

# Spinning Higher Dimensional Einstein-Yang-Mills black holes

Sushant G. Ghosh<sup>a, b \*</sup> and Uma Papnoi<sup>a †</sup>

<sup>a</sup> *Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi 110025, India and*

<sup>b</sup> *Astrophysics and Cosmology Research Unit, School of Mathematical Sciences, University of Kwa-Zulu-Natal, Private Bag 54001, Durban 4000, South Africa*

(Dated: May 30, 2013)

We construct a Kerr-Newman-like spacetimes starting from higher dimensional (HD) Einstein-Yang-Mills black holes via complex transformations suggested by Newman-Janis. The new metrics are HD generalization of Kerr-Newman spacetimes which has a geometry precisely that of Kerr-Newman in 4D corresponding to Yang-Mills (YM) gauge charge, but the sign of charge term gets flipped in the HD spacetimes. It is interesting to note that gravitational contribution of YM gauge charge, in HD, is indeed opposite (attractive rather than repulsive) that of Maxwell charge. The effect of YM gauge charge on the structure and location of static limit surface and apparent horizon is discussed. We find that static limit surfaces become less prolate with increase in dimensions and are also sensitive to YM gauge charge thereby affecting the shape of ergosphere. We also analyze some thermodynamical properties of these BHs.

PACS numbers: 04.70.Bw, 12.10.-g, 12.15.-y

## I. INTRODUCTION

The Kerr metric [1] is an explicit exact solution of Einstein field equations describing a spinning black hole (BH) in four dimensional (4D) spacetime. It is well known that BH with non-zero spinning parameter, i.e., Kerr BH enjoys many interesting properties distinct from its non-spinning counterpart, i.e., from Schwarzschild BH [2]. However, there is a surprising connection between the two BHs of Einstein theory, and is analyzed by Newman and Janis [3]. They demonstrated that applying a complex coordinate transformation, it was possible to construct both the Kerr and Kerr-Newman solutions starting from the Schwarzschild metric and Reissner-Nordström metric respectively [3]. The Kerr-Newman describes the exterior of a spinning massive charged BH [4]. The Newman Janis Algorithm (NJA) is successful in analysing several spinning BH metric starting from their non-spinning counterparts [5–17]. For a review on the NJA (see, e.g., [18]). However, The NJA has often be considered that there is arbitrariness in the choice of complex function of term in the original seed metric [19]. But Schiffer et. al. [20] gave a very elegant mathematical proof as why Kerr metric can be considered as complex transformation of Schwarzschild metric.

It is rather well established that higher dimensions provide a natural playground for the string theory and they are also required for its consistency [21, 22]. Even from the classical standpoint, it is interesting to study the higher dimensional (HD) extension of Einstein's theory, and in particular its BH solutions [23]. There seems to be a general belief that endowing general relativity with a tunable parameter namely the spacetime dimen-

sion should also lead to valuable insights into the nature of the theory, in particular into its most basic objects: BHs. For instance, 4D BHs are known to have a number of remarkable features, such as uniqueness, spherical topology, dynamical stability, and the laws of BH mechanics. One would like to know which of these are peculiar to 4D, and which are true more generally? At the very least, such probings into HD will lead to a deeper understanding of classical BHs and of what space-time can do at its most extreme. There is a growing realization that the physics of HD BHs can be markedly different, and much richer than its counterpart in 4D [2, 24–26]. It is of interest to consider models based on different interacting fields including the Yang-Mills (YM). In general, it is difficult to tackle Einstein-Yang-Mills (EYM) equations because of the non-linearity both in the gauge fields as well as in the gravitational field. The solutions of the classical YM fields depend upon the particular *ansatz* one chooses. Wu and Yang [27] found static spherically symmetric solutions of the YM equations in flat space for the gauge group  $SO(3)$ . A curved spacetime generalization of these models has been investigated by several authors (see, e.g., [28]). Indeed Yasskin [28] has presented an explicit procedure based on the Wu-Yang *ansatz* [27] which gives the solution of EYM rather trivially. Using this procedure, Mazharimousavi and Halilsoy [29–31] have found a sequence of static spherically symmetric HD EYM BH solutions. The remarkable feature of this *ansatz* is that the field has no contribution from gradient; instead, it has pure YM non-Abelian component. It, therefore, has only the magnetic part.

The main purpose of this work is to apply NJA to HD EYM BH metric previously discovered in [29–31] and spinning HD EYM BH metric is obtained. This result shows that NJA works well also in HD spacetime. We further discuss the properties of the spinning HD EYM BH such as horizons and ergosphere. Spinning BH solutions in higher dimensions are known as Myers-Perry

---

\*Electronic address: sghosh2@jmi.ac.in, sgghosh@gmail.com

†Electronic address: uma.papnoi@gmail.com

BHs [26]. The thermodynamical quantities associated with the spinning HD EYM BH are also calculated. Further we demonstrate that the thermodynamical quantities of this BH go over to corresponding quantities of Myers-Perry BH and Kerr BH.

## II. HIGHER DIMENSIONAL STATIC BLACK HOLE IN EINSTEIN-YANG-MILLS THEORY

We consider  $(N+1)(N+2)/2$  parameter Lie group with structure constant  $C_{(\beta)(\gamma)}^{(\alpha)}$ . The gauge potentials  $A_a^{(\alpha)}$  and the YM fields  $F_{ab}^{(\alpha)}$  are related through the equation

$$F_{ab}^{(\alpha)} = \partial_a A_b^{(\alpha)} - \partial_b A_a^{(\alpha)} + \frac{1}{2\sigma} C_{(\beta)(\gamma)}^{(\alpha)} A_a^{(\beta)} A_b^{(\gamma)}. \quad (1)$$

Then one can choose the gravity and gauge field action (EYM), which in  $(N+3)$ -dimensions reads [29, 30, 32]:

$$\mathcal{I}_G = \frac{1}{2} \int_M dx^{N+3} \sqrt{-g} \left[ R - \sum_{\alpha=1}^{(N+1)(N+2)/2} F_{ab}^{(\alpha)} F^{(\alpha)ab} \right]. \quad (2)$$

Here,  $g = \det(g_{ab})$  is the determinant of the metric tensor,  $R$  is the Ricci Scalar and  $A_a^{(\alpha)}$  are the gauge potentials. We note that the internal indices  $\{\alpha, \beta, \gamma, \dots\}$  do not differ whether in covariant or contravariant form. We introduce the Wu-Yang *ansatz* in  $(N+3)$ -dimension [29–31] as

$$\begin{aligned} A^{(\alpha)} &= \frac{Q}{r^2} (x_i dx_j - x_j dx_i) \\ 2 &\leq i \leq N+2, \\ 1 &\leq j \leq i-1, \\ 1 &\leq (\alpha) \leq (N+1)(N+2)/2, \end{aligned} \quad (3)$$

where the super indices  $\alpha$  is chosen according to the values of  $i$  and  $j$  in order and we choose  $\sigma = Q$  [29–31]. The Wu-Yang solution appears highly non-linear because of mixing between spacetime indices and gauge group indices. However, it is linear as expressed in the non-linear gauge fields because purely magnetic gauge charge is chosen along with position dependent gauge field transformation [28]. The YM field 2 form is defined by the expression

$$F^{(\alpha)} = dA^{(\alpha)} + \frac{1}{2Q} C_{(\beta)(\gamma)}^{(\alpha)} A^{(\beta)} \wedge A^{(\gamma)}. \quad (4)$$

The integrability conditions

$$dF^{(\alpha)} + \frac{1}{Q} C_{(\beta)(c)}^{(\alpha)} A^{(\beta)} \wedge F^{(\gamma)} = 0, \quad (5)$$

as well as the YM equations

$$d * F^{(\alpha)} + \frac{1}{Q} C_{(\beta)(\gamma)}^{(\alpha)} A^{(\beta)} \wedge * F^{(\gamma)} = 0, \quad (6)$$

are all satisfied. Here  $d$  is exterior derivative,  $\wedge$  stands for wedge product and  $*$  represents Hodge duality. All these are in the usual exterior differential forms notation.

Variation of the action with respect to the space-time metric  $g_{ab}$  yields the EYM equations

$$G_{ab} = T_{ab}. \quad (7)$$

where the gauge stress-energy tensor is

$$T_{ab} = \sum_{\alpha=1}^{(N+1)(N+2)/2} \left[ 2F_a^{(\alpha)\lambda} F_{b\lambda}^{(\alpha)} - \frac{1}{2} F_{\lambda\sigma}^{(\alpha)} F^{(\alpha)\lambda\sigma} g_{ab} \right]. \quad (8)$$

In general, it is difficult to solve EYM Eq. (7). However, the Wu-Yang *ansatz* [27] facilitate in obtaining the solution.

The metric for the HD EYM BH [31] obtained using Wu-Yang *ansatz* [27] is given by

$$ds^2 = f(r) dt^2 - f(r)^{-1} dr^2 - r^2 d\Omega_{N+1}^2, \quad (9)$$

with

$$f(r) = 1 - \frac{\mu}{r^N} - \frac{N}{(N-2)} \frac{Q^2}{r^2}, \quad N \neq 2,$$

where

$$\begin{aligned} d\Omega_{N+1}^2 &= d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \sin^2 \theta_1 \sin^2 \theta_2 d\theta_3^2 + \dots \\ &+ \left[ \left( \prod_{j=1}^N \sin^2 \theta_j \right) d\theta_{N+1}^2 \right], \end{aligned}$$

where,  $\mu$  is the integration constant which can be related to mass  $M$  and  $D = N+3$  is spacetime dimensions. Since  $T_{ab}$  go as  $r^{-4}$  (same as for Maxwell field in 4D), interestingly for all  $D \geq 6$ . That is why its contribution in  $f(r)$  is same for all  $D \geq 6$  as in Reissner-Nordström. There is, however, an important difference in the sign before  $Q^2/r^2$  term. In 4D case, it is exactly like Reissner-Nordström BH, i.e., positive. In contrast, to 4D, the sign before  $Q^2/r^2$  is negative for  $D \geq 6$ . On the other hand, if YM gauge charge is switched off ( $Q = 0$ ), the metric (9) reduces to well known Schwarzschild-Tangherlini metric [2]. In addition if  $N = 1$ , one may note that it reduces to Schwarzschild metric. When  $N = 1$ , the metric (9) is exactly Reissner-Nordström BH with  $Q$  as YM gauge charge.

## III. SPINNING HD EYM BH VIA NEWMAN-JANIS ALGORITHM

We want to derive axially symmetric spinning analogue of the static spherically symmetric EYM BH adapting Newman-Janis [3] complex transformation. Newman et al. [3] discovered curious derivations of stationary, spinning metric solutions from static, spherically symmetric

solutions in 4D Einstein theory. In order to derive spinning HD EYM BH, we start with the non-spinning version of the HD EYM BH metric (9), with  $f(r)$  given by

$$f(r) = 1 - \frac{\mu}{r^N} - \frac{Q_0^2}{r^2}, \quad Q_0^2 = \frac{N}{(N-2)}Q^2,$$

as a seed solution to construct its spinning counterpart. Following Newman and Janis [3] the first step is to write the metric (9) in advanced Eddington-Finkelstein coordinates by the following coordinate transformation:

$$du = dt - f(r)^{-1}dr, \quad (10)$$

we obtain

$$ds^2 = f(r)du^2 + 2dudr - r^2d\Omega_{N+1}^2. \quad (11)$$

The metric (11) can be written in terms of a null vieltrrad  $Z^a = (l^a, n^a, m_1^a, \bar{m}_1^a, m_2^a, \bar{m}_2^a, \dots, m_{(N+1)/2}^a, \bar{m}_{(N+1)/2}^a)$ , [3, 14] as

$$g^{ab} = l^a n^b + l^b n^a - m_1^a \bar{m}_1^b - m_2^a \bar{m}_2^b, \dots, \quad (12)$$

$$-m_{(N+1)/2}^a \bar{m}_{(N+1)/2}^b,$$

where null vieltrrad are

$$l^a = \delta_r^a,$$

$$n^a = \left[ \delta_u^a - \frac{1}{2} \left( 1 - \frac{\mu}{r^N} - \frac{Q_0^2}{r^2} \right) \delta_r^a \right],$$

$$m_k^a = \frac{1}{\sqrt{2}r \sin \theta_1 \sin \theta_2, \dots, \sin \theta_{(k-1)}} \left( \delta_{\theta_k}^a + \frac{i}{\sin \theta_k} \delta_{\theta_{(k+1)}}^a \right),$$

where  $D = N + 3$ , is spacetime dimension and  $k$  is used to denote the number of vector which take values  $1, 2, 3, \dots, (N+1)/2$ , e.g., in 6D  $k$  is 2 which corresponds to following vector

$$m_1^a = \frac{1}{\sqrt{2}r} \left( \delta_{\theta_1}^a + \frac{i}{\sin \theta_1} \delta_{\theta_2}^a \right),$$

$$m_2^a = \frac{1}{\sqrt{2}r \sin \theta_1} \left( \delta_{\theta_2}^a + \frac{i}{\sin \theta_2} \delta_{\theta_3}^a \right).$$

Here we assume  $N$  is odd, i.e., spacetime dimension  $D$  is even. However, the final result is independent of this assumption. Note  $l^a$  and  $n^a$  are real,  $m_k^a, \bar{m}_k^a$  are mutual complex conjugate. This vieltrrad is orthonormal and obeying metric conditions.

$$l_a l^a = n_a n^a = (m_k)_a (m_k)^a = (\bar{m}_k)_a (\bar{m}_k)^a = 0,$$

$$l_a (m_i)^a = l_a (\bar{m}_i)^a = n_a (m_k)^a = n_a (\bar{m}_k)^a = 0,$$

$$l_a n^a = 1, \quad (m_k)_a (\bar{m}_k)^a = 1.$$

Now we allow for some  $r$  factor in the null vectors to take on complex values. Following [5, 11], we rewrite the null

vectors in the form

$$l^a = \delta_r^a,$$

$$n^a = \left[ \delta_u^a - \frac{1}{2} \left( 1 - \frac{\mu}{2r^{N-1}} \left[ \frac{1}{r} + \frac{1}{\bar{r}} \right] - \frac{Q_0^2}{r\bar{r}} \right) \delta_r^a \right],$$

$$m_k^a = \frac{1}{\sqrt{2}\bar{r} \sin \theta_1 \sin \theta_2 \dots \sin \theta_{(k-1)}} \left( \delta_{\theta_k}^a + \frac{i}{\sin \theta_k} \delta_{\theta_{(k+1)}}^a \right),$$

with  $\bar{r}$  being the complex conjugate of  $r$ . In 4D there is only one possible spinning axisymmetric spacetime, and there is therefore only one angular momentum parameter. In HD there are several choices of spinning axis and there is a multitude of angular momentum parameters, each referring to a particular spinning plane. We concentrate on the simplest case for which there is only one angular momentum parameter, that we shall denote by  $a$ . Next we perform the similar complex coordinate transformation, in the HD, as used by Newman and Janis [3] by defining a new set of coordinates  $(u', r', \theta'_i)$ , where  $i = 1, \dots, (N+1)/2$  by the relations

$$x'^a = x^a + ia(\delta_r^a - \delta_u^a) \cos \theta_1 \rightarrow \begin{cases} u' = u - ia \cos \theta_1, \\ r' = r + ia \cos \theta_1, \\ \theta'_i = \theta_i \end{cases} \quad (13)$$

Simultaneously let null vieltrrad vectors  $Z^a$  undergo a transformation  $Z^a = Z'^a \partial x'^a / \partial x^b$  in the usual way, we obtain

$$l^a = \delta_r^a,$$

$$n^a = \left[ \delta_u^a - \frac{1}{2} \left( 1 - \frac{\mu}{r^{N-2}\Sigma} - \frac{Q_0^2}{\Sigma} \right) \delta_r^a \right],$$

$$m_k^a = \frac{1}{\sqrt{2}(r + ia \cos \theta_1) \sin \theta_1 \sin \theta_2, \dots, \sin \theta_{(k-1)}} \times \left( ia(\delta_u^a - \delta_r^a) \sin \theta_1 + \delta_{\theta_k}^a + \frac{i}{\sin \theta_k} \delta_{\theta_{(k+1)}}^a \right),$$

where we have dropped the primes. From the new null vieltrrad, a new metric is discovered using (13), which can be written as

$$ds^2 = \left( 1 - \frac{\mu}{r^{N-2}\Sigma} - \frac{Q_0^2}{\Sigma} \right) du^2 + 2dudr - 2a \sin^2 \theta_1 dr d\theta_2$$

$$- \Sigma d\theta_1^2 - \left[ (r^2 + a^2) + \left( \frac{\mu}{r^{N-2}\Sigma} + \frac{Q_0^2}{\Sigma} \right) a^2 \sin^2 \theta_1 \right]$$

$$\times \sin^2 \theta_1 d\theta_2^2 - 2a \left( \frac{\mu}{r^{N-2}\Sigma} + \frac{Q_0^2}{\Sigma} \right) \sin \theta_1^2 dud\theta_2$$

$$- r^2 d\Omega_{N-1}^2, \quad (14)$$

where  $\Sigma = r^2 + a^2 \cos^2 \theta_1^2$ . Thus, we have obtained spinning BH corresponding to HD EYM BH. Also note that the derived HD metric (14) via NJA is in Kerr like coordinates [6]. A further simplification is made on taking

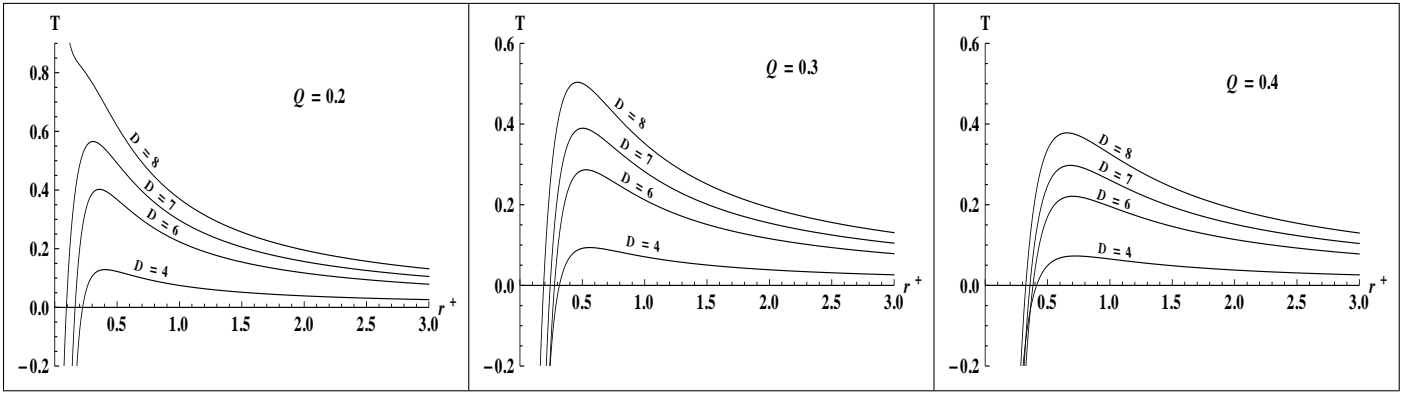


FIG. 1: The temperature profile shown as function of  $r^+$  for different dimensions  $D$  with three different values of YM gauge charge parameter  $Q$ .

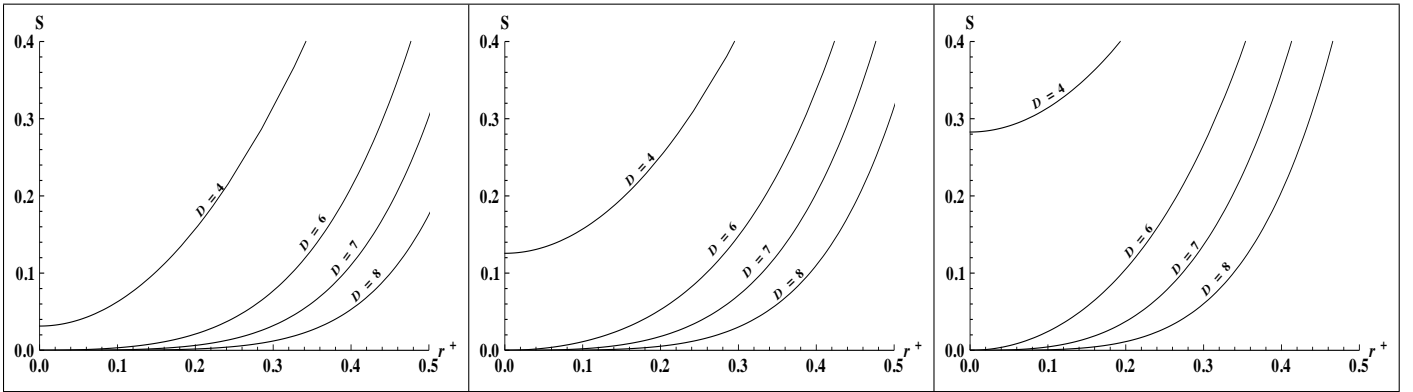


FIG. 2: The behaviour of entropy as a function of horizon radius  $r^+$  for different dimensions  $D$  with three different values of rotation parameter  $a$ .

coordinate transformation as in Ref. [6]. This transformation leaves only one off-diagonal element and we arrive at the following:

$$\begin{aligned}
 ds^2 = & \left( \frac{\Delta - a^2 \sin^2 \theta_1}{\Sigma} \right) dt^2 - \frac{\Sigma}{\Delta} dr^2 + 2a \\
 & \times \left[ 1 - \left( \frac{\Delta - a^2 \sin^2 \theta_1}{\Sigma} \right) \right] dt d\theta_2 - \Sigma d\theta_1^2 \\
 & - \left[ \Sigma + a^2 \sin^2 \theta_1 \left( 2 - \frac{\Delta - a^2 \sin^2 \theta_1}{\Sigma} \right) \right] \\
 & \times \sin^2 \theta_1 d\theta_2^2 - r^2 \cos^2 \theta_1 d\Omega_{N-1}^2, \quad (15)
 \end{aligned}$$

where on substituting back the value of  $Q_0$  in terms of  $Q$ ,  $\Delta$  reads

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{N-2}} - \frac{N}{(N-2)} Q^2. \quad (16)$$

Eq. (15) is in Boyer-Lindquist coordinates. Here, we have also introduced

$$\begin{aligned}
 \Delta &= a^2 \sin^2 \theta_1 + \Sigma G(r, \theta_1), \\
 G(r, \theta_1) &= \frac{\Delta - a^2 \sin^2 \theta_1}{\Sigma}.
 \end{aligned}$$

Thus, we are able to generate HD axisymmetric solution starting with HD static spherically symmetric EYM BH solutions using the approach originally proposed by Newman-Janis [3], i.e., we have an explicit YM gauge charged HD spinning BH solution. One can see that HD metric (15) as behaviour of a metric produce from spinning charged source. In the limit  $N = 1$ , the geometry of solution (15) is precisely of the Kerr-Newman form [6] and the charge that determines the geometry is YM gauge charge. Thus we have exact HD Kerr-Newman like solution, but  $Q$  corresponds to magnetic charge. Hereafter, we refer the solution (15) spinning HD EYM BH solutions. For vanishing YM gauge charge  $Q = 0$ , one recovers Myers-Perry BH solution discussed in [26]. The Reissner-Nordström BH are recovered in the limit  $a = 0$  and  $N = 1$ . The HD EYM BH [31] are discovered for the vanishing spinning parameter  $a = 0$ .

Eq. (15) has parameters  $\mu$  and  $a$  which are respectively related to mass ( $M$ ) and angular momentum ( $J$ ) via relations :-

$$M = \frac{(N+1)}{16\pi} A_{N+1} \mu, \quad J = \frac{1}{8\pi} A_{N+1} \mu a,$$

and

$$\frac{M}{J} = \frac{(N+1)}{2}a. \quad (17)$$

The determinant  $g$  of the metric (15) gives

$$\sqrt{-g} = \sqrt{\gamma}\Sigma r^{N-1} \sin\theta \cos^{N-1}\theta, \quad (18)$$

and  $A_{N+1}$  is the area of unit  $(N+1)$  sphere which is given by

$$A_{N+1} = \int_0^{2\pi} d\theta_2 \int_0^\pi \sin\theta_1 \cos^{N-1}\theta_1 d\theta_1 \quad (19)$$

$$\times \prod_{i=3}^{N-1} \int_0^\pi \sin^{(N-1)-i}\theta_i d\theta_i = \frac{2\pi^{(N+2)/2}}{\Gamma(N+2)/2}.$$

The angular velocity at the horizon is given by

$$\Omega_H = \frac{a}{r^{+2} + a^2}. \quad (20)$$

Area of the event horizon (EH) for metric (15) can be given by the standard definition of the horizon area [33] as:

$$A_H = \int_{\rho(r=r^+)} \sqrt{\eta} d\theta_1 d\theta_2 d^{N-1}\rho,$$

which trivially solves to

$$A_H = r^{+N-1}(r^{+2} + a^2)A_{N+1}.$$

The entropy of the BH typically satisfies the area law of the entropy which states that the entropy of the BH is one fourth of the area of EH [34]. The horizon area and the surface gravity of the solution are related to the entropy and the temperature, respectively,  $S = A_H/4$  and  $T = \kappa/2\pi$ . Thus the expression for entropy and temperature of the spinning HD EYM BH on the horizon are

$$S = \frac{r^{+N-1}(r^{+2} + a^2)A_{N+1}}{4},$$

$$T = \frac{(N-2)(r^{+2} + a^2) + 2r^{+2} - NQ^2}{4\pi r^+(r^{+2} + a^2)}.$$

In the appropriate limit the physical quantities derived above reproduces the corresponding qualities of Myers-Perry BH when  $Q = 0$ , of Kerr-Newman BH if  $N = 1$  and when both  $Q = 0$  and  $N = 1$ , we get these quantities associated with Kerr BH. In Figs. (1) and (2), we plot temperature and entropy of the spinning HD EYM BH respectively. It is interesting to note from Fig. (1) that at low value of YM gauge charge temperature is at maximum in HD but as  $Q$  is increased temperature starts increasing from a minimum value and on reaching at a maximum value, starts decaying. We note that the rate of decrease in temperature slows down with the increase

in  $Q$ . The entropy for 4D case,  $N = 1$ , is always positive even for vanishing horizon radius,  $r^+ = 0$ , but it is zero in HD case for  $r^+ = 0$ . The dependence of entropy on the horizon radius,  $r^+$ , is shown in Fig. (2) which also confirms the area law for our solution, i.e., entropy is increasing with the radius of horizon.

#### IV. HORIZON PROPERTIES

It is natural to discuss not only spinning BH solutions but their various properties. It is known that the structure of a spinning BH is much different from that of a stationary BH. The EH of a spinning BH is smaller than the EH of an otherwise identical but non-spinning one. Similar to Kerr solutions in asymptotically flat spacetimes the above metric has two types of horizons like hypersurface: a stationary limit surface (SLS) and an EH. In a spinning BH, the ergosphere is associated with the stationary limit, the location at which spacetime is flowing at the speed of light, making stationary particles that would be travelling at the speed of light. Within the stationary limit, no particles can remain at rest, even though they are outside the EH. We shall explore the two horizons SLS and EH of spinning HD EYM BH, and also discuss the effects which comes from the YM gauge charge and also due to spacetime dimensions.

Let us now address the horizon properties of the solution, beginning with SLS. The SLS is the boundary of the region in which an observer travelling a long time-like curve can follow the orbits of the asymptotic time translation Killing vector  $\partial/\partial t$  and so remain stationary with respect to infinity. Beyond this surface any observer following the orbits of  $\partial/\partial t$  must go faster than the speed of light, i.e., he can not remain stationary with respect to an observer at infinity. On this surface the Killing vector  $\partial/\partial t$  is null. They are surfaces of infinite redshift, and for the spinning HD EYM BH requires that prefactor  $g_{tt}$  of the  $dt^2$  term in metric vanish. It follows that SLS will satisfy

$$r^N + \left[ a^2 \cos^2\theta - \frac{N}{(N-2)}Q^2 \right] r^{N-2} - \mu = 0. \quad (21)$$

On the other hand, surfaces at which the radial coordinate  $r$  flips its signature is an EH, i.e., the spacetime (15) has a horizon where [21] the four-velocity of an observer tends to zero, or the surface  $r=\text{constant}$  becomes null. EH is a solution of  $\Delta = 0$  and thus it must satisfy

$$r^N + \left[ a^2 - \frac{N}{(N-2)}Q^2 \right] r^{N-2} - \mu = 0. \quad (22)$$

##### 1. 4D Case

When  $N = 1$ , i.e., in 4D, recalling that  $\mu = 2M$ , we recover the well-known results for the Kerr-Newman met-

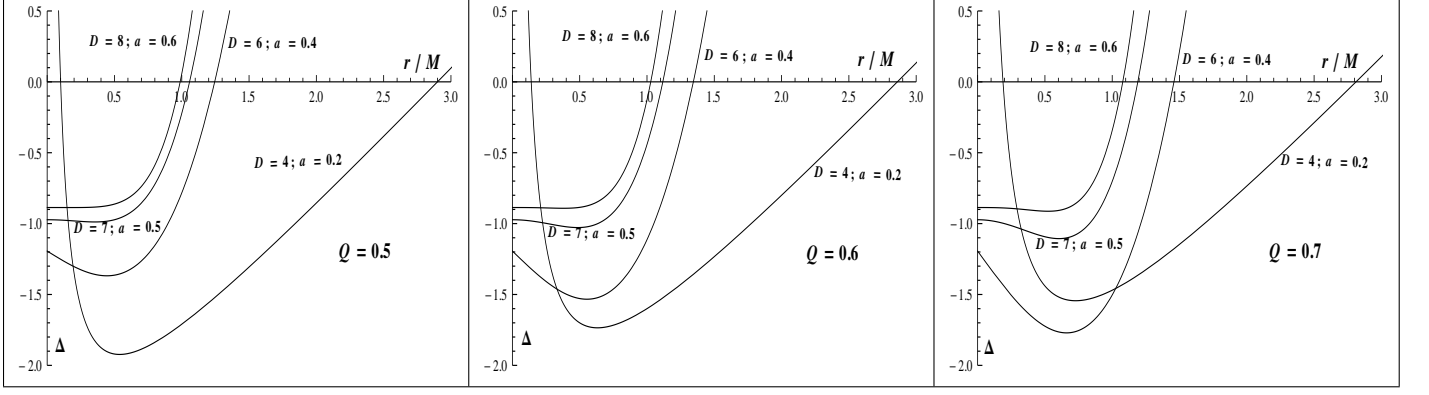


FIG. 3: Plot of  $\Delta(r)$  to show the behaviour of horizon of spinning HD EYM BH for different dimensions  $D$  with three different values of YM gauge charge parameter  $Q$ .

ric:

$$\begin{aligned}
 ds^2 = & \left(1 - \frac{2Mr}{\Sigma} + \frac{Q^2}{\Sigma}\right) dt^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 \\
 & - \left[ (r^2 + a^2) + \left(\frac{2Mr}{\Sigma} - \frac{Q^2}{\Sigma}\right) a^2 \sin^2 \theta \right] \sin^2 \theta d\phi^2 \\
 & - 2a \left(\frac{2Mr}{\Sigma} - \frac{Q^2}{\Sigma}\right) \sin^2 \theta dt d\phi,
 \end{aligned} \quad (23)$$

with  $\theta_1 = \theta$ ,  $\theta_2 = \phi$  and in (15)  $\mu = 2M$ . The Eq. (22), for  $N = 1$ , is simplified to

$$\Delta = r^2 + a^2 - 2Mr + Q^2, \quad (24)$$

which admit solution  $r^\pm$ , identified as outer and inner EH. The EH of Eq. (24) are

$$r^\pm_{EH} = M \pm \sqrt{(M^2 - Q^2) - a^2}. \quad (25)$$

If  $a^2 < (M^2 - Q^2)$ , there exist two horizons, when  $a^2 \rightarrow (M^2 - Q^2)$ , two horizons coincides, i.e., the extremal case and if  $a^2 > (M^2 - Q^2)$ , then there exists no horizon, i.e., one has naked singularity. For SLS,  $N = 1$  in Eq. (21), it reduces to

$$r^2 - 2Mr + a^2 \cos^2 \theta + Q^2 = 0, \quad (26)$$

which trivially solves to

$$r^\pm_{SLS} = M \pm \sqrt{(M^2 - Q^2) - a^2 \cos^2 \theta}. \quad (27)$$

These are regular outer and inner SLSs for a Kerr-Newman BH when  $a^2 \cos^2 \theta < (M^2 - Q^2)$ , and further in the non-spinning limit  $a \rightarrow 0$ , both SLS and EH coincides to

$$r^\pm = M \pm \sqrt{M^2 - Q^2}, \quad (28)$$

which are outer and inner EH of Reissner-Nordström BH. Thus the Kerr-Newman BH, in the limit  $a \rightarrow 0$ , degenerates to Reissner-Nordström BH.

## 2. 6D Case

Eq. (22) for  $N = 3$  or  $6D$  case reduces to

$$r^3 + (a^2 - 3Q^2)r - \mu = 0, \quad (29)$$

which gives EH as

$$r^+_{EH} = \frac{(27\mu + \sqrt{729\mu^2 + \delta})^{1/3}}{3 \times 2^{1/3}} - \frac{2^{1/3}(a^2 - 3Q^2)}{(27\mu + \sqrt{729\mu^2 + \delta})^{1/3}},$$

with  $\delta = 4(3a^2 - 9Q^2)^3$ .

Eq. (21) reduces to

$$r^3 + (a^2 \cos^2 \theta - 3Q^2)r - \mu = 0, \quad (30)$$

which can be solved to

$$r^+_{SLS} = \frac{(27\mu + \sqrt{729\mu^2 + \delta})^{1/3}}{3 \times 2^{1/3}} - \frac{2^{1/3}(a^2 \cos^2 \theta - 3Q^2)}{(27\mu + \sqrt{729\mu^2 + \delta})^{1/3}},$$

with  $\delta = 4(3a^2 \cos^2 \theta - 9Q^2)^3$ .

## 3. 7D Case

Eq. (22), for  $7D$ , reduces to

$$r^4 + (a^2 - 2Q^2)r^2 - \mu = 0. \quad (31)$$

So, we get the EH as

$$r^+_{EH} = \sqrt{\frac{1}{2}(2Q^2 - a^2) + \sqrt{4\mu + \delta_1}},$$

with  $\delta_1 = a^4 + 4Q^2 - 4a^2Q^2$ .

Eq. (21) can be written as

$$r^4 + (a^2 \cos^2 \theta - 2Q^2)r^2 - \mu = 0, \quad (32)$$

which admits solution

$$r^+_{SLS} = \sqrt{\frac{1}{2}(2Q^2 - a^2 \cos^2 \theta) + \sqrt{4\mu + \delta_2}},$$

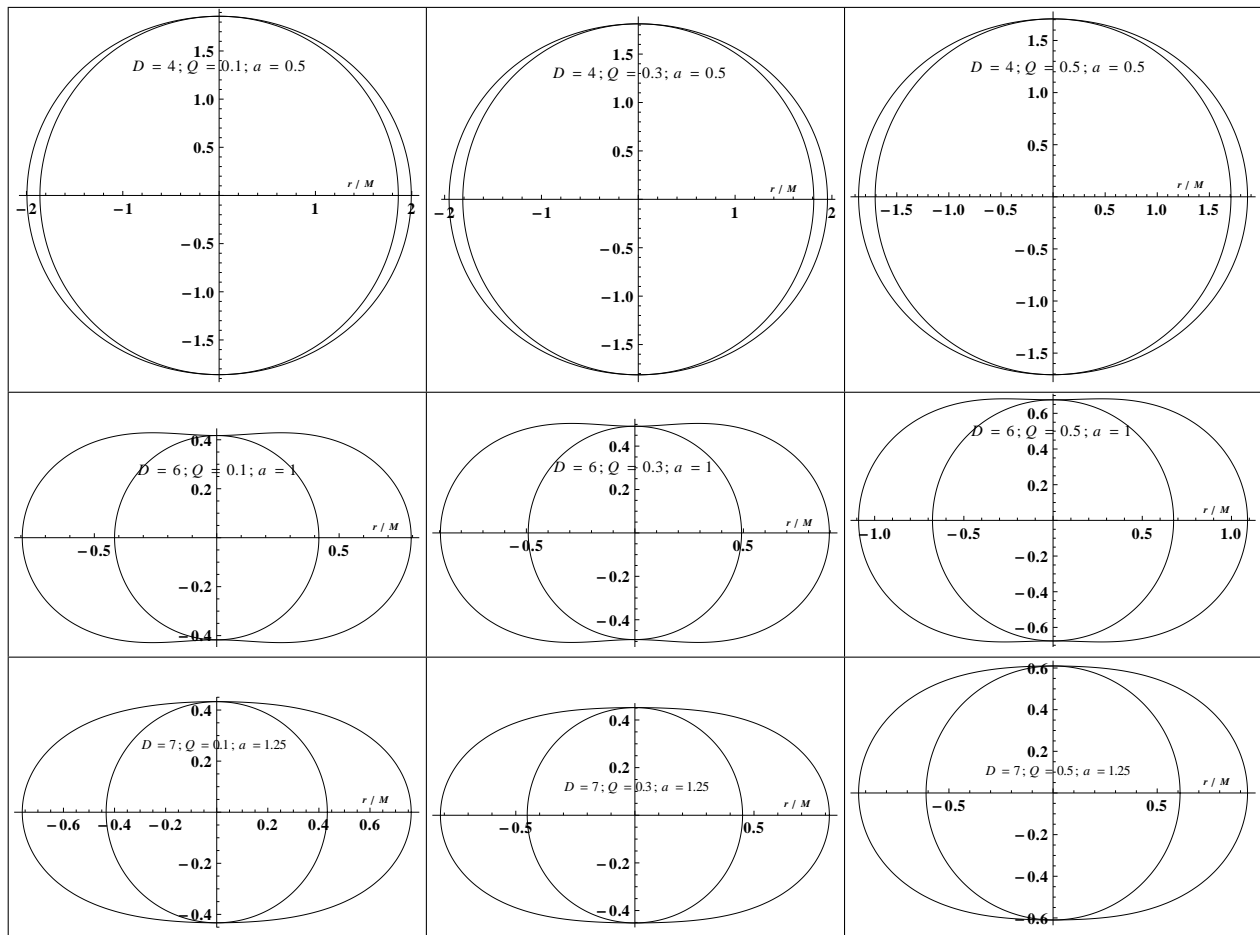


FIG. 4: Plots showing cross section of SLS and EH and the variation of the ergosphere for different dimensions  $D$  with three different values of YM gauge charge parameter  $Q$ .

with  $\delta_2 = a^4 \cos^4 \theta - 4a^2 \cos^2 \theta + 4Q^2$ .

Thus, the SLS and EH depends on the spacetime dimension. For HD we note that

$$\lim_{r \rightarrow 0} \Delta(r) = -\mu < 0, \quad \lim_{r \rightarrow +\infty} \Delta(r) = \infty, \quad \Delta'(r) \geq 0. \quad (33)$$

It is seen that  $\Delta'(r) \geq 0$  for  $D \geq 6$ . However it is seen that Eqs. (21) and (22) has just one positive root for HD ( $D \geq 6$ ), i.e., just one EH and SLS in HD. This means that there is no extremal spinning BH when  $D \geq 6$ . The effect of YM gauge charge on horizon is shown in Fig. (3). It turns out that the radius of EH is decreasing with spacetime dimension. On the other hand, it increases with the value of YM gauge charge. One gets same result for SLS and we do not present them here.

### A. Ergosphere

For the Schwarzschild and Reissner-Nordström BH, it is possible that a traveller can approach arbitrarily close to the EH whilst remaining stationary with respect to infinity. This is not the case for the Kerr/Kerr-Newman

BH. The spinning BH drags the surrounding region of spacetime causing the traveller to spin regardless of any arbitrarily large thrust that he can provide. The ergoregion is the region in which this happens and is bounded by the ergosphere. On plotting the SLS and EH for spinning HD EYM BH, it can be verified that the SLS always lies outside the EH for all dimensions  $D$ . The ergosphere is plotted in Figs. (4) and (5). The ergosphere is defined to be the place where the vectors  $\partial t$  parallel to the *time-axis* are not timelike but spacelike. An ergosphere, thus, is a region of spacetime where no observer can remain still with respect to the coordinate system in question. Thus the ergosphere of a spinning HD EYM BH, as in Kerr/Kerr-Newman BH, is bounded by the EH on the inside and an oblate spheroid SLS, which coincides with the EH at the poles and is noticeably wider around the equator. It is the region of spacetime where timelike geodesics remain stationary and timelike particles can have negative energy relative to infinity. It is theoretically possible to extract energy and matter from the BH from ergosphere [35]. In Fig. (4) we have shown the dependence of the shape of the ergosphere on the YM gauge charge. It is noticed that for  $D = 4$  the shape

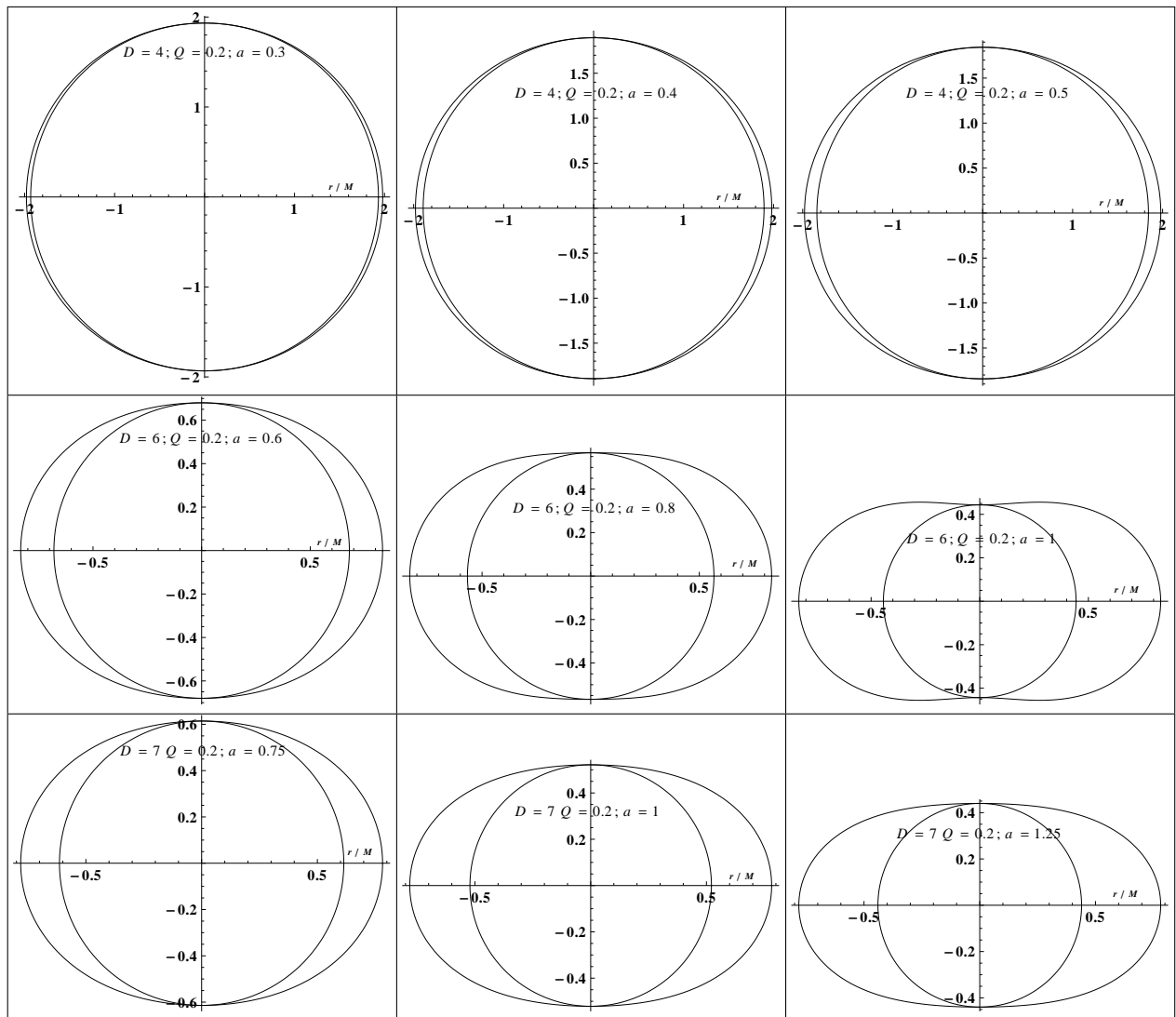


FIG. 5: Plots showing cross section of SLS and EH and the dependence of the shape of ergosphere for different dimensions  $D$  with three different values of rotation parameter  $a$ .

of the ergosphere is increasing with the increase in YM gauge charge while decreasing for  $D \geq 6$ . This shows that shape of ergosphere is sensitive to the YM gauge charge. In Fig. (5) the variation of the shape of ergosphere with  $a$  is shown. We note that the relative shape of the ergosphere becomes more prolate thereby increasing area of the ergosphere with rotation parameter  $a$ , i.e., the faster the BH rotates, the more the ergosphere grows. This may have direct consequence on the Penrose's energy extraction process [35].

## V. CONCLUSION

In this paper, we have extended NJA in order to construct spinning BH from HD EYM BH. The method does not use field equation but works on the HD spherical solution to generate spinning solutions. The algorithm is

very useful since it directly allows us to construct spinning BH in HD, which otherwise could be extremely cumbersome. Originally, the NJA was applied to Reissner-Nordström solution to obtain Kerr-Newman solution [3]. The metric (15) is stationary, axisymmetric and asymptotically flat. It depends on mass, YM gauge charge and spinning parameter which reduces to Kerr BH [1] ( $N = 1, Q = 0$ ) and Myers Perry BH [26] ( $Q = 0$ ). The solution in 4D ( $N = 1$ ) has precisely the geometry of Kerr-Newman [5], but the charge that determines the geometry is YM gauge charge. Also, it is easy to check that the metric (15) in 4D ( $N = 1$ ) is solution of EYM Eq. (7). Thus, we can say that a spinning BH solution of Einstein Maxwell equations is also solution of EYM, but the charge  $Q$  is the YM gauge charge and not electric charge. However, this is not true in HD. Our spinning HD EYM BH solution deviate from HD Kerr-Newman

[36] because  $\Delta(r)$  for the latter is given by

$$\Delta_{KN}(r) = r^2 + a^2 - \frac{\mu}{r^{N-2}} + \frac{Q^2}{r^{2(N-1)}}, \quad (34)$$

The corresponding  $\Delta(r)$  in spinning HD EYM BH is given by Eq. (16). The difference in the last term of (16) and (34) is because of the fact that the charge term  $Q^2/r^2$  in the solution (9) is dimension independent, while it would go as  $Q^2/r^{2(N-1)}$  in HD Reissner-Nordström. It may be noted HD Reissner-Nordström can be used to obtain HD Kerr-Newman using NJA [6]. The two kinds of horizon like surfaces viz. SLS and EH are studied. In 4D, it turns out that there exist two horizon like surfaces corresponding to two positive roots which are identified as inner and outer horizons. However, in HD there exist only one positive root and thereby only one SLS and EH. It is interesting to see that the structure of SLS, EH and ergosphere are sensitive to YM gauge charge parameter  $Q$ .

The physical properties of the solutions have not yet

been fully investigated; this being very severe job. However, we are currently on this project. We have also shown that the presence of YM gauge charge decreases the temperature with increase in gauge charge parameter  $Q$ . Such a change could have a significant effect in the thermodynamics of a BH. Hence, It will be of interest to see how YM gauge charge affects the thermodynamics by deriving Smarr-like relation and the first law, and also stability analysis. Further analysis of these solutions and the role of YM gauge charge and spacetime dimension in energy extraction process remains interesting issue to explore in the future.

### Acknowledgments

Authors would like to thank University Grant Commission (UGC) for major research project grant NO. F-39-459/2010(SR) and to IUCAA, Pune for kind hospitality while part of this work was being done.

- 
- [1] R. P. Kerr, Phys. Rev. Lett. D **11**, 237 (1963).
  - [2] F. R. Tangherlini, Nuovo Cim. **27**, 636 (1963).
  - [3] E. T. Newman and A. I. Janis, J. Math. Phys. **6**, 915 (1965).
  - [4] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
  - [5] E. T. Newman, R. Couch, K. Chinnapared, A. Exton, A. Prakash and R. Torrence, J. Math. Phys. **6**, 918 (1965).
  - [6] D. Y. Xu, Class. Quant. Grav. **5**, 871 (1988).
  - [7] S. P. Drake and R. Turolla, Class. Quant. Grav. **14**, 1883 (1997).
  - [8] H. Kim, Phys. Rev. D **59**, 064002 (1999).
  - [9] S. Yazadjiev, Gen. Rel. Grav. **32**, 2345 (2000).
  - [10] S. P. Drake and P. Szekeres, Gen. Rel. Grav. **32**, 445 (2000).
  - [11] D. J. Cirilo Lombardo, Class. Quant. Grav. **21**, 1407 (2004).
  - [12] F. Caravelli and L. Modesto, Class. Quant. Grav. **27**, 245022 (2010).
  - [13] L. Modesto and P. Nicolini, Phys. Rev. D **82**, 104035 (2010).
  - [14] S. Capozziello, M. De laurentis and A. Stabile, Class. Quant. Grav. **27**, 165008 (2010).
  - [15] T. Johannsen and D. Psaltis, Phys. Rev. D **83**, 124015 (2011).
  - [16] C. Bambi and L. Modesto, Phys. Lett. B **721**, 329 (2013).
  - [17] E. N. Glass and J. P. Krisch, [arXiv:0405143 [gr-qc]].
  - [18] R. d'Inverno, *Introducing Einstein's Relativity*, Oxford, UK: Clarendon (1992) 383 p.
  - [19] E. J. Flaherty, *Hermitian and Kahlerian Geometry in Relativity Lecture notes in Physics*, **46**, Springer-Verlag, Berlin (1976).
  - [20] M. M. Schiffer, R. J. Adler, J. Mark and C. Scheffield, J. Math. Phys. **14**, 52 (1973).
  - [21] A. Strominger, and C. Vafa, Phys. Lett. B, **379**, 99, (1996).
  - [22] R. Emparan and S. R. Harvey, *Black Holes in Higher Dimensions, Living Rev. Relativity* **11**, (2008).
  - [23] P. Kanti, Int. J. Mod. Phys. A, **19**, 4899, (2004).
  - [24] A. Chados and S. Detweiler, Gen. Relativ. Gravit. **14**, 879 (1982).
  - [25] A. Das and A. DeBenedictis *Progress of Theo. Phy.* **108**, 119 (2002).
  - [26] R. C. Myers and M. J. Perry, Annals Phys. **172**, 304 (1986).
  - [27] T. T. Wu and C. N. Yang, in *Properties of Matter Under Unusual conditions*, edited by H. Mark and S. Fernbach (Interscience, New York, 1969), p. 349.
  - [28] P. B. Yasskin, Phys. Rev. D **12**, 2212, (1975).
  - [29] S. Habib Mazharimousavi and M. Halilsoy, Phys. Rev. D **76**, 087501 (2007).
  - [30] S. Habib Mazharimousavi and M. Halilsoy, JCAP **0812**, 005 (2008).
  - [31] S. H. Mazharimousavi and M. Halilsoy, Phys. Lett. B **659**, 471 (2008).
  - [32] S. G. Ghosh and N. Dadhich, Phys. Rev. D **82**, 044038 (2010).
  - [33] R. Banerjee, B. R. Majhi, S. K. Modak and S. Samanta, Phys. Rev. D **82**, 124002 (2010).
  - [34] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
  - [35] R. Penrose and R. M. Floyd, Nature **229**, 177 (1971).
  - [36] A. N. Aliev, Phys. Rev. D **74**, 024011 (2006).