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An Alternative Cosmology

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1. Introduction

The subject of astroparticle physics has originated from an alliance of convenience between particle physicists and cosmologists. For the particle physicists the expected dividends are briefly as follows.

- (i) The big bang cosmology provides a setting when the universe has temperature as high as $10^{15} - 10^{19}$ GeV when the dynamics of grand unification, supersymmetry and quantum gravity would be applicable. Thus cosmology offers the only working scenario for these theories.
- (ii) The various brands of esoteric non baryonic dark matter as yet not discovered in the particle laboratories may play a vital role in structure formation in the universe. Thus cosmological information on large scale structure and the radiation background can help place limits on masses, lifetimes etc. of these particles.

For the big bang cosmologists the inputs from particle physics are also expected to bring the following benefits :

- (i) The large scale structure of the universe and its relationship to the homogeneity of the microwave background appear to be easier to understand in terms of the nature of matter and its interaction in the very early universe.
- (ii) The transition of the universe from the big bang to the stage of primordial nucleosynthesis is not yet, understood properly thus leaving such issues like photon/baryon number ratio, the origin of nucleons etc. to be explained by ideas from particle physics.

However, I wish to begin by highlighting the speculative nature of this alliance. So far as particle physics is concerned there is *no laboratory experiment* which can test the full dynamics of grand unification. Thus *without* big bang cosmology, these ideas would largely remain pure speculation. So far as cosmology is concerned, the discussions of what the universe was like 10^{-43} s - 10^{-32} s after the big bang would also be purely speculative without very high energy particle physics. Thus neither component of the alliance can stand on its own as a completely scientific theory.

I think this point needs to be underscored if only to alert the student to the maxim that two speculations when matched together for consistency do not make

hard facts ! They would keep their minds open for other possible theories of cosmology that seek to explain all the observed phenomena without recourse to the above astroparticle alliance.

I will first highlight, as a cosmologist, my reservations on big bang cosmology, as the last word on the origin and evolution of the universe. I will then describe an alternative theory that is more readily testable than the big bang cosmology. Particle physicists may rest assured that their ideas will find an even greater applicability in this new alternative than in the standard big bang model.

2. The Strengths and Weaknesses of the Big Bang Cosmology

Let me first summarize the strong points of big bang cosmology, for which the theory is held to be so popular.

a) Historically the theory is credited with three predictions which were subsequently confirmed by observations. These are as follows :

1922-1924 : A. Friedmann [1, 2] constructed models of the expanding universe which were vindicated by Hubble's 1929 observations of nebular redshifts that were found to increase in proportion to distances of the nebulae from us [3]. The Friedmann models indeed predict such a relationship.

1946-1948 : G. Gamow and his colleagues [4, 5] calculated the primordial abundances in a hot big bang; these, so far as light nuclei are concerned, were subsequently (in the 1960s and 1970s) observationally verified.

1948 : R. A. Alpher and R. C. Herman [6] had predicted in 1948 that we should observe a relic radiation of temperature ~ 5 K from the early hot epochs. The discovery of the microwave background by A. A. Penzias and R. W. Wilson in 1965 confirmed this expectation [7]. The isotropy and the black body spectrum observed subsequently are consistent with the above relic interpretation.

Thus since 1965 the big bang cosmology has enjoyed the accolade of a 'theory along the right lines'.

- b) General relativists were very impressed by the global singularity theorems of Penrose, Hawking and Geroch [for details, see 8] in the late 1960s that spacetime singularity *a-la-big bang* was inevitable provided certain normal equations of energy-momentum positivity were obeyed by the universe. This led to the conviction that the big bang was not an artefact of a special kind of solution from general relativity but rather a *generic* feature of the theory.
- c) Astronomical observations of galaxies, radio sources, quasars etc. have claimed that the universe has been evolving [see for example, 9-10]. The idea of an evolutionary universe is consistent with that of big bang and hence such claims

are often taken as supporting the big bang hypothesis.

- (d) The astroparticle physicists are impressed by the big bang claim that there cannot be more than 3 neutrino flavours in view of the limits set by observations of primordial helium [11], for, their accelerator experiments have also led them to the same conclusion.

The run of big bang successes, however, ends here ! In fact in the late 1970s, great expectations were roused by the astroparticle alliance that the respective ends of physics and cosmology were within sight [12-14]. Hard facts and theoretical brainstorming have now shown that holy grail to be distant if not altogether quite elusive. Indeed, the remarkable advances of observational astronomy in the 1980s have shown the universe to be a much more complicated structure than assumed in the big bang model. Let us now see where the big bang falls short of expectations.

(i) How universal is Hubble's law ?

Hubble's law and the interpretation of redshift as arising from the expanding universe are taken for granted. Yet, there are several features which are puzzling with regard to redshift of extragalactic objects. These puzzles may not all be serious but they do contain some hitherto unknown fact of nature. Taken at face value they cannot be fitted into the simple expanding universe interpretation.

(ii) Is the microwave background primordial ?

The relic interpretation of the microwave background rests on its accurately measured spectrum, and to a lesser extent, on its observed anisotropies (see lectures by E. Wright in the School). Now spectrum of a black body radiation can be produced in other ways also : it does not necessarily have to be a relic of the hot big bang. I will demonstrate this here. Anisotropies of the type found by COBE [15] have served to eliminate a large class of theories attempting to form structures in a big bang universe. Those that still survive do so by a delicate tuning of parameters and could hardly be considered 'proved' by COBE.

Thus there is a strong case in the post-COBE phase to look for a non-relic interpretation of the microwave background, for a theory that explains its origin, spectrum and anisotropies without recourse to a hot big bang. Indeed, the long line of unsuccessful theories of structure formation in a big bang universe is evidence enough for the uncommitted to try other scenarios.

(iii) Does an evolving universe imply big-bang ?

In the 1950s and the early 1960s cosmological debate ran along simple lines with the observers trying to decide (from their data) between the big bang and the steady state cosmologies. In the latter cosmology, the universe, in the large is supposed to be unchanging with time. So observational studies were devoted towards deciding whether the universe was indeed similar in the past to what it is now or whether it has evolved. As a choice between the two theories therefore 'evolution'

was considered favouring big bang.

However, today, when there is no other theory in the field, what does evolution tell us? The objects for which evolution is claimed have redshifts not exceeding ~ 4 . Thus the claim for big bang is made by extrapolating from $z = 4$ to $z = \infty$, indeed a big jump in the argument. As I will show in my alternative cosmology, there are other ways of understanding the data and that the conclusion of big bang can be easily avoided.

(iv) Can big bang relate to high energy astrophysics?

The evidence of galactic and extragalactic astronomy increasingly and insistently shows that there is an ongoing explosive activity in the universe that feeds such powerful cosmic sources as radio galaxies, X-ray sources, quasars, gamma ray bursts, active galactic nuclei etc. This activity is certainly more recent than the big bang and cannot be linked to it. In the 1960s there was a half-hearted attempt to look upon these phenomena as 'delayed bangs' and 'lagging cores' of the main big bang event. This did not get very far.

Instead, today's popular paradigm is that of a massive black hole surrounded by an accretion disc. The disc is supposed to get hot by friction/viscosity and radiate. Also, the material falling into the disc somehow manages (in part) to escape as a jet along the axis of rotation. Considerable efforts have gone into modelling this scenario. However, when it comes to hard facts there is hardly any direct evidence. Neither the black hole nor the disc is seen; instead of the infall to a black hole we rather see outpouring of matter and energy. Several epicycles are needed to explain these observations. Nor do such explosive phenomena have any relationship to the primordial explosion, the big bang. Ideally, we need a theory which links the high energy phenomena in astrophysics directly to the cosmological properties. The big bang cosmology does not provide any scenario for this type of linkage.

3. Creation of Matter as the Primary Cosmological Phenomenon

Although I have highlighted the QSOs, AGN, etc. as recently discovered phenomena, there has always been indications from the early part of this century that violent activity with outpouring of matter and energy has been going on in the nuclear regions of galaxies. Back in 1929 Sir James Jeans [16] had this to say:

"The type of conjecture which presents itself somewhat insistently is that the centres of the nebulae are of the nature of 'singular points' at which matter is pouring into our universe from some other, and entirely extraneous dimension, so that, to a denizen of our universe, they appear as points at which matter is being continuously created."

In the early 1960s V. A. Ambartsumian [17] had been insisting that the key phenomena of high energy astrophysics were the explosive manifestations of matter production in centres of galaxies. However, the 'fashionable' viewpoint centred

around blackholes and accretion discs and the idea was not seriously followed up, even though a framework for describing matter creation already existed in the steady state cosmology. We will take it up now.

First let us look at the Hilbert action principle which describes general relativity and hence the standard hot big bang model, to see where and how it fails to describe matter creation in a self consistent matter. The action is given by

$$A = - \sum_a \int_{\Gamma_a} m_a c ds_a + \int_v \frac{c^3}{16\pi G} R \sqrt{-g} d^4x, \quad \dots(1)$$

where v is an arbitrary spacetime volume with coordinates $x^i (i=0, 1, 2, 3, x^0 \text{ timelike, } x^\mu, \mu = 1, 2, 3 \text{ spacelike})$, R the curvature scalar, g the determinant of the metric tensor g_{ik} , a, b, \dots labels of particles whose worldlines $\Gamma_a, \Gamma_b, \dots$ cross v , with ds_a the element of proper time and m_a the mass of a^{th} particle. c is the speed of light and G the gravitational constant.

The variation

$$\delta A / \delta g^{ik} = 0 \quad \dots(2)$$

leads to the Einstein field equations

$$R_{ik} - \frac{1}{2} g_{ik} R = - \frac{8\pi G}{c^4} T_{ik} \quad \dots(3)$$

where T_{ik} is the energy tensor of matter. The field equations guarantee matter energy conservation via the identity.

$$T_{ik} ; k \equiv 0. \quad \dots(4)$$

Thus prima facie there is no scope for matter creation in this framework. How then does the big bang manifest itself? The sequence of arguments is as follows. First we simplify the ten interlinked partial differential equations (3) by the special cosmological assumption of the Weyl postulate and the cosmological principle. (I will not go into this standard work here, but refer the readers to standard text books, e.g.[18].) The result is the Robertson-Walker line element

$$ds^2 = c^2 dt^2 - S^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \quad \dots(5)$$

Here (r, θ, ϕ) are the comoving coordinates of a typical fundamental cosmological observer and t is his proper time, also known as the cosmic time. This time is used to order the various large scale events in the universe chronologically. The event of big bang itself is given by $t=0$, when $S=0$.

The cosmological discussions are confined to $t > 0$ and one is not supposed to extend the worldlines of the fundamental observers to the past of $t = 0$. Notice, however, that the conservation laws consequent on the action principle (2) break-down at $t = 0$. Thus the action principle is also supposed to operate at $t > 0$.

In theoretical physics the general procedure follows is the reverse of what is done here. The standard practice is to have an overriding symmetry principle determining the specific solutions. If a specific solution fails the mathematical criteria of continuity, etc., it is probed further and modified and made respectable. Here in cosmology, a solution that fails at $S = 0$ is made the starting point and the general principle is sacrificed in its favour! Clearly, the correct recourse should be to see how the physics near $S = 0$ can be treated so that such contradictions don't arise. This approach leads us to a more satisfactory theory of creation of matter. Let us see how we need to modify the existing framework to accommodate matter creation.

To (1) add two extra terms to get :

$$\mathcal{A} = -\sum_a \int_{\Gamma_a} m_a c ds_a + \int_V \frac{c^3}{16\pi G} R \sqrt{-g} d^4x - \frac{1}{2c} \int_V C_i C^i \sqrt{-g} d^4x + \sum_a \int_{\Gamma_a} C_i dd^i \quad \dots(6)$$

where C is a scalar field and $C_i = \partial C / \partial x^i$, f is a coupling constant. The last term of (6) is manifestly path-independent and so, at first sight it appears to contribute no new physics. The first impression, however, turns out to be false if we admit the existence of broken worldlines.

Thus, if the worldline of particle a begins at point A , then the variation of \mathcal{A} with respect to that worldline gives

$$m_a \frac{da^i}{ds_a} = g^{ik} C_k \quad \dots(7)$$

at A . In other words, the C -field balances the energy-momentum of the created particle.

The field equations likewise get modified to

$$R_{ik} - \frac{1}{2} g_{ik} R = -\frac{8\pi G}{c^4} \left[T_{ik} + T_{ik}^c \right] \quad \dots(8)$$

where

$$T_{ik}^c = -f \left\{ C_i C_k - \frac{1}{2} g_{ik} C^l C_l \right\} \quad \dots(9)$$

Thus the energy conservation law is

$$T_{m;k}^{ik} + T_{m;k}^c = -T_{m;k}^{ik} = f C^i C^k ;_k \quad \dots(10)$$

That is, matter creation via a nonzero left hand side of (10) is possible while conserving the overall energy and momentum.

The C -field tensor has negative stresses which lead to the expansion of spacetime, as in the case of inflation. The formalism described here is essentially that used by Hoyle and Narlikar [19-21] in the 1960s to produce inflation type solution (which, of course predated Guth's inflationary cosmology by 15 years!). More recently Hoyle, Burbidge and I [22-24] have used a conformally invariant set of field equations, instead of the above relativistic ones, to arrive at essentially the same picture. Since the formalism used here is more familiar to students of particle physics and general relativity, I will proceed with it to the next step.

From (7) we therefore get a necessary condition for creation as

$$C_i C^i = m_a^2 c^4; \quad \dots(11)$$

This is the 'creation threshold' which must be crossed for particle creation. Now this can happen near a massive object, as can be seen from the following simple example.

The Schwarzschild solution for a massive object \mathcal{M} of radius $R > 2GM/c^2$ is

$$ds^2 = c^2 dt^2 \left(1 - \frac{2GM}{c^2 r} \right) - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad \dots(12)$$

for $r \geq R$. Now if the C -field does not seriously change the geometry, we would have at $r \gg R$,

$$C = \alpha, \quad C^i = \frac{\partial C}{\partial r} \equiv 0. \quad \dots(13)$$

If we continue this solution closer to $r = R$, we find that

$$C^i C_i = \left(1 - \frac{2GM}{c^2 r} \right)^{-1} \frac{\alpha^2}{c^2}. \quad \dots(14)$$

In other words $C_i C^i$ increases towards the object and can become arbitrarily large if $r = 2GM/c^2$. So it is possible for the creation threshold to be reached near a massive collapsed object even if $C_i C^i$ is below the threshold far away from the object. In this way massive collapsed objects can provide new sites for "matter creation."

4. A cosmological Solution

Since the C -field is a global cosmological field, we expect the creation phenomenon to be globally cophased. Thus, there will be phases when the creation activity is large, leading to generation of the C -field strength in large quantities.

However, the C-field growth because of its large negative stresses leads to a rapid expansion of the universe and a consequent drop in its background strength. When that happens creation is reduced and takes place only near the most collapsed massive objects thus leading to a drop in the intensity of the C-field. The reduction in C-field slows down the expansion, even leading to local contraction and so to a build-up of the C-field strength. And so on!

We can describe this up and down type of activity as an oscillatory solution superposed on a steadily expanding de Sitter type solution of the field equations as follows. For the line element (5) the equations (8)-(10) give

$$3 \frac{\dot{S}^2 + kc^2}{S^2} = 8\pi G (\rho - \frac{1}{2} f \dot{C}^2), \quad \dots(15)$$

$$2 \frac{\dot{S}^2}{S} + \frac{\dot{S}^2 + kc^2}{S^2} = 4\pi G f \dot{C}^2. \quad \dots(16)$$

The de Sitter type solution is given by

$$k = 0, \quad \dot{C} = \text{constant}, \quad \rho = \text{constant}$$

The oscillatory solution is given by

$$k = +1, \quad \dot{C} \propto 1/S^3, \quad \rho \propto 1/S^3. \quad \dots(18)$$

Thus (15) becomes, in the latter case

$$\dot{S}^2 = -c^2 + \frac{A}{S} - \frac{B}{S^4}, \quad A, B = \text{constant}. \quad \dots(19)$$

Here the oscillatory cycle will typically have a period $Q \ll P$.

Although the exact solution of (19) will be difficult to obtain, we can use the following approximate solution of (18) and (19) to describe the short-term and long-term cosmological behaviour:

$$S(t) = \exp\left(\frac{t}{P}\right) \left\{ 1 + \alpha \cos \frac{2\pi t}{Q} \right\}. \quad \dots(20)$$

Note that although the universe has a secular expanding behaviour, because $|\alpha| < 1$, it also executes non-singular oscillations around it. We can determine α and our present epoch $t = t_0$ by the standard tests of cosmology which include (i) the redshift magnitude relation (ii) the counts of radio sources and (iii) the angular size redshift relation.

For example, in the redshift magnitude relation for galaxies magnitude m increases with redshift z but afterwards the redshift decreases and m also decreases! This happens because we are now encountering galaxies belonging to the previous cycle of oscillation. This behaviour is different from that of standard hot big bang cosmology, and hence more precise astronomical observations of the future should be able to distinguish between the two models.

5. The Origin of Nuclei and the Microwave Background

We have as yet not said what particle is being created by the C-field. The answer is, the Planck particle whose mass is

$$m_p = \sqrt{\frac{3h}{4\pi G}} \sim 10^{-5} g \quad \dots(21)$$

This particle, however, has a very short lifetime

$$\tau_p \sim \sqrt{\frac{Ch}{c^5}} \sim 5 \times 10^{-44} s \quad \dots(22)$$

It decays ultimately the baryon octet and radiation. Most members of the octet except n and p are also short-lived and decay into protons. Only the neutron and the proton live long enough to combine into helium nuclei. Thus approximately 25 % by mass (2 out of 8 baryons) combine to form helium.

A more careful calculation gives the helium mass fraction to be around 23%, with a tiny fraction of 1-2 % in the form of metals. This type of nucleosynthesis also generates ^2H , ^3H , ^3He , ^6Li , ^7Li , ^9Be , ^{10}B etc. in small amounts that are in agreement with the observations. In fact, the light nuclear abundances in this cosmology lead to a better agreement with observations than in the big bang model.

There is one further important consequence. In the big bang model the required production of deuterium imposes a stringent upper limit on the present day baryon density. This limit forces us to assume that the dark matter component of the universe must be largely nonbaryonic. In the QSSC, there is no such density limit from deuterium abundance and thus the dark matter component can be baryonic. We will discuss this point further in the following section.

What about the microwave background? The QSSC obtains it in the following way. First, each Planck particle decay is like a fireball: it produces lot of energy, including baryons ($\sim 10^{19}$ per Planck particle) and radiation. Rather than the hot big bang, the Planck fireball can provide several interesting studies in astroparticle physics. Further, since the Planck fireballs are repeated phenomena, rather than the 'once only' type situation of the hot big bang cosmology, they are amenable to a more exhaustive scientific study.

Bulk of the fireball energy goes into expansion. However, some radiation remains as relic of the fireball. Together with the starlight generated in the preceding oscillatory cycles this energy is to be thermalized to provide the microwave background. Does it provide enough radiant energy to give a 2.7 K background? Is the background thoroughly thermalized to produce a black body spectrum? Also, is it homogeneous to the extent given by COBE and other measurements?

The answer to all these questions is in the affirmative as we shall now demonstrate.

Let us consider the present day energy density of intergalactic starlight which comes largely from old star populations. Taking into consideration the usual estimates of 10^{-14} erg cm^{-3} for starlight in the visible spectral region, we get the total (in all wavelengths) starlight energy density as $\sim 2 \times 10^{14}$ erg cm^{-3} . Using (20) with the present epoch as $t_0 = 0.85 Q$ we estimate the average starlight production rate per unit volume per unit time as

$$\epsilon = 1.14 \times 10^{-13} \text{ erg } cm^{-3} Q^{-1} \quad \dots(23)$$

Thus the total amount of starlight produced from the previous oscillatory minimum at $t = -0.5 Q$ to $t = 0.5 Q$ (the last minimum) is

$$\epsilon \int_{-0.5Q}^{0.5Q} \frac{dt}{1+z} = 4.56 \times 10^{-13} \text{ erg } cm^{-3} \quad \dots(24)$$

Now the weakening [due to the expansion factor (exp t/P) in each cycle] of energy density W_{\min} at the oscillatory minimum is $4Q/P \times W_{\min}$. This must be replenished by fresh thermalization of starlight produced. Thus in the steady state we get for $P = 20 Q$,

$$W_{\min} = 1.14 \times 10^{-13} \times \frac{P}{Q} = 2.28 \times 10^{-12} \text{ erg } cm^{-3} \quad \dots(25)$$

This is with respect to coordinate volume. To convert it to proper volume we need to introduce the redshift effect from $t = 0.5 Q$ to $t = t_0$, which is a factor 0.1734. Thus the present energy density is

$$W_{\text{present}} = W_{\min} \times 0.1734 = 3.96 \times 10^{-13} \text{ erg } cm^{-3} \quad \dots(26)$$

which corresponds to a black body temperature of 2.68 K - very close to that observed!

Thus quantity-wise the starlight from several past generations of stars is sufficient to maintain a steady background of radiation, provided, some agency is available to thermalize it. The agency proposed is dust in the form of metallic needles, mostly of iron which absorb the ambient radiation and reradiate it in the microwaves. Provided this has gone on long enough, the radiation spectrum will approach the black body form. Calculation shows that indeed the thermalization has occurred through as many as 10^3 absorptions and remissions by iron whiskers-sufficient to ensure an extremely close approximation to the black body curve. The iron whiskers are typically ~ 1 mm in length and 10^{-6} cm in radius of cross section. The iron itself is produced partly from stellar nucleosynthesis in supernovae. The required density in the form of such whiskers is only $\sim 10^{-35}$ g cm^{-3} well within the observed cosmic abundances of iron.

The background produced will be very smooth with a patchiness of density of the order of 10^{-5} . Fluctuations of density and temperature of this or larger order get smoothed out by redistribution of iron grains by the radiation pressure. On smaller scales the dynamical smoothness-restoring forces are too small to make the radiation smooth. Thus, the COBE finding of $\Delta T/T \sim 10^{-5}$ is consistent with the above picture. Moreover, the characteristic scale of 10^{26} cm at the oscillatory minimum will expand to $\sim 5 \times 10^{26}$ cm at present, giving a characteristic angular scale for the above patchiness to be $\sim 10^\circ$, in conformity with the COBE scale of angular inhomogeneity.

6. The MCEs and Astrophysics

The minicreation events (MCEs) have several points of contact with astrophysics. I will briefly enumerate a few :

- (i) **Gravity wave sources** : The explosive creation near compact massive objects makes them potential sources of gravity waves, provided the events are sufficiently anisotropic. Narlikar and Das Gupta [25] have shown that such events can be detected by the laser interferometric detectors being planned worldwide. Further, the gravity wave background created by such MCEs can also significantly affect the timing mechanism of millisecond pulsars.
- (ii) **High energy sources** : The explosive nature of energy generation in QSOs and AGN as well as in the gamma ray burst sources makes the MCEs ideal candidates for these energy sources. This is in keeping with Ambartsumian's conjecture.
- (iii) **The age of the universe** : According to QSSC the universe is infinitely old but the average age of astronomical objects is $1/3P \sim 3 \times 10^{11}$ yrs. This makes many clusters much older than hitherto assumed. Even our Galaxy has age of this order with several generations of stars formed, evolved and burst out. Thus the dark matter component in the Galaxy may be largely made of burnt out stars.
- (iv) **Hierarchy of structures** : The largest structure to form in the MCEs is the so called supercluster of mass $\sim 10^{15} - 10^{16} M_\odot$. There are, however, MCEs on smaller scales going right down to galactic nuclei with masses $\sim 10^6 - 10^7 M_\odot$. It is, however, the former that keep the universe going steady state at all times.

7. Concluding Remarks

This approach is intended more in the spirit of opening up the field of cosmology to other ideas than those provided by the standard big bang cosmology. Thus the QSSC, like any scientific theory is to be judged vis-a-vis the big bang cosmology by how it performs in explaining the observed large scale features of the

universe and what testable future predictions it can make.

Even in this alternative cosmology there is considerable scope for inputs from high energy particle physics, in particular (i) in giving a more fundamental description to the C-field formalism (which is described here only classically) and (ii) in working out the details of how the created Planck particle decays to baryons.

Thus the area of astroparticle physics continues to be relevant in this cosmology also !.

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UNIFORM CONVERGENCE AND INTEGRATION

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ABSTRACT

In this note we characterize those measure spaces (X, Σ, μ) for which wherever a sequence (f_n) of integrable functions converges uniformly to a function f then f is also integrable and those measure spaces for which whenever a sequence (f_n) of integrable functions uniformly converges to an integrable function f then the sequence $\int f_n d\mu$ converges to $\int f d\mu$. We also provide a proof of monotone convergence theorem using uniform convergence.

Introduction :

Let (X, Σ, μ) be a measure space with Σ a σ -algebra and μ a measure. It is well known [S] that if $\mu(X)$ is finite then whenever a sequence (f_n) of real valued integrable functions converges uniformly to a real valued function f then f is integrable and the sequence $(\int f_n d\mu)$ converges to $\int f d\mu$. P. R. Halmos [H, p. 111] showed, by giving two examples over \mathbb{N} , that neither integrability nor integral is preserved for the limit function under uniform convergence. These examples can be adapted verbatim over \mathbb{R} yielding that in the space of Lebesgue integrable functions over \mathbb{R} , uniform convergence neither preserves integrability nor integral.

Hence it seems worthwhile to characterize :

- (1). those measure spaces for which whenever a sequence (f_n) of integrable real valued functions converges uniformly to a real valued function f then f is also integrable. This is provided in Theorem 1.5; and
- (2). those measure spaces for which whenever a sequence (f_n) of integrable real valued functions converges uniformly to an integrable real valued function f then the