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EVOLUTION OF THE CORRELATION FUNCTION FOR A CLASS OF PROCESSES INVOLVING NON LOCAL SELF - REPLICATION

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ABSTRACT

A large class of evolutionary processes can be modeled by a rule that involves self-replication of some physical quantity with a non local rescaling. We show that a class of such models are exactly solvable — in the discrete as well as continuum limit — and can represent several physical situations as varied from the formation of galaxies in some cosmological models to growth of bacterial cultures. This class of models, in general, has no steady state solution and evolve unstably as $t \to \infty$ for generic initial conditions. They can however exhibit (unstable) power law correlation function in the continuum limit, for an intermediate range of times and length scales.

Subject headings: cosmology, galaxy formation

1. INTRODUCTION

Observations show that the two point correlation function of the galaxies is an approximate power law over a range of scales. This result, known for decades now, defies a simple "first-principle" explanation because of the complexity of the physical processes involved. In the conventional big-bang cosmology the dominant contribution to the energy density of the universe is in the form of nonbaryonic dark matter and the visible galaxies (made of baryons) form due to complex processes of cooling and fragmentation within the dark matter halos. It is possible to model non linear gravitational clustering of *dark matter halos* by a scaling ansatz (see eg. Padmanabhan (1996)) and show that a power law correlation is expected (in a spatial and temporal interval) for scale invariant initial conditions. This result arises from the fact that Newtonian gravitational dynamics in a $\Omega = 1$ universe does not have any preferred scale which is of interest to cosmology. But to translate this result to baryonic structures is not easy and at present the only explanation for the power law correlation of galaxies arises from numerical simulations.

There are, however, alternate (and less popular) models for galaxy formation in quasi-steady state cosmology (QSSC) in which the process is addressed without invoking gravitational instability explicitly (Nayeri et. al. 1999). It has been claimed, based on numerical simulations, that these models also lead to power law correlation function for galaxy distribution. This result is of intrinsic interest even to conventional cosmologists — who may not accept the alternate model — because it provides a scenario to probe the relationship between (i) the conventional scenario of galaxy formation via gravitational instability (accepted by most of the working cosmologists) and (ii) the power law correlation function for the galaxies. If a completely different model can lead to power law correlation function, then it is clear that observations of galaxy correlation function, by itself, cannot be used as a discriminator between the theories.

Since the "rule" used for creating galaxies in QSSC is simple and explicit, it is indeed possible to study this process completely analytically. We provide such an analysis in this paper and show that the model is intrinsically unstable. However, the galaxies produced by this rule *will* exhibit a power law correlation function for a range of intermediate time scales.

The rest of the paper is arranged as follows. In the next section, we provide an analytic description of the model for galaxy formation in the QSSC, which was used in (Nayeri et. al. 1999). This is done in terms of discrete time steps to parallel the previous work. In section 3 we discuss the continuum limit of this model that lends itself to a completely tractable analysis. In section 4 we describe a very general class of processes which obey the same equations as described here and a broader class of applications. The last section gives the conclusions.

2. A MODEL FOR STRUCTURE FORMATION - DISCRETE VERSION

A non standard model for galaxy formation in QSSC is built along the following lines: Consider a set of points in the 3-dimensional space which represents the location of galaxies at a given instant of time. We now generate a set of new galaxies near each one of the galaxies ("non-local self-replication") by a physical process which should quite minimally involve some extra negative energy field. The key ingredient of the nonstandard cosmological models is the existence of some "creation field" so that galaxies could generate new galaxies nearby (see Nayeri et. al. (1999)). Let the probability for any given new galaxy to be located at a distance $\mathbf{l}$ from an old galaxy be $W(\mathbf{l})d^3\mathbf{l}$. This process will increase the number of galaxies as the universe evolves. In QSSC, a balance between creation and quasi-steady state structure is maintained by rescaling the size of the universe in each cycle. Mathematically, this is taken into account by rescaling each of the 3 dimensions by a factor $\bar{\mu} > 1$ thereby increasing the volume of available space. (If we create *one* new galaxy near an old one with a probability $W(\mathbf{l})$, then we will be doubling the number of galaxies during the self replication. The rescaling should be such that the volume doubles giving $\bar{\mu}^3 = 2$. For future convenience, we shall keep $\bar{\mu}$ as an arbitrary constant.) We now select a subset of galaxies in the central region such that the total number of galaxies remain the same. This

step renormalizes the process back to the original situation so that the process can now be repeated with the new subset of galaxies. In the specific case of number of galaxies doubling per cycle, we should take the central region containing half the volume. It is obvious that such a process of galaxy formation will lead to correlations between the galaxies since new galaxies are created close to the old ones with a probability $W(\mathbf{l})$. The key question is how the correlation function scales with the distance scale. We will now provide a mathematical analysis of this problem.

The evolutionary rule described above can be stated mathematically in the form:

$$Q(n+1,\mathbf{x}) = \frac{1}{(1+\lambda)\bar{\mu}^3}\Big[Q(n,\mathbf{x}/\bar{\mu}) + \lambda\int Q(n,\frac{\mathbf{x}}{\bar{\mu}}-\mathbf{l})W(\mathbf{l})d\mathbf{l}\Big] \quad (1)$$

where $\lambda, \bar{\mu}$ are constants with $\lambda > 0$ and $\bar{\mu} > 1$; $W(\mathbf{l})$ is a probability function normalized to unity for integration over all $\mathbf{l}$. We shall also assume that Q is normalized in such a way that its integral over all $\mathbf{x}$ is unity. This gives us the conditions

$$\int d\mathbf{x}\ Q = 1 = \int d\mathbf{l}\ \ W(\mathbf{l}). \quad (2)$$

It is obvious that equation (1) preserves these conditions under evolution because of the explicit normalization chosen on the right hand side. In the context of the quasi steady state cosmological model, Q will be the ratio between the number density of galaxies and the mean density. The normalization in (2) preserves the quasi steady state condition between different cycles under the simultaneous action of matter creation and expansion.

While equation (1) was motivated from a particular model for galaxy formation, the rest of the analysis only uses this equation and is independent of the assumptions which go into it. In section 4 we will comment on a wider class of phenomena which could be modeled by such an equation.

Let us now consider the solutions to equation (1). Linearity in Q suggests switching to the Fourier space variables $f(n,\mathbf{k})$ with

$$f(n,\mathbf{k}) = \int d\mathbf{x}\ Q(n,\mathbf{x})e^{i\mathbf{k}.\mathbf{x}}; \quad f(n,0) = 1. \quad (3)$$

Equation (1) now reduces to a simple form

$$f(n+1,\mathbf{k}) = \frac{f(n,\bar{\mu}\mathbf{k})}{(1+\lambda)}\left[1+\lambda W(\mathbf{k})\right] \quad (4)$$

where we have denoted by the same symbol $W(\mathbf{k})$ the Fourier transform of the probability function. Given the form of this probability function, this equation iteratively determines the evolution of f.

While the equation is fairly simple in structure, it is not easy to find its general solution. Note that the conservation condition (2) — which demands $f(n,0) = 1$ for all n — is satisfied in this case because $W(\mathbf{k}=0) = 1$ for a normalized probability distribution. This also shows that solutions of the form $f(n,\mathbf{k}) = A(\mathbf{k})\exp(\alpha n)$, with some constant α, are unacceptable for $\alpha \neq 0$ because they will violate the normalization condition. Further, since we have the freedom to choose the initial condition $f(0,\mathbf{k}) = f_{\rm in}(\mathbf{k})$, any general solution to equation (4) must contain one arbitrary function; it is difficult to obtain such a *general* solution.

A special class of solutions to equation (4) will correspond to a steady state such that $f(n+1,\mathbf{k}) = f(n,\mathbf{k}) = f_{\rm s}(\mathbf{k})$. This function satisfies the equation

$$f_s(\mathbf{k}) = f_s(\mathbf{k}\bar{\mu})\left[\frac{1+\lambda W(\mathbf{k})}{1+\lambda}\right]. \quad (5)$$

Two trivial solutions to this equation correspond to $f_s(\mathbf{k}) = (0,\infty)$ and, in fact, we shall see later that the most generic initial conditions will drive the system to either of these two limits by our process. The only nontrivial solution which exists can be found by the iteration in the form

$$f_s(\mathbf{k}\bar{\mu}) = \frac{f_s(\mathbf{k})(1+\lambda)}{1+\lambda W(\mathbf{k})} = \frac{f_s(\frac{\mathbf{k}}{\bar{\mu}})(1+\lambda)^2}{[1+\lambda W(\mathbf{k})]\left[1+\lambda W(\frac{\mathbf{k}}{\bar{\mu}})\right]}$$
$$= f_s\left(\frac{\mathbf{k}}{\bar{\mu}^N}\right)\prod_{n=0}^{N}\frac{(1+\lambda)}{1+\lambda W(\mathbf{k}/\bar{\mu}^n)}. \quad (6)$$

Taking the logarithms, followed by the limit $N\to\infty$, and using $f_s(0) = 1$ we get the result

$$\ln f_s(\mathbf{k}\bar{\mu}) = \sum_{n=0}^{\infty}\ln\left[\frac{(1+\lambda)}{1+\lambda W(\mathbf{k}/\bar{\mu}^n)}\right]. \quad (7)$$

To study the properties of the solution we note that $W(k/\bar{\mu}^n) \approx 1$ for all $n > \log_{\bar{\mu}}(kL)$. This is true for a wide class of probability distributions with some characteristic scale L, and $W(k)$ usually decreases for $kL \gg 1$. We will also assume that $W(k)$ depends only on the magnitude of $\mathbf{k}$ because of the statistical isotropy of the process (though our results are easily generalizable to other cases). It follows that each of the denominators in (7) is close to unity for all $n > \log_{\bar{\mu}}(kL)$ making all the terms negligibly small for $n > \log_{\bar{\mu}}(kL)$. So we need to sum the series in (7) only up to $n = n_c \leq [\log_{\bar{\mu}}(kL)]$ with the integer value, lower than the bound, taken for n_c. The result of the sum depends on the form of W at large k. The simplest case corresponds to assuming the W vanishes for $kL \gg 1$ which is exact if we take

$$W(\mathbf{k}) = \theta(1-kL) \quad (8)$$

where $\theta(z) = 1$ for $z > 0$ and zero otherwise. Then the asymptotic solution is given by

$$f_s(\mu k) = (1+\lambda)^{(n_c+1)} = (kL)^{\gamma}; \qquad \gamma = \frac{\ln(1+\lambda)}{\ln\bar{\mu}} \quad (9)$$

(The equivalence of the two forms follows from simple algebra and definition of n_c.) This is a power law solution with the index determined essentially by the two parameters of the problem.

Before proceeding further, let us consider the effect of the normalization condition (2) more closely. This is of some interest because one might feel that such a normalization is unnecessarily restrictive and a wider class of phenomena can be modeled by relaxing this condition. It turns out, however, that our qualitative considerations are not affected by this constraint. Relaxing this normalisation can be done most conveniently by replacing the factor $(1+\lambda)$ in the denominator of the right hand side of (1) by $(1+\lambda_1)$ where λ_1 is a constant different from λ. This will change equation (4) to the form:

$$f(n+1,\mathbf{k}) = \frac{f(n,\bar{\mu}\mathbf{k})}{(1+\lambda_1)}\left[1+\lambda W(\mathbf{k})\right]. \quad (10)$$

Setting $\mathbf{k}=0$ and using $W(0)=1$, we get

$$f(n+1,0)=f(n,0)\left(\frac{1+\lambda}{1+\lambda_1}\right)=f(1,0)\left(\frac{1+\lambda}{1+\lambda_1}\right)^n. \tag{11}$$

Since the integral over all space of $Q(n,\mathbf{x})$ is just $f(n,0)$ we find that

$$\int d\mathbf{x}\, Q(n+1,\mathbf{x})=\left(\frac{1+\lambda}{1+\lambda_1}\right)^n\int d\mathbf{x}\, Q(1,\mathbf{x}). \tag{12}$$

This shows how the total number of galaxies changes with time if $\lambda\neq\lambda_1$. For example, a cosmological model in which the mean densities of galaxies decreases with time can be modeled with $\lambda_1>\lambda$ and interpreting Q as the number density of galaxies. In this case, equation (10) admits solutions of the form $f(n,\mathbf{k})=A(\mathbf{k})\exp(\alpha n)$ with

$$A(\mathbf{k})=A(\bar{\mu}\mathbf{k})\left(\frac{1+\lambda W(\mathbf{k})}{1+\lambda_1}\right)e^{-\alpha}. \tag{13}$$

This equation has the same form as (5) with the factor $(1+\lambda)$ replaced by $(1+\lambda_1)e^{\alpha}$ so that the solution is

$$A(\mathbf{k}\bar{\mu})=A\left(\frac{\mathbf{k}}{\bar{\mu}^N}\right)\prod_{n=0}^{N}\frac{(1+\lambda_1)e^{\alpha}}{1+\lambda W(\mathbf{k}/\bar{\mu}^n)}. \tag{14}$$

Taking the logarithms, followed by the limit $N\to\infty$, we get the result

$$\ln A(\mathbf{k}\bar{\mu})=\sum_{n=0}^{\infty}\ln\left[\frac{(1+\lambda_1)e^{\alpha}}{1+\lambda W(\mathbf{k}/\bar{\mu}^n)}\right]+\ln A(0). \tag{15}$$

The convergence of the product in the right hand side (14) is now more tricky. If $W(\mathbf{k}/\bar{\mu}^n)$ becomes close to unity for sufficiently large n, then we will pick up a factor $p=(1+\lambda_1)(1+\lambda)^{-1}e^{\alpha}$ in each of the terms. Unless this factor is unity, the product in (14) will either diverge or vanish. Thus we get the condition $p=1$ for convergence thereby making equation (14) identical to (5). Thus, when the normalization condition in (2) is relaxed, we again get the same $k-$dependence as in (9) with an extra time (n) dependence of the form

$$\exp(\alpha n)=\left(\frac{1+\lambda}{1+\lambda_1}\right)^n \tag{16}$$

which takes into account the condition (12). This clearly shows that nothing changes qualitatively as far as the spatial dependence is concerned by relaxing (2).

We thus have three possible steady state solutions $f_s=[0,\infty,(kL)^{\gamma}]$ *none* of which incorporates *arbitrary* initial conditions. Obviously if the system is started at any of these solutions, it will stay in it without any evolution. The question arises as to whether any of them acts as a fixed point for the system evolving from a *nontrivial* initial condition or even random Poisson initial conditions for which $|f_{in}(k)|^2=1$. While this question is difficult to analyze in the discrete model, it can be answered using a much more detailed description of the system in the continuum limit.

3. A MODEL FOR STRUCTURE FORMATION - CONTINUUM LIMIT

In the continuum limit we need to study the system at two infinitesimally separated moments in time t and $t+\Delta t$ and obtain a partial differential equation for the evolution of $Q(t,x)$ or — equivalently — for $f(t,k)$. We will also need to change the parameters λ and $\bar{\mu}$ to $\lambda\Delta t$ and $(1+\mu\Delta t)$ respectively for consistency; this has the effect of making the *rate* of creation and *rate* of stretching finite, as it should. Equation (5) now becomes

$$f(t+\Delta t,\mathbf{k})=\frac{f(t,\mathbf{k}(1+\mu\Delta t))}{1+\lambda}\left[1+\lambda W(k)\right]. \tag{17}$$

We have assumed that W depends only on $|\mathbf{k}|$ making f also depend only on $|\mathbf{k}|$. Expanding the equation retaining up to linear terms in Δt, and using the result

$$k^{\alpha}\frac{\partial f(k)}{\partial k^{\alpha}}=k\frac{\partial f}{\partial k} \tag{18}$$

we get the final partial differential equation satisfied by f to be

$$\frac{\partial f}{\partial t}-\mu k\frac{\partial f}{\partial k}=-\lambda f(1-W). \tag{19}$$

The general solution to this equation is straightforward to obtain; we find that

$$\ln f(t,k)=G(ke^{\mu t})+\frac{\lambda}{\mu}\int_0^k\frac{dq}{q}\left[1-W(q)\right] \tag{20}$$

where G is an arbitrary function of its argument with the condition $G(0)=0$. [This condition incorporates our normalization condition (2). This condition can be relaxed in exactly the same manner as in the discrete case; however, as we shall see later, our conclusions do not change.] This function - in turn - can be expressed in terms of the initial condition for the problem $f(t=0,k)\equiv f_{in}(k)$, which is assumed to be known. Doing this we can write the solution in the form

$$f(t,k)=f_{\rm in}(ke^{\mu t})\exp\left[-\frac{\lambda}{\mu}\int_k^{ke^{\mu t}}\frac{dq}{q}\left[1-W(q)\right]\right]. \tag{21}$$

We shall now study the properties of the solution.

Let us first consider the simple case in which W is given by equation (8). In this case the solution is found to be

$$f(t,k)=\begin{cases} f_{in}(ke^{\mu t}) & (k\le L^{-1}e^{-\mu t})\\ f_{in}(ke^{\mu t})e^{-\lambda t}(kL)^{-\lambda/\mu} & (L^{-1}e^{-\mu t}\le k\le L^{-1})\\ f_{in}(ke^{\mu t})e^{-\lambda t} & (L^{-1}\le k)\end{cases} \tag{22}$$

At any finite t, there is a range of k values for which the power spectrum (which is proportional to $|f|^2$) is a power law with the index $(-2\lambda/\mu)$. But note that as $t\to\infty$ the solution decays exponentially at all scales for a wide class of initial conditions (including Poisson distribution with $f_{in}=1$). More generally, if $f_{in}(k)\propto k^{\beta}$, then the solution in the three ranges go as

$$f(t,k)\propto\begin{cases} k^{\beta}\exp(\beta\mu t)\\ k^{(\beta-\frac{\lambda}{\mu})}\exp\left[\mu(\beta-\frac{\lambda}{\mu})t\right]\\ k^{\beta}\exp\left[\mu(\beta-\frac{\lambda}{\mu})t\right].\end{cases} \tag{23}$$

For generic values of the parameters (β,λ,μ) *all* the solutions tend to either zero or infinity at late times. The only exception is if we choose the initial spectrum with $\beta=(\lambda/\mu)$. Then, for $kL<1$, we get a pure power law $k^{(\lambda/\mu)}$. This is precisely the solution we found in the discrete case [see equation (9)], since in the continuum limit, we can set

$$\gamma\equiv\left.\frac{\ln(1+\lambda)}{\ln\bar{\mu}}\right|_{\rm discrete}\to\frac{\ln(1+\lambda\Delta t)}{\ln(1+\mu\Delta t)}=\frac{\lambda}{\mu}. \tag{24}$$

It must be emphasised that only a very special choice can lead to a nontrivial steady state solution. Even with this special choice we get a solution which has *no* time dependence at all. It is, however, possible to have a power law solution that is valid for a large range of k at any finite t but with amplitude decreasing exponentially.

Equation (21) can be explicitly integrated for several cases of W. A particularly simple case is the one with

$$W(\mathbf{l}) \propto \exp(-l/L); \qquad W(\mathbf{k}) = \frac{1}{1+k^2L^2}. \tag{25}$$

In this case the solution is given by

$$f(t,k) = f_{\rm in}(ke^{\mu t}) \left[\frac{1+k^2L^2}{1+k^2L^2e^{2\mu t}}\right]^{\frac{\lambda}{2\mu}}. \tag{26}$$

As $\mu t \to \infty$, the solution goes to

$$f(t\to\infty,k) \simeq f_{in}(\infty)\left(1+k^2L^2\right)^{\lambda/2\mu}(kL)^{-\lambda/\mu}e^{-\lambda t} \tag{27}$$

which is again in conformity with the results obtained above, thereby showing that they were not an artifact of the sharp cutoff assumed in equation (8).

Finally, we briefly mention the results in the continuum limit when the normalisation condition (2) is relaxed. In this case, the factor $(1+\lambda)$ in the denominator on the right hand side of equation (17) will be replaced by a new constant $(1+\lambda_1)$. This changes equation (19) and the solution (21) to

$$\frac{\partial f}{\partial t} - \mu k\frac{\partial f}{\partial k} = -\lambda f\left(\frac{\lambda_1}{\lambda} - W\right), \tag{28}$$

$$f(t,k) = f_{\rm in}(ke^{\mu t})\exp\left[-\frac{\lambda}{\mu}\int_k^{ke^{\mu t}}\frac{dq}{q}\left[\frac{\lambda_1}{\lambda} - W(q)\right]\right]. \tag{29}$$

Writing the factor inside the integral as

$$\left(\frac{\lambda_1}{\lambda} - W\right) = (1-W) + \frac{(\lambda_1-\lambda)}{\lambda}, \tag{30}$$

it is trivial to see that the solution in this case is the same as the solution with $\lambda_1 = \lambda$ multiplied by the factor $\exp[-(\lambda_1-\lambda)t]$. That is,

$$f(t,k)|_{\lambda_1\neq\lambda} = f(t,k)|_{\lambda_1=\lambda}\exp[-(\lambda_1-\lambda)t]. \tag{31}$$

This is exactly what one would expect based on our results in the discrete limit in which we found that the solution gets multiplied by a factor in (16). In the continuum limit, λ_1 and λ gets replaced by $\lambda_1\Delta t$ and $\lambda\Delta t$ and n becomes $t/\Delta t$. In the limit of $\Delta t \to 0$ the extra factor in (16) becomes

$$\begin{aligned}\lim_{\Delta t\to 0}\left(\frac{1+\lambda\Delta t}{1+\lambda_1\Delta t}\right)^{\frac{t}{\Delta t}} &= \lim_{\Delta t\to 0}\exp\left[\left(\frac{t}{\Delta t}\right)\ln\left(\frac{1+\lambda\Delta t}{1+\lambda_1\Delta t}\right)\right] \\ &= \exp[-(\lambda_1-\lambda)t]\end{aligned} \tag{32}$$

which is precisely the extra factor in (31). Thus, even in the continuum limit, our qualitative conclusions do not change when the condition (2) is relaxed.

4. A GENERIC CLASS OF PHYSICAL PROCESSES WITH NON-LOCAL SELF-REPLICATION

While the discussion above was modeled and motivated by galaxy formation in QSSC, the process described here has a much broader range of applicability. The fact that our results were obtained for a model that does not involve gravity explicitly (for example, neither Newtonian gravitational constant nor the fact that gravitational force varies as a power law is used in the "rule" mentioned above), suggests that results of the above kind could be quite general.

In fact, there exists several natural phenomena that are described by power law correlations (see eg. Mandelbrot (1983)). More often than not, such a correlation function seem to arise in a manner that does not depend critically on the details of the underlying dynamical model. It would be interesting to see whether one can provide a mathematical model with a minimal set of assumptions which can reproduce the power law correlation. One such minimal set of assumptions can be extracted from the above analysis and we can show that models based on these assumptions will have a generic behaviour.

Consider a dynamical process in which some physical quantity $Q(t,\mathbf{x})$ evolves in time in a manner which depends on its value non locally. Such an evolution can be treated in the discrete version in terms of a rule which allows one to compute $Q(n+1,\mathbf{x})$ in terms of $Q(n,\mathbf{x})$ where n represents the discretised version of time with, say, $t = n\tau$ and τ representing a convenient time interval. For example, the amount of bacteria in a culture, trees in an orchard, buildings in a city or the number of galaxies in some region of the universe could be studied by such prescription. To be more specific, we shall consider processes of the following kind: We start with a set of points in the D-dimensional space which represents the location of bacteria or galaxies or trees, say. We now generate a set of new points near each one of the points ("non-local self-replication"). (The bacteria creating new bacteria nearby or trees generating new trees nearby or even cities leading to the formation of new cities nearby seem reasonable.) Let the probability for any given new point to be located at a distance $\mathbf{l}$ from an old point is $W(\mathbf{l})d^D\mathbf{l}$. Next, we rescale each of the D dimensions by a factor $\bar\mu > 1$ thereby increasing the volume of available space. (This is useful in the case of growing bacterial culture or a jungle of trees or cities in order to avoid boundary effects which will limit the process. In the cosmological model mentioned above, this is a natural consequence of the expansion of the universe.) Finally we select a subset of particles in the central region such that the total number of particles remains the same. This step renormalizes the process back to the original situation so that the process can now be repeated with the new subset of points. This evolutionary rule can be stated mathematically in the form:

$$\begin{aligned}Q(n+1,\mathbf{x}) = \frac{1}{(1+\lambda)\bar\mu^D}\Big[&Q(n,\mathbf{x}/\bar\mu) \\ &+ \lambda\int Q(n,\frac{\mathbf{x}}{\bar\mu}-\mathbf{l})W(\mathbf{l})d\mathbf{l}\Big]\end{aligned} \tag{33}$$

where $\lambda, \bar\mu$ are constants with $\lambda > 0$ and $\bar\mu > 1$; D is the dimension of the space in which the vector $\mathbf{x}$ lives, and $W(\mathbf{l})$ is a probability function normalized to unity for integration over all $\mathbf{l}$. This is same as equation (1) with $\bar\mu^3$ replaced by $\bar\mu^D$. We shall also assume that Q is normalized in such a way that its integral over all $\mathbf{x}$ is unity. As discussed above, this can be easily relaxed if required.

It must be stressed that we can consider equation (33) as the basic postulate of this analysis rather than any physical

model described in the last paragraph (involving bacteria, trees, cities, galaxies ....). In particular: (i) we need not restrict to any specific form of $W(\mathbf{l})$ or a choice for dimension D; (ii) it is also not necessary to identify the vector $\mathbf{x}$ with the position vector in real space. The equation (1) can also describe quite effectively the power transfer in Fourier space when the vector $\mathbf{x}$ is actually identified with a Fourier space vector. There are phenomena, like fluid turbulence, in which our analysis can be applied by using the power spectrum as the basic variable in $\mathbf{k}$ space. The essential postulate will then be that power at nearby wave numbers is generated with a given probability. This could provide a tool for attacking a wide class of nonlinear phenomena.

5. CONCLUSIONS

Two point correlations functions which exhibit power law behaviour are very prevalent in nature. The analysis here suggests that two ingredients – which we have called (i) non local self replication and (ii) rescaling – can lead to such correlation functions fairly generically.

The first of these two ingredients (non local self replication) allows similar entities to be created at nearby locations in some space. This could be as varied as bacterium creating new bacterium nearby by cell division, trees creating trees nearby by seeding or galaxies creating fresh galaxies because of the existence of a creation field. If we take the space to be the Fourier domain, then transfer of power from a given wave number k to nearby wave numbers will also constitute such a process. This variety shows that the ingredient (i) is fairly generic and natural.

An immediate consequence of self replication as defined here is the development of correlations. This is because we have tacitly assumed that the members of the second generation are preferentially (in the probabilistic sense) produced near the original parents. This will naturally lead to the second generation to be correlated with the first, spatially. If the process takes place in the Fourier domain, then the correlation will arise in the space of wave numbers and the consequent relationship in real space will be more complicated; but even in this case correlation will definitely be established. (Of course, in principle one can also introduce negative correlation in the model by choosing the probability distribution to be an increasing function of distance near the origin.)

In the context of conventional galaxy formation due to gravitational instability, the development of correlation is more indirect and dynamical while the process described here is direct and kinematical. Ultimately, the kinematics of the model, encoded in the probability distribution function $W(\mathbf{l})$, need to be connected to an underlying model (say, the creation field in the case of QSSC) in order to provide the dynamical basis. While this is the basic paradigm in physics, it must be stressed that it suffers from two well known difficulties: (a) If the physical process is sufficiently complicated (fluid turbulence, galaxy formation ...), it just may not be possible to provide such a dynamical underpinning. (b) Modeling different processes separately could lead one to miss the existence of a very general description for widely different classes of phenomena. Much of the initial attraction for fractals in the description of natural phenomena originated from its promise to provide such a unifying perspective. The mathematical formalism developed here should be viewed against such a backdrop.

The second ingredient that we have used (rescaling) essentially serves to renormalise the scales and is, in fact, very similar to ideas used in the theory of renormalization group. Once self replication takes place, the system has become denser on the whole and in order to concentrate on the intrinsic correlation, it is necessary to renormalise the system back to the original state. In all the examples which we have it is always possible to link this rescaling with a tangible physical process.

The combination of these two ingredients very nicely and naturally leads to a system with higher level of correlation at each time step. In fact, at least in the case of galaxy formation, the combined effect of these two phenomena is very similar to actual gravitational attraction between the particles.

The key result of the analysis is that processes with these two ingredients are inherently unstable in the sense that the correlation function either grows without bound or decays to zero (with the system becoming more and more dilute) as $t \to \infty$. Once again such an instability is reminiscent of similar phenomena seen in self gravitating systems though we have not used any gravitational dynamics. There is one special initial condition which leads to a static solution but as we discussed in the text, this is rather too special. The interest in these two systems lies in the intermediate time scale during which it could exhibit a power law correlation function very generically. Once again the situation is similar to galaxy formation due to gravitational instability in which the observed power law correlation function will exist *only* during a limited temporal and spatial window in the numerical simulations. Even in the case of fractals, it is generally known that one has to introduce cut-off in spatial and temporal scales in order to maintain power law correlation functions.

In conclusion, it is interesting how a fairly simple mathematical model could lead to an approximate description of a wide class of phenomena. Only further investigations for specific contexts will determine whether such a unified treatment is of some value or whether these ideas are destined to remain as mathematical curiosities.

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