^b = E_a^b + \tilde{E}_a^b - (E+\tilde{E})u_a u^b - \tilde{E} g_a^b - \frac{1}{2}(\epsilon_{amn} H^{mn} u^b + \epsilon^{bmn} H_{mn} u_a) \tag{2.6}$$

where $E = E_a^a$ and $\tilde{E} = \tilde{E}_a^a$.

The electrogravity-duality transformation is defined by an interchange of the active and passive parts of the electric field,

$$E_{ab} \longleftrightarrow \tilde{E}_{ab}, \; H_{ab} \longrightarrow -H_{ab}. \tag{2.7}$$

To see the effect of this transformation on vacuum solutions, note that the vacuum field equations, $R_{ab} = 0$, are in general equivalent to

$$E \text{ or } \tilde{E} = 0, \; H_{[ab]} = 0 = E_{ab} + \tilde{E}_{ab} \tag{2.8}$$

which is symmetric in E_{ab} and $\tilde{E}_{ab}$. Thus the vacuum field equations (2.8) are invariant under the duality transformation (2.7).

III. SCHWARZSCHILD DUAL

To set the notation and to aid the discussion of the static, charged black hole solution in string theory and its dual, we briefly study how one arrives at the dual of the Schwarzschild solution [6]. Birkoff's theorem tells us that the Schwarzschild solution, characterized by its mass, is the unique spherically symmetric solution to Einstein's vacuum field equations. Any spherically symmetric metric can be cast in the form:

$$ds^2 = -c^2(r,t)dt^2 + a^2(r,t)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{3.1}$$

A natural choice for the timelike vector u^a in this case is the timelike unit normal to t =constant hypersurfaces. Then, a subset of the conditions in Eqs. (2.8) that ensure a vacuum solution, namely, $H_{[ab]} = 0$ and $E_\theta^\theta + \tilde{E}_\theta^\theta = 0$ imply that $ac = 1$. A supplementary requirement of $\tilde{E} = 0$ yields $a = (1 - 2M/r)^{-1/2}$. This leads to the Schwarzschild solution. Here, it is important to note that we did not require to impose the remaining condition in Eqs. (2.8), namely, $E_r^r + \tilde{E}_r^r = 0$, in order to obtain this solution. In fact, in this case it is implied by the rest. We thus have a choice for introducing some matter distribution in the r-direction without affecting the Schwarzschild solution. We modify the vacuum field equations (2.8) to read as

$$H_{[ab]} = 0 = \tilde{E} \quad \text{and} \quad E_{ab} + \tilde{E}_{ab} = \lambda w_a w_b \tag{3.2}$$

where λ is a scalar and w_a is a spacelike unit vector parallel to the (radial) acceleration, namely, $\dot{u}_a = u^b \nabla_b u_a$. The above equations once again admit the Schwarzschild solution as the unique spherically symmetric solution with $\lambda = 0$.

We now perform the duality transformation (2.7) on the above set of equations (3.2) to write:

$$H_{[ab]} = 0 = E\,, \quad E_{ab} + \tilde{E}_{ab} = \lambda w_a w_b\,. \tag{3.3}$$

Its general solution is given by the metric (3.1) with

$$c = a^{-1} = (1 - 2k - \frac{2M}{r})^{1/2} \ , \tag{3.4}$$

which is the Barriola-Vilenkin solution [7] for a Schwarzschild particle with global monopole charge parameter, $\sqrt{2k}$. Again we obtain $ac = 1$. Then, the condition $E = 0$ yields $c = (1 - 2k - 2M/r)^{1/2}$ and $\lambda = 2k/r^2$. This has non-zero stresses given by

$$T_t^t = T_r^r = \frac{2k}{r^2}. \tag{3.5}$$

Just like the Schwarzschild solution, the monopole solution (3.4) is also the unique solution of Eq. (3.3). Note that in the above case, since the spacetime is not vacuum, we had to interchange the Ricci and Einstein tensors to effect the duality transformation. In fact, for SSS line-elements there is a general prescription to obtain the dual solution. For details, we refer the reader to Ref. [8].

IV. GLOBAL MONOPOLE

Global monopoles are stable topological defects. They are supposed to be produced when global symmetry is spontaneously broken in phase transitions in the early Universe [9]. A global monopole is described by an isoscalar triplet, ψ^a, with $a = 1, 2, 3$. The associated Lagrangian density is [7]:

$$L_m = \frac{1}{2}(\nabla\psi^a)^2 + \frac{\lambda}{4}(\psi^a\psi_a - \eta^2)^2 \ . \tag{4.1}$$

Such a system has a global $O(3)$ symmetry and offers topologically non-trivial self-supporting solutions. The global monopole is obtained by implementing the ansatz that $\psi^a(r) = \eta f(r) x^a/r$, where $x_a x^a = r^2$. Here η is a constant whose value defines the energy scale of symmetry breaking.

For a given spacetime metric, the stress tensor associated with a global monopole can be inferred from the above Lagrangian density in a standard manner. Consider the SSS metric (3.1) with c and a independent of t. Then x^a is interpreted as a "Cartesian" coordinate, and the field equation for ψ^a reduces to the following equation for $f(r)$.

$$\frac{1}{a^2}f'' + \left[\frac{2}{ra^2} + \frac{1}{2c^2}\left(\frac{c^2}{a^2}\right)'\right] f' - \frac{2f}{r^2} - \lambda\eta^2 f(f^2 - 1) = 0 \, . \tag{4.2}$$

The stress-tensor components of the monopole are:

$$\begin{aligned} T_t^t &= \frac{\eta^2 f'^2}{2a^2} + \frac{\eta^2 f^2}{r^2} + \frac{1}{4}\lambda\eta^4(f^2-1)^2 \ , \\ T_r^r &= -\frac{\eta^2 f'^2}{2a^2} + \frac{\eta^2 f^2}{r^2} + \frac{1}{4}\lambda\eta^4(f^2-1)^2 \ , \\ T_\theta^\theta &= T_\phi^\phi = \frac{\eta^2 f'^2}{2a^2} + \frac{1}{4}\lambda\eta^4(f^2-1)^2 \, . \end{aligned} \tag{4.3}$$

The monopole core is defined by values of r for which $f(r) \approx 1$. Outside and at large distances from the monopole core the stresses would approximate to [10]

$$T_r^r \approx T_t^t \approx \frac{\eta^2}{r^2}, \quad T_\theta^\theta = T_\phi^\phi \approx 0 \ , \tag{4.4}$$

which is precisely of the form given in Eq. (3.5).

The dual solution to flat spacetime can be obtained as follows. Note that flat spacetime is a solution to the following equations of motion:

$$\tilde{E}_{ab} = 0 = H_{[ab]} \ , \quad E_{ab} = \lambda w_a w_b \ , \tag{4.5}$$

which are solved to give $c = a = 1$. As before, the condition $\lambda = 0$ is implied by the fact that such a solution corresponds to an isotropic spacetime. Its dual is the solution of the equation dual to (4.5), which reads as

$$E_{ab} = 0 = H_{[ab]}, \quad \tilde{E}_{ab} = \lambda w_a w_b \tag{4.6}$$

yielding the general solution,

$$c' = a' = 0 \Longrightarrow c = 1, \ a = (1 - 2k)^{-1/2} = \text{constant}. \tag{4.7}$$

The resulting spacetime is non-flat and represents a global monopole of zero mass. Note that such a spacetime is the same as the one described by Eq. (3.4) in the limit of vanishing mass M. This could as well be considered as a spacetime of constant relativistic potential. Note that it is non-flat but would be gravity-free in the Newtonian limit. It can hence be envisioned as "minimally" curved spacetime [5,6]. The electrogravity-duality thus generates topological defects in the vacuum/flat solutions of the Einstein equation, which is a remarkable property of this transformation [6].

To summarize, the above procedure for obtaining solutions dual to any known spacetime solution would work as long as there occurs a free equation in the field equations (2.8) that is not used in finding that solution. Note that this holds for all solutions in the family of charged Kerr black holes [11,12] as well as for the NUT solution [13]. Then the dual set admits a solution similar to the original one, but with a topological defect, namely, global monopole charge.

V. 4D DILATON GRAVITY

In the spirit of the Barriola-Vilenkin solution (3.4), one may expect analogous solutions to exist even in some scalar-tensor theories of gravity. A particular class of candidates among these classical theories, which are posed as leading alternatives to general relativity, are the 4D low-energy effective theories derived from heterotic string theory. Here, we consider the specific case of 4D dilaton gravity action coupled to a $U(1)$ gauge field, which has charged black hole solutions [14,15] (see Refs. [16,17] for reviews):

$$S = \frac{1}{16\pi} \int d^4x \, \sqrt{-\bar{g}} \left\{ e^{-2\phi} [\bar{R} + 4(\bar{\nabla}\phi)^2 - 2\Lambda] - \bar{F}^2 \right\} \ , \tag{5.1}$$

where $\bar{g}_{\mu\nu}$ is the string metric, $\bar{R}$ is the 4D Ricci scalar, Λ is a cosmological constant and $\bar{F}_{\mu\nu}$ is the Maxwell field associated with a U(1) subgroup of $E_8 \times E_8$. Here, we shall consider the case where $\Lambda = 0$. The conformal transformation $g_{\mu\nu} = e^{-2\phi}\bar{g}_{\mu\nu}$ can be implemented to recast the above action in the "Einstein-Hilbert" form:

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\left[R - 2(\nabla\phi)^2 - e^{-2\phi}F^2\right] \quad , \tag{5.2}$$

where $g_{\mu\nu}$ is the Einstein-frame metric. The corresponding equations of motion are:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 2\nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}(\nabla\phi)^2 - 2e^{-2\phi}F_{\mu\lambda}F_\nu^\lambda + \frac{1}{2}g_{\mu\nu}e^{-2\phi}F^2 = 0 \quad , \tag{5.3a}$$

$$2\nabla^2\phi + e^{-2\phi}F^2 = 0 \quad , \tag{5.3b}$$

$$\nabla^\nu(e^{-2\phi})F_{\mu\nu} = 0 \, . \tag{5.3c}$$

Taking the trace of (5.3a) gives:

$$R = 2(\nabla\phi)^2 \quad , \tag{5.4}$$

which, together with (5.3a) and (5.3c), implies

$$R_{\mu\nu} = 2\nabla_\mu\phi\nabla_\nu\phi + 2e^{-2\phi}F_{\mu\lambda}F_\nu^\lambda + g_{\mu\nu}\nabla^2\phi \, . \tag{5.5}$$

This equation will play a pivotal role below in our study of charged black hole solutions and their duals.

If a matter action, say, corresponding to the Lagrangian density (4.1) of a global monopole, were present in Eq. (5.2), then a matter stress tensor following from such a term would contribute to the right-hand side of the field equation (5.3a). Since it is not known how the dilaton couples to the monopole field ψ^a, one can obtain different theories incorporating ψ^a based on the choice of this coupling. Global monopoles in 4D dilaton gravity have been studied for some choices of coupling and for both massive and massless dilaton in Ref. [18]. We will later consider solutions to action (5.2) coupled to ψ^a by adding to it the matter action

$$S_m = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\,L_m \quad , \tag{5.6}$$

where L_m is given in Eq. (4.1). Such a choice of coupling is different from those considered in Ref. [18].

We begin by briefly recalling how the above equations of motion are solved to obtain the charged black hole solution. Consider the spherically symmetric static (SSS) metric

$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + h^2d\omega^2 \quad , \tag{5.7}$$

where ν, λ, and h are functions of the radial coordinate r only. For such a metric the only non-vanishing components of the Ricci tensor are the diagonal elements. For Eq. (5.5) to have an SSS solution, the following conditions on R_{tt} and R_{rr} must be obeyed:

$$R_{tt} = e^{2(\nu-\lambda)}\left\{\nu'' + \nu'^2 - \lambda'\nu' + \frac{2\nu'}{r}\right\} - 2\nu' e^{2(\nu-\lambda)}\left[\frac{1}{r} - \frac{h'}{h}\right] = e^{2\nu-2\phi}\frac{Q^2}{h^4} \;, \tag{5.8a}$$

$$R_{rr} = \left\{-\nu - \nu'^2 + \lambda'\nu' + \frac{2\lambda'}{r}\right\} - 2\lambda'\left[\frac{1}{r} - \frac{h'}{h}\right] - \frac{2h''}{h} = \left[2\phi'^2 - e^{2\lambda-2\phi}\frac{Q^2}{h^4}\right] . \tag{5.8b}$$

Similarly, the field equation for the component $R_{\theta\theta}$,

$$\left\{1 + e^{-2\lambda}\left[r(\lambda' - \nu') - 1\right]\right\} + e^{-2\lambda}\left[(hh' - r)(\lambda' - \nu') - (hh'' + h'^2) + 1\right] = e^{-2\phi}\frac{Q^2}{h^2} \;, \tag{5.9}$$

must also be satisfied. Spherical symmetry ensures that the $R_{\phi\phi}$ equation implies the same condition on ν, λ, and h as in Eq. (5.9). If $Q = 0$, then the right-hand side of the above equation vanishes. Therefore, the following expressions,

$$e^{2\nu} = e^{-2\lambda} = 1 - 2m/r \,, \quad \text{and} \quad h = r \tag{5.10}$$

constitute a solution to the above equations. This simply corresponds to the Schwarzschild metric, which indeed is a solution to the equations of motion (5.3) with $F_{\mu\nu} = 0$ and $\phi =$ constant.

Finding an SSS metric as a solution to (5.8) and (5.9) for $Q \neq 0$ is also straightforward. Note that ν and λ given in Eq. (5.10) makes the braces on the left-hand sides of Eqs. (5.8) and (5.9) vanish. Thus, the problem reduces to finding an h that makes the remaining term on the left-hand sides of Eqs. (5.8) and (5.9) equal to their right-hand sides, respectively. Such an h exists and is given by

$$h^2 = r^2\left(1 - \frac{Q^2}{mr}\right) . \tag{5.11}$$

This, therefore, constitutes the charged black hole solution of 4D dilatonic gravity. The corresponding fields are:

$$ds^2 = -\left(1 - \frac{2m}{r}\right)dt^2 + \left(1 - \frac{2m}{r}\right)^{-1}dr^2 + r^2\left(1 - \frac{Q^2}{mr}\right)d\omega^2 . \tag{5.12}$$

$$e^{-2\phi} = \left(1 - \frac{Q^2}{mr}\right) = U(\phi) \;, \tag{5.13}$$

$$F_{rt} = \frac{Q}{r^2} . \tag{5.14}$$

In obtaining the above solution from the equations of motion, we have assumed that $\phi \to 0$ as $r \to \infty$.

The effect of the electrogravity-duality transformation on the field equations (5.3a) is to modify them by the addition of the asymptotic form of the global-monopole stress tensor (3.5) on its right-hand side. This is completely analogous to what happens in the case of the Schwarzschild black hole in GR (see section III). It, therefore, follows that the following new choices for ν and λ will solve such a set of equations:

$$e^{2\nu} \to e^{2\tilde{\nu}} = 1 - 8\pi\eta^2 - \frac{2\tilde{m}}{r} \ ,$$
$$e^{2\lambda} \to e^{2\tilde{\lambda}} = 1 - 8\pi\eta^2 - \frac{2\tilde{m}}{r} \ , \tag{5.15}$$

where $\tilde{m}$ is just an integration constant. In other words, for the choice in (5.15), the braces on the lhs of Eqs. (5.8) and (5.9) are exactly equal to the components of the global-monopole stress-tensor. This suggests the possibility that there exists an $h \to \tilde{h}$ and $\phi \to \tilde{\phi}$, for which $\tilde{\nu}$ and $\tilde{\lambda}$ solve the field equations (5.8) and (5.9) modified by the presence of source terms arising from the global monopole stress tensor.

In fact, it turns out that such a choice for h is available. This can be understood by noting that $\tilde{\nu}$ and $\tilde{\lambda}$ can be cast in the same form as (5.10):

$$e^{2\tilde{\nu}} = e^{-2\kappa}\left(1 - \frac{2m}{r}\right) = e^{-2\tilde{\lambda}} \ , \tag{5.16}$$

where $e^{-2\kappa} = (1 - 8\pi\eta^2)$ and $\tilde{m} = e^{-2\kappa}m$. Using such scaling relations between tilded and untilded variables, it is easy to see that

$$\tilde{h}^2 = r^2\left(1 - \frac{Q^2}{\tilde{m}r}\right) \ , \quad \text{and} \quad e^{-2\tilde{\phi}} = \left(1 - \frac{Q^2}{\tilde{m}r}\right) \ . \tag{5.17}$$

Calling $\tilde{m} = M$, we finally arrive at the metric of the spacetime dual to the charged dilatonic black holes:

$$ds^2 = -\left(1 - 8\pi\eta^2 - \frac{2M}{r}\right)dt^2 + \left(1 - 8\pi\eta^2 - \frac{2M}{r}\right)^{-1}dr^2 + r^2\left(1 - \frac{Q^2}{Mr}\right)d\omega^2 \ . \tag{5.18}$$

This is a solution to the modified equations of motion. The corresponding dilaton and $U(1)$ field solutions are given by Eqs. (5.17) and (5.14), respectively. The resulting field configuration solves the equations of motion (5.3). In the limit $Q = 0$, one recovers the Barriola-Vilenkin spacetime. This is expected since in that limit the dilaton field acquires a constant value. Consequently, 4D dilaton gravity reduces to general relativity. Additionally, if $M = 0$, then the above metric describes a locally flat spacetime with a global monopole charge, which is the electrogravity dual of flat spacetime.

It, however, remains to be shown that the stress tensor (3.5) is indeed the asymptotic form of the global monopole stress tensor arising from (5.6) for the above spacetime metric. To see that this is indeed true, we cast the above metric as

$$\begin{aligned} ds^2 = & -\left(1 - \frac{4M^2}{Q^2 + \sqrt{1 + \frac{4M^2\rho^2}{Q^4}}}\right)dt^2 \\ & + \frac{1}{4}\left(1 + \frac{Q^4}{4M^2\rho^2}\right)^{-1}\left(1 - \frac{4M^2}{Q^2 + \sqrt{1 + \frac{4M^2\rho^2}{Q^4}}}\right)^{-1}d\rho^2 + \rho^2 d\omega^2 \ , \end{aligned} \tag{5.19}$$

where $\rho^2 = r^2 - rQ^2/M$. This metric is of the same form as Eq. (3.1) with r replaced by ρ there. Using Eqs. (4.3) to compute the matter stress-tensor components gives

$$T^t_t \approx T^\rho_\rho \approx \eta^2/\rho^2 \,, \quad T^\theta_\theta = T^\phi_\phi \approx 0 \;\;, \tag{5.20}$$

outside the monopole core. In the limit of large r these go over to the expected stress tensor components (4.4) of a global monopole. Moreover, it is straightforward to verify that the field equation for $f(\rho)$, which is given by Eq. (4.2) with r replaced by ρ, can be solved asymptotically with $f(\rho) \approx 1$ outside the core.

VI. DISCUSSION

An important observation made in Ref. [5] was that as long as one of the set of Einstein field equations remains unused in obtaining a particular solution, one can obtain a dual solution, which is different from the original, by modifying that equation. The modification is to introduce the term on the right in Eqs. (3.2), where the spacelike unit vector w_a is along the 4-acceleration vector. By this prescription solutions dual to all isolated sources have been obtained. The next question is: How good are Eqs. (3.2) as "non-vacuum" field equations? The first part of the equation implies vanishing of energy density (i.e., $\tilde{E} = 0$) and of energy flux (i.e., $H_{[ab]} = 0$). This means that they cannot have a physically meaningful non-vacuum solution. In the case of spacetimes corresponding to non-localized sources, such as plane gravitational waves, the Kasner solution, the Weyl and the Levi-Civita metrics, it turns out that Eqs. (3.2) (with $\lambda w_a w_b$ replaced by $\lambda(g_{ab} + u_a u_b)$ for the homogeneous case) admits them as solutions, and so does its dual equation [19]. Thus, such solutions describing non-localized sources are electrogravity self-dual.

Since electrogravity duality transformation only involves the Riemann tensor and hence is quite general. It should be applicable in other metric theories as well. Here we have seen its application to black hole solutions in the 4D low-energy effective heterotic string theory. The procedure works exactly along the lines of GR and we obtain the dual solution to static charged stringy black hole which describes the black hole with the global monopole charge. Thus we can make the general statement that *by implementing the electrogravity duality transformation we can always put a global monopole charge on a static spherically symmetric black hole in GR or in 4D dilaton gravity.* This is because as in GR, even in certain string-inspired 4D low-energy effective theories, the black hole sector of their solution space is determined by only a subset of the corresponding field equations. In fact this is true even in lower dimensional Einstein-gravity, e.g., the theory corresponding to the 3D Einstein-Hilbert action involving a cosmological constant term. Hence, these theories, which have played an important role in this decade in the understanding of black hole physics and related quantum aspects, also hold the promise of harboring yet unknown solutions with topological defects that may play an important role in alternative cosmological models. We are currently involved in studying these solutions [20].

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