A PROPOSAL FOR S-DUALITY BREAKING

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WITH ASHOK DAS

• SYMMETRIES ARE GUIDING PRINCIPLES TO UNDERSTAND LAWS OF FUNDAMENTAL INTERACTI ONS.

• EXACT SYMMETRIES LEAD TO CONSERVATION LAWS.

• BROKEN SYMMETRIES ALSO PLAY IMPORTANT ROLES.

(i) BROKEN GLOBAL SYMMETRIES , ISOSPIN AND ITS GENERALIZATION, $\mathbf{SU(3)}$, PCAC, CURRENT ALGEBRA IN LOW ENERGY STRONG INTERACTIONS. APPROXIMATE SYMMETRIES HAVE INTERESTING CONSEQUENCES.

(ii) SPONTANEOUSLY BROKEN GAUGE SYMMETRIES, HIGG'S PHENOMENA.

(ii) LOCAL SYMMETRIES ARE PRINCIPAL PARADIGMS FOR THE LAWS OF FUNDAMENTAL INTERACTIONS SUCH AS ELECTRO-WEAK THEORY, QCD AND EINSTEIN'S THEORY.

STANDARD MODEL OF PARTICLE PHYSICS: $\mathbf{SU(3)_C \otimes SU(2) \otimes U(1)}$

QCD IS VERY WELL TESTED EXPERIMENTALLY IN THE PERTURBATIVE REGIME.

THE PREDICTIONS OF E-W THEORY ARE IN EXCELLENT AGREEMENT WITH DATA.

THERE ARE STILL SOME MISSING PARTICLES: (i) THE HIGGS, (ii) AXION AND (iii) SUSY PARTNERS - IF SUSY IS TO RESOLVE GAUGE HIERARCHY PROBLEM.
HIGGS MAY BE FOUND IN LHC AND WE MAY SEE SUSY PARTICLES.

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INTRODUCTION

◇ **AXION IS A MYSTERIOUS PARTICLE, IS INTERESTING AND IMPORTANT.**

◇ **IT WAS INTRODUCED TO SOLVE THE STRONG CP PROBLEM. IT IS EXPECTED TO BE VERY LIGHT AND IS VERY WEAKLY INTERACTING.**

◇ **RECALL: IN QUARK MODEL OF HADRONS, WITH THREE FLAVOURS, THE LOW ENERGY DESCRIPTION OF MESONS IS QUITE WELL UNDERSTOOD BY AN EFFECTIVE $\mathbf{SU(3)} \otimes \mathbf{SU(3)}$ WHICH IS SPONTANEOUSLY BROKEN AND THE MESONS ARE INTERPRETED AS GOLDSTONE PARTICLES.**
EIGHT OF THE MESONS ARE 'LIGHT' TAKING INTO ACCOUNT EFFECTS OF FLAVOURS: HOWEVER THE NINETH MESON, η', IS MUCH HEAVIER AND THIS IS THE $\mathbf{U(1)}$ PROBLEM.

◇ **'t Hooft PROVIDED A RESOLUTION OF THIS ISSUE. HE ARGUED THAT QCD VACUUM IS QUITE COMPLICATED DUE TO THE PRESENCE OF INSTANTON SOLUTIONS AND THE QCD LAGRANGIAN (EFFECTIVELY) HAS AN EXTRA TERM**

$$\mathcal{L} = -\mathrm{Tr}\left(\frac{\mathbf{1}}{\mathbf{4g^2}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + \frac{\theta\mathbf{g^2}}{\mathbf{16}\pi^2}\mathbf{F}_{\mu\nu}\widetilde{\mathbf{F}}^{\mu\nu}\right),$$

g IS THE YANG-MILLS COUPLING CONSTANT, θ , A PARAMETER, IS CALLED THE VACUUM ANGLE.

$$\widetilde{\mathbf{F}}^{\mu\nu} = \frac{\mathbf{1}}{\mathbf{2}}\epsilon^{\mu\nu\lambda\rho}\mathbf{F}_{\lambda\rho},$$

THE SECOND TERM, A PSEUDOSCALAR DENSITY, MULIPLYING VACUUM ANGLE , IN THE LAGRANGIAN DOES NOT CONTRIBUTE TO EQUATION OF MOTION.

- **IN THE STANDARD MODEL, QCD AND E-W THEORY TAKEN TOGETHER, THE PHASE OF THE MASS-MATRIX ALSO CONTRIBUTES TO THE PSEUDOSCALAR DENSITY. THEREFORE, θ, SHOULD BE UNDERSTOOD AS**

$$\theta_{\mathrm{eff}} = \theta + \mathrm{Arg}\,\det\mathbf{M}$$

THE PRESENCE OF SUCH A TERM VIOLATES P AND T (THEREFORE, CP).

- **CONSEQUENCE: IT INDUCES AN ELECTRIC DIPOLE MOMENT OF NEUTRON.**
- **THERE ARE STRINGENT EXPERIMENTAL BOUND ON THE DIPOLE MMENT AND THIS SEVERELY RESTRICTS**

$\theta < 10^{-9} - 10^{-10}$.

NOW ON WE DENOTE θ_{eff} *AS* θ

• WITHOUT ANY NATURAL JUSTIFICATION WHY θ IS SO SMALL ?

PECCEI-QUINN PROPOSAL:

ACCORDING TO PQ , θ, IS PROMOTED TO A DYNAMICAL PSEUDOSCALAR FIELD, χ .

THE THEORY IS ENDOWED WITH AN ADDITIONAL GLOBAL $\mathbf{U(1)}$ SYMMETRY. THE SYMMETRY SPONTANEOUSLY BROKEN THROUGH CHOICE OF A POTENTIAL SO THAT χ DEVELOPS A VEV AND IT IS THE θ PARAMETER.

• SINCE VEV CAN BE ADJUSTED TO BE SMALL ONE CAN RESOLVE THE SO CALLED STRONG CP PROBLEM THROUGH PQ MECHANISM - AXION IS THE GOLDSTONE BOSON OF PQ SYMMETRY.

• PREDICTIONS OF ORIGINAL P-Q MODEL HAVE BEEN RULED OUT BY EXPERIMENTS AND MANY VARIATIONS OF THE MODEL HAVE APPEARED.

THE ESSENTIAL FEATURES ARE: AXION IS WEAKLY INTERACTING, LONG LIVED VERY LIGHT PSEUDOSCALAR PARTICLE. THE PARAMETERS ARE:

$$\mathbf{m}_\chi \simeq \mathbf{10^{-4}}\mathrm{eV}, \quad \mathbf{f}_\chi \simeq \mathbf{10^{9}} - \mathbf{10^{12}}\mathrm{GeV},$$

$\mathbf{f}_\chi$: **DECAY CONSTANT. THE AXION COUPLING TO THE PSEUDOSCALAR DENSITY IS**

$$\mathcal{L}_\chi = -\frac{\zeta}{\mathbf{f}_\chi}\frac{\mathbf{g^2}}{\mathbf{16}\pi^\mathbf{2}}\,\mathrm{Tr}\left(\chi \mathbf{F}_{\mu\nu}\widetilde{\mathbf{F}}^{\mu\nu}\right),$$

ζ : **MODEL DEPENDENT PARAMETER.**
VEV OF χ IS RELATED TO θ-PARAMETER THROUGH A MULTIPLICATIVE CONSTANT.

- **SO FAR THERE IS NO DIRECT EXPERIMENTAL EVIDENCE IN FAVOUR OF AXION.**

S-DUALITY SYMMETRY IN FIELD THEORY

WE CAN REWRITE THE YANG-MILLS LAGRANGIAN WITH θ TERM AS

$$\mathcal{L} = -\frac{1}{16\pi}\,\mathrm{Im}\Big(\mathrm{Tr}(\pm\,\mathbf{i}\tau\mathbf{F}^{\mu\nu\pm}\widetilde{\mathbf{F}}^{\pm}_{\mu\nu})\Big) \tag{1}$$

WHERE $\tau = \pm\frac{\theta g^2}{2\pi} + \frac{4i\pi}{g^2}$

$\mathbf{F}^{\pm}_{\mu\nu} = \mathbf{F}_{\mu\nu} \pm \mathbf{i}\widetilde{\mathbf{F}}_{\mu\nu}$.

NOTE THAT θ HAS PERIOD 2π.

THE EQUATIONS OF MOTION ARE INVARIANT UNDER

$\tau \to \tau + 1$ AS WELL AS $\tau \to -\frac{1}{\tau}$.

THERE IS MORE: UNDER FRACTIONAL TRANSFORMATION

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

AND

$$\tau\mathbf{F}^{+}_{\mu\nu} \to (a\tau + b)\mathbf{F}^{+}_{\mu\nu}, \quad \bar{\tau}\mathbf{F}^{-}_{\mu\nu} \to (a\bar{\tau} + b)\mathbf{F}^{-}_{\mu\nu}$$

$a, b, c, d \in \mathbf{Z}$ AND $ad - bc = 1$ AND "BAR" OVER τ DENOTES COMPLEX CNJUGATION. AND THIS IS IS DISCRETE S-DUALITY TRANSFORMATION: $SL(2, \mathbf{Z})$.

REMARKS:

(I) THE PRECEDING ACTION IS NOT INVARIANT UNDER THE DUALITY GROUP.

(II)THE S-DUALITY SYMMETRY IS MANIFEST IN THE Y-M SECTOR AT THE LEVEL OF EQUATIONS OF MOTION. IT WILL BE GRATIFYING IF WE CAN PROMOTE IT TO FULLY INTERACTING THEORY - INCLUDING FERMIONS.

(III)S-DUALITY SYMMETRY APPEARS IN SOME SUPERSYMMETRIC FIELD THEORIES WITH LARGE NUMBER OF SUPERSYMMETRIES, IN SOME SUPERGRAVITY THEORIES AS WELL AS IN STRING THEORIES.

(IV)WE CONSIDER CONTINUOUS S-DUALITY TRANSFORMATIONS (SEE LATER) AND INTRODUCE ONLY ONE ADDITIONAL SCALAR FIELD - A PARTNER OF THE AXION. THIS SCALAR COUPLES TO THE Y-M SCALAR DENSITY JUST AS AXION COUPLES TO PSEUDOSCALAR DNSITY.

(IV) THE COUPLING IS S-DUALITY INVARIANT AT THE LEVEL OF EQUATIONS OF MOTION.

(V) WE DENOTE S-DUALITy PARTNER OF χ AS DILATON, ϕ. NO STRING THEORETIC ARGUMENT IS INVOKED IN OUR DISCUSION OF S-DUALITY IN WHT FOLLOWS.

(VI) IT SUFFICES IF ϕ IS A VERY LIGHT, WEAKLY

INTERACTING SCALAR - SUCH A SCALAR HAS BEEN INTRODUCED IN THE CONTEXT OF JORDAN-BRANS-DICKE GRAVITY.

SINCE IT IS PARTNER OF AXION, IT SHOULD SHARE MOST OF THE ATTRIBUTES OF AXION.

- **WE BELIEVE: S-DUALITY IS A FUNDAMENTAL SYMMETRY OF NATURE ALTHOUGH IT IS BROKEN**
- **OUR GOAL IS PRAGMATIC AND APPROACH IS PHENOMENOLOGICAL.**

DETOUR ON S-DUALITY: •S-DUALITY HAS BEEN A VERY FERTILE DOMAIN OF RESEARCH IN QUANTUM FIELD THEORY, SUPERSYMMETRIC THEORIES, SUPERGRAVITY AND STRING THEORY.

•THERE ARE MANY ROBUST RESULTS, ESPECIALLY IN THEORIES WITH LARGE NUMBER OF SUPERSYMMETRIES.

•IN THE CONTEXT OF STRING THEORIES S-DUALITY HAS PLAYED A CRUCIAL ROLE IN UNRAVELLING NONPERTURBATIVE ASPECTS OF STRING THEORY.

• A HOST OF VERY IMPORTANT RESULTS IN STRINGY BLACK HOLE PHYSICS ARE EXPLOITED USING THIS SYMMETRY.

THE MODEL

• WE INTEND TO CONSTRUCT S-DUALITY INVARIANT ACTIONS FOR THE INTERACTING FIELDS.

• OUR S-DUALITY GROUP IS $\mathbf{SL(2,R)}$ IT IS ISOMORPHIC TO $\mathbf{SU(1,1)}$.

• THE LIE ALGEBRA IS GIVEN BY

$$[\mathbf{T_1},\mathbf{T_2}]=-\mathbf{i}\mathbf{T_3},\quad [\mathbf{T_2},\mathbf{T_3}]=\mathbf{i}\mathbf{T_1},\quad [\mathbf{T_3},\mathbf{T_1}]=\mathbf{i}\mathbf{T_2},$$

• NOTE THE SIGN OF THE FIRST COMUTATOR WHICH IS SIGNAL THAT $\mathbf{SL(2,R)}$ IS A NONCOMPACT

•IT IS NOT POSSIBLE TO HAVE FINITE DIMENSIONAL UNITARY REPRESENTATIONS OF $\mathbf{SL(2,R)}$ UNLIKE $\mathbf{SU(2)}$. WE CANNOT CHOOSE ALL THE FINITE DIMENSIONAL GENERATORS, $\{\mathbf{T_i}\}$ TO BE HERMITIAN.

• IT SUFFICES FOR TO CONSIDER THE TWO DIMENSIONAL REPRESENTATION AND OUR CHOICE OF THE GENERATORS IS:

$\mathbf{T_1}=\frac{1}{2}\sigma_1,\quad \mathbf{T_2}=-\frac{\mathbf{i}}{2}\sigma_3,\quad \mathbf{T_3}=\frac{1}{2}\sigma_2$

EXPLICITLY

$$\mathbf{2T_1}=\begin{pmatrix}0 & i\\ i & 0\end{pmatrix},\quad \mathbf{2T_2}=\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix},\quad \mathbf{2T_3}=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad (2)$$

AND $\mathbf{T_i}^{\dagger}\mathbf{T_3}=\mathbf{T_3}\mathbf{T_i}$. $\{\sigma_{\mathbf{i}}\}$ REPRESENT THE PAULI MATRICES.

AN ALTERNATIVE REPRESENTATION CAN BE CHOSEN FOR THE GENERATORS:

$\overline{\mathbf{T}}_1=\frac{\mathbf{i}}{2}\sigma_1,\quad \overline{\mathbf{T}}_2=\frac{\mathbf{i}}{2}\sigma_2,\quad \overline{\mathbf{T}}_3=\frac{1}{2}\sigma_3$

THESE TWO SETS ARE RELATED TO ONE ANOTHER BY A UNITARY TRANSFORMATION:
$\mathcal{S}\mathbf{T_i}\mathcal{S}^\dagger = \overline{\mathbf{T}}_\mathbf{i}, \quad \mathcal{S} = \frac{1}{\sqrt{2}}(\mathbf{1} - \mathbf{i}\sigma_1)$
[A $\mathbf{2 \times 2}$ MATRIX BELONGING TO $\mathbf{SL(2, R)}$ CAN BE EXPRESSED AS:

$$\mathbf{\Omega = e^{-i\alpha_k T_k}} \tag{3}$$

WHERE $\alpha_\mathbf{k}, \mathbf{k = 1, 2, 3}$ THREE REAL PAPAMETERS. SINCE THE GENERATORS ARE ALL IMAGINARY THE TRANSFORMATION MATRIX IS REAL. MOREOVER, IT FOLLOWS FROM PROPERTIES OF THE GENERATORS THAT
$\mathbf{\Omega^\dagger T_3 \Omega = \Omega^T T_3 \Omega = T_3}, \quad \mathbf{\Omega^T T_3 = T_3 \Omega^{-1}}$
$\mathbf{2T_3}$ CAN BE THOUGHT OF AS THE METRIC IN THE GROUP SPACE- $\mathbf{(2T_3)^2 = 1}$.
UNDER A FINITE $\mathbf{SL(2, R)}$ TRANSFORMATION, A VECTOR (DOUBLET FOR US) TRANSFORMS AS

$$\mathbf{\Psi \to \Omega\Psi} \tag{4}$$

WHILE A MATRIX IN THE ADJOINT REPRESENTATION TRANSFORMS AS

$$\mathbf{M \to \Omega M \Omega^T} \tag{5}$$

AS IS WELL KNOWN THE AXION-DILATON DOUBLET PARAMETRIZE THE COSET, $\frac{SL(2,R)}{U(1)}$ IN THE FORM

$$\mathbf{V} = \begin{pmatrix} e^{-\phi} + \chi^2 e^{\phi} & \chi e^{\phi} \\ \chi e^{\phi} & e^{\phi} \end{pmatrix} \tag{6}$$

SINCE $\mathbf{V} \in \frac{SL(2,R)}{U(1)}$, WE CAN EXPAND THIS MATRIX AS LINEAR COMBINATION OF UNIT MATRIX AND THE THREE GENERATORS. FOR THE CASE AT HAND,

$$\mathbf{V} = \mathbf{v_0 1} - \mathbf{2iv_1T_1} + \mathbf{2iv_2T_2} \tag{7}$$

WITH

$$\mathbf{v_0} = \frac{1}{2}(\mathbf{e}^{-\phi} + \chi^2\mathbf{e}^{\phi} + \mathbf{e}^{\phi}), \tag{8}$$

$$\mathbf{v_1} = \chi\mathbf{e}^{\phi}, \tag{9}$$

$$\mathbf{v_2} = \frac{1}{2}(\mathbf{e}^{-\phi} + \chi^2\mathbf{e}^{\phi} - \mathbf{e}^{\phi}) \tag{10}$$

SINCE $\mathbf{V} \in \mathbf{SL(2,R)}$, $\det \mathbf{V} = \mathbf{1}$, THEN $\{\mathbf{v_0}, \mathbf{v_1}, \mathbf{v_2}\}$ SATISFY THE CONSTRAINT

$$\mathbf{v_0^2} - \mathbf{v_1^2} - \mathbf{v_2^2} = \mathbf{1} \tag{11}$$

AND V SATISFIES

$$\mathbf{V^T} = \mathbf{V}, \text{ AND } \mathbf{VT_3} = \mathbf{T_3V^{-1}} \tag{12}$$

NOTE: AXION AND DILATON AS DEFINED SO FAR CARRY ZERO CANONICAL DIMENSION, UNLIKE CONVENSIONAL DIMENSION OF A SCALAR FIELD.

WE CAN ASSIGN USUAL DIMENSION, 1, BY RESCALING THEM

$$\phi \to \mathbf{f_s}^{-1}\phi, \quad \chi \to \mathbf{f_s}^{-1}\chi \tag{13}$$

$\mathbf{f_s}$: IDENTIFIED WITH SCALE OF SYMMETRY BREAKING.

IN THIS FORMULATION, IT IS NOT POSSIBLE

TO WRITE INTERACTION LAGRANGIAN OF AXION-DILATON-YANG-MILLS FIELDS IN A MANIFESTLY S-DUALITY AND LORENTZ INVARIANT FORM. HOWEVER, RESULTING EQUATIONS OF MOTION FROM OUR ACTION ARE DUALITY AND LORENTZ INVARIANT.

AXION-DILATON ACTION

$$\mathcal{L}_{(\chi\phi)} = -\frac{\mathbf{f_s}^2}{\mathbf{4}} \mathrm{Tr}\, \partial_\mu \mathbf{V}^{-1} \partial^\mu \mathbf{V} \tag{14}$$

THIS ACTION REDUCES TO THE CONVENTIONAL AXION-DILATON ACTION WHEN WE WRITE V IN TERMS OF ITS FIELD CONTENTS.

$$\mathcal{L}_{\chi\phi} = \frac{\mathbf{f_s}^2}{\mathbf{2}} \Big(\partial_\mu \phi \partial^\mu \phi + \mathbf{e}^{\mathbf{2}\phi} \partial_\mu \chi \partial^\mu \chi \Big), \tag{15}$$

THE S-DUALITY INVARIANCE OF THE ABOVE ACTION LEADS TO CONSERVED CURRENTS.

$$\mathbf{J}^\mu_{(\chi\phi)} = -\mathbf{2i}\mathbf{V}^{-1}\partial^\mu \mathbf{V} \tag{16}$$

IN COMPONENT FORM: $\mathbf{J}^{\mu}_{(\chi\phi)\,\mathbf{i}} = \mathrm{Tr}\,\mathbf{T_i J}^{\mu}_{(\chi\phi)}$. AXION-DILATON-FERMION COUPLING STEP I. WE INTEND TO CONSTRUCT S-DUALITY INVARIANT FREE FERMION ACTION.

CONSIDER A FOUR COMPONENT SPINOR, ψ MAY BE DIRAC OR MAJORANA.

STEP II. CONSTRUCT A DOUBLET OF $\mathbf{SL(2,R)}$ AS

$$\Psi = \frac{\mathbf{1}}{\mathbf{2\sqrt{2}}}\begin{pmatrix}\big((1-\gamma_5)+i(1+\gamma_5)\big)\psi\\ \big((1+\gamma_5)+i(1-\gamma_5)\big)\psi\end{pmatrix} \tag{17}$$

$\gamma_5 = \mathbf{i}\gamma^0\gamma^1\gamma^2\gamma^3$. UNDER $\mathbf{SL(2,R)}$, Ψ TRANSFORMS AS

$$\Psi \to \Omega\Psi \tag{18}$$

THE DIRAC ADJOINT OF THIS DOUBLET WHICH TRANSFORMS INVERSELY UNDER LORENTZ TRANSFORMATION ASSUMES THE FORM

$$\overline{\Psi} = \frac{\mathbf{1}}{\mathbf{2\sqrt{2}}}\left(\,\bar\psi\big((1+\gamma_5)-i(1-\gamma_5)\big) \quad \bar\psi\big((1-\gamma_5)-i(1+\gamma_5)\big)\,\right)$$

REMARK: WE MAY GENERATE GRADIENT COUPLING OF THE AXION-DILATON DOUBLET WITH FERMIONS FROM THE FREE FERMION LAGRANGIAN BY THE TRICK OF REDEFINING THE FERMIONIC FIELD AS IS KNOWN FROM THE PCAC DAYS.

$$\Psi \to \mathbf{e}^{\mathbf{i g_{2Y}V(2T_3)}}\Psi, \qquad \widetilde{\overline{\Psi}} \to \widetilde{\overline{\Psi}}\mathbf{e}^{-\mathbf{i g_{2Y}V(2T_3)}}. \tag{19}$$

NOTE THAT THE NONDERIVATIVE INTERACTION CANNOT BE GENERATED BY FERMION FIELD REDEFINATION - CHECK EXPLICITLY.

- **FROM THE POINT OF VIEW OF S-DUALITY TRANSFORMATION IT IS MORE USEFUL TO DEFINE AN ALTERNATIVE ADJOINT OF THE DOUBLET,** $\boldsymbol{\Psi}$.

THIS TRANSFORMS INVERSELY UNDER BOTH LORENTZ AND S-DUALITY TRANSFORMATION AND CAN BE UTILIZED TO CONSTRUCT DUALITY INVARIANT FREE FERMION LAGRANGIAN AND INTERACTION LAGRANGIAN.

$$\widetilde{\overline{\boldsymbol{\Psi}}} = \overline{\boldsymbol{\Psi}}(\mathbf{2T_3}) \tag{20}$$

- **UNDER AN** $\mathbf{SL(2,R)}$ **TRANSFORMATION**

$$\widetilde{\overline{\boldsymbol{\Psi}}} \to \overline{\boldsymbol{\Psi}}\boldsymbol{\Omega}^{\mathbf{T}}(\mathbf{2T_3}) \tag{21}$$

RECALL $\boldsymbol{\Omega}^{\mathbf{T}}\mathbf{T_3} = \mathbf{T_3}\boldsymbol{\Omega}^{-\mathbf{1}}$. **THUS**

$$\widetilde{\overline{\boldsymbol{\Psi}}} \to \widetilde{\overline{\boldsymbol{\Psi}}}\boldsymbol{\Omega}^{-1} \tag{22}$$

THE FREE FERMION LAGRANGIAN IS

$$\mathcal{L}_{(\psi)} = \mathbf{i}\widetilde{\overline{\boldsymbol{\Psi}}}\gamma_{\mathbf{5}}\partial\!\!\!/\boldsymbol{\Psi} \tag{23}$$

REWRITTEN IN TERMS OF FOUR COMPONENT FERMIONS: $\mathcal{L}_{(\psi)} = \mathbf{i}\bar{\psi}\partial\!\!\!/\psi$ **NOTE THAT THE FREE FERMION ACTION CAN BE WRITTEN IN A MANIFESTLY S-DUALITY FORM.**

THE ASSOCIATED CURRENT IS

$$\mathbf{J}^{\mu}_{(\psi)\mathbf{i}} = \widetilde{\overline{\boldsymbol{\Psi}}}\mathbf{T_i}\boldsymbol{\Psi} \tag{24}$$

THEREFORE, THE S-DUALITY CURRENT IS

$$\mathbf{J}^{\mu}_{\mathbf{i}} = \mathbf{J}^{\mu}_{(\chi\phi)\mathbf{i}} + \mathbf{J}^{\mu}_{(\psi)\mathbf{i}} \tag{25}$$

INTERACTIONS LET US CONSIDER INTERAC-

TION TERMS FOR AXION AND DILATON WITH QCD FERMIONS IN AN S-DUALITY INVARIANT FORM.

HERE WE HAVE TWO POSSIBILITIES: (i) NON-DERIVATIVE TYPE OR (ii) DERIVATIVE TYPE. FOR THE FORMER WE CHOOSE

$$\mathcal{L}_{1\mathbf{Y}} = -\mathbf{i}\mathbf{g}_{1\mathbf{Y}}\Lambda_{\mathrm{QCD}}\,\widetilde{\overline{\Psi}}\gamma_5\mathbf{V}(2\mathbf{T}_3)\Psi \tag{26}$$

THIS IS MANIFESTLY $\mathbf{SL(2,R)}$ INVARIANT SINCE

$$\mathcal{L}_{1\mathbf{Y}} \to -\mathbf{i}\mathbf{g}_{1\mathbf{Y}}\Lambda_{\mathrm{QCD}}\,\widetilde{\overline{\Psi}}\gamma_5\mathbf{V}(2\mathbf{T}_3)\Omega^{-1}\Omega\Psi \tag{27}$$

HERE THE YUKAWA COUPLING CONSTANT IS CHOSEN DIMENSIONLESS, CONSEQUENTLY THE INTERACTION STRENGTH INVOLVES A MASS SCALE. HERE THE ONLY MASS SCALE IS Λ_{QCD}

WRITTEN IN COMPONENT FIELDS

$$\mathcal{L}_{1\mathbf{Y}} = -\frac{\mathbf{g}_{1\mathbf{Y}}\Lambda_{\mathbf{QCD}}}{2}(\mathbf{e}^{-\phi} + \chi^2\mathbf{e}^{\phi} - \mathbf{e}^{\phi})\bar{\psi}\psi + \mathbf{i}\mathbf{g}_{1\mathbf{Y}}\Lambda_{\mathbf{QCD}}\,\chi\mathbf{e}^{\phi}\bar{\psi}\gamma_5\psi$$

THE GRADIENT TYPE INTERACTION IS INTRODUCED AS FOLLOWS:

$$\mathcal{L}_{2\mathbf{Y}} = -\mathbf{g}_{2\mathbf{Y}}\widetilde{\overline{\Psi}}(\partial\!\!\!/\mathbf{V})(2\mathbf{T}_3)\Psi \tag{28}$$

AND IN COMPONENT FORM

$$\mathcal{L}_{2Y} = -\frac{g_{2Y}}{2}(-e^{-\phi} + \chi^2 e^{\phi} + e^{\phi})\bar{\psi}\gamma_\mu\psi\partial^\mu\phi - g_{2Y}\chi e^{\phi}\bar{\psi}\gamma_\mu\psi\partial^\mu\chi$$

IT IS POSSIBLE TO GENERATE GRADIENT TYPE AXION-DILATON-FERMION INTERACTION FROM THE FREE FERMION LAGRANGIAN THROUGH REDEFINITION OF FERMION IELD AS IS KNOWN FROM THE PCAC DAYS.

$$\mathbf{\Psi \rightarrow e^{ig_{2Y}V(2T_3)}\Psi, \quad \widetilde{\overline{\Psi}} \rightarrow \widetilde{\overline{\Psi}} e^{-ig_{2Y}V(2T_3)}.} \tag{29}$$

S-DUALITY BREAKING

◇ **THERE IS NO COMPELING EXPERIMENTAL EVIDENCE IN FAVOUR OF EXACT S-DUALITY SYMMETRY IN NATURE FROM THE SCALES PROBED SO FAR.**

◇ **THIS SYMMETRY NEEDS TO BE BROKEN. IF AXION AND DILATON ARE S-DUALITY PARTNERS THEIR MASSES SHOULD BE SAME OF ORDER.**

◇ **(a) LIGHT WEAKLY INTERACTING AXION IS NOT OBSERVED IN LABORATORY AS WELL AS IN COSMOLOGICAL EXPERIMENTS-** $\mathbf{m_\chi \sim 10^{-4}}$ **eV. (b)THERE ESTIMATES OF MASS OF A LIGHT SCALAR (JBD) FROM CORRECTIONS TO GRAVITATIONAL FORCE AND LIMIT ON MASS IS OF SAME ORDER AS AXION.**

◇ **JUST A COINCIDENCE?**

◇ **WE ARE ENCOURAGED BY THESE COINCIDENCES TO PROPOSE THAT S-DUALITY IS A SYMMETRY AND INVESTIGATE.**

◇ **THE SYMMETRY BREAKING MECHANISM IS PARALLEL TO PCAC.**

◇ **HERE WE HAVE NONCOMPACT** $\frac{\mathbf{SL(2,R)}}{\mathbf{U(1)}}$

◇ **WE WANT TO GIVE SUITABLE VEV TO AXION SO THAT IT IS IDENTIFIED WITH THE** $\theta -$ PARAMETER

◇ **RECALL THAT $\mathbf{v_0^2 - v_1^2 - v_2^2 = 1}$ THIS DEFINES SURFACE OF A PSUDOSPHERE, AdS IN GROUP SPACE, CAN BE DESCRIBED BY CIRCLE ON THE UPPER HALF PLANE OF (REIMANN CIRCLE) ON COMPLEX PLANE.**

$$\mathbf{v_0} = \cosh\mathbf{\Sigma}, \quad \mathbf{v_1} = \sinh\mathbf{\Sigma}\cos\xi, \quad \mathbf{v_2} = \sinh\mathbf{\Sigma}\sin\xi$$

AND THE CONSTRAINT IS AUTOMATIC. AXION AND DILATON ARE RELATED TO THE NEW VARIABLES AS FOLLOWS

$$e^{-\phi} = \frac{1}{(v_0 - v_2)} = \frac{1}{(\cosh\Sigma - \sinh\Sigma\sin\xi)}$$
$$\chi = \frac{v_1}{(v_0 - v_2)} = \frac{\sinh\Sigma\cos\xi}{(\cosh\Sigma - \sinh\Sigma\sin\xi)}.$$

THEREFORE, WE CAN GIVE VEV TO BOTH DILATON AND AXION CONSISTENT WITH THE CONSTRAINT:

$$\langle\mathbf{\Sigma}\rangle = \mathbf{\Sigma_0}, \quad \langle\xi\rangle = \xi_\mathbf{0}$$

$\mathbf{\Sigma_0}$ AND $\xi_\mathbf{0}$ **ARBITRARY CONSTANTS AND THESE TRANSLATE INTO VEV'S**

$$\langle\mathbf{e}^{\phi}\rangle = \mathbf{e}^{\phi_\mathbf{0}} = \cosh\mathbf{\Sigma_0} - \sinh\mathbf{\Sigma_0}\sin\xi_\mathbf{0},$$

$$\langle\chi\rangle = \chi_\mathbf{0} = \mathbf{e}^{-\phi_\mathbf{0}}\sinh\mathbf{\Sigma_0}\cos\xi_\mathbf{0}$$

WE OBSERVE THAT

$\langle\mathbf{v_0}\rangle = \cosh\mathbf{\Sigma_0}$, $\langle\mathbf{v_1}\rangle = \sinh\mathbf{\Sigma_0}\cos\xi_\mathbf{0}$, $\langle\mathbf{v_2}\rangle = \sinh\mathbf{\Sigma_0}\sin\xi_\mathbf{0}$

• A DIRECT CONSEQUENCE OF GIVING VEV'S IS THAT NONDERIVATIVE AND GRADIENT COUPLINGS INTRODUCED ABOVE LEAD TO TRILINEAR INTERACTIONS INVOLVING AXION, DILATON AND FERMIONS. • THE NONDERIVATIVE YUKAWA COUPLING WILL GENERATE MASSES FOR QCD FERMIONS DUE SPONTANEOUS S-DUALITY BREAKING. SHIFTING AROUND VEV'S IN A CONSISTENT MANNER i.e.

$$\phi \longrightarrow \phi_0 + \frac{\phi}{\mathbf{f_s}}, \quad \chi \longrightarrow \chi_0 + \frac{\chi}{\mathbf{f_s}}, \quad \mathbf{e}^{-\phi} \longrightarrow \mathbf{e}^{-(\phi_0 + \frac{\phi}{\mathbf{f_s}})} \tag{30}$$

THE NONDERIVATIVE TYPE INTRACTION LAGRANGIAN IS]

$$\mathcal{L}_{1Y} = -\mathbf{m_f}\bar{\psi}\psi + \mathbf{im}\bar{\psi}\gamma_5\psi - \frac{\mathbf{g_{1Y}}\Lambda_{\mathbf{QCD}}}{\mathbf{f_s}}(<\mathbf{v_2}>\phi + <\mathbf{v_1}>\chi)\bar{\psi}\psi$$
$$+ i\frac{g_{1Y}\Lambda_{QCD}}{f_s}e^{\phi_0}(\chi + \chi_0\phi)\bar{\psi}\gamma_5\psi + \ldots$$

WITH

$$\mathbf{m_f} = \mathbf{g_{1Y}}\Lambda_{\mathbf{QCD}} <\mathbf{v_2}>, \qquad \mathbf{m} = \mathbf{g_{1Y}} <\mathbf{v_1}> \tag{31}$$

THE GRADIENT TYPE INTERACTION IS NOW GIVEN BY

$$\mathcal{L}_{2Y} = -\frac{\mathbf{g_{2Y}}}{2\mathbf{f_s}}(-\mathbf{e}^{-\phi_0} + \chi_0^2\mathbf{e}^{\phi_0} + \mathbf{e}^{\phi_0})\bar{\psi}\gamma_\mu\psi\partial^\mu\psi$$
$$-\frac{\mathbf{g_{2Y}}}{\mathbf{f_s}}\chi_0\mathbf{e}^{\phi_0}\bar{\psi}\gamma_\mu\psi\partial^\mu\chi + \ldots.$$

◇ AXION-NUCLEAN-NUCLEAN COUPLING CONSTANT HAS BEEN ESTIMATED IN THE PAST IN VARIOUS MODELS USING INPUTS FROM QCD AND IS

$$\mathbf{g_{NN\chi}} \approx \mathbf{10^{-12}} \tag{32}$$

UP TO MULTIPLICATIVE CONSTANT $O(1)$. WE TAKE $\mathbf{g_{1Y}},\ \mathbf{g_{2Y}}$
TO BE SAME ORDER OF MAGNITUDE. IF THE TRILINEAR COUPLING CONSTANT IS OF THIS STRENGTH, THEN $\mathbf{m_f},\ \mathbf{m}$ WILL DEPEND OF PARAMETERS $\mathbf{\Sigma_0},\ \xi_0$

◇ NOTE THAT $\chi_0 \sim \theta$ IS CONSTRAINED TO BE VERY SMALL. THUS ξ_0 SHOULD BE NEAR $\frac{\pi}{2}$
THUS WE ESTIMATE THAT

$$m_f \approx g_{1Y}\Lambda_{QCD}\mathrm{sinh}\Sigma_0 \approx 10^{-11}\mathrm{GeV}, \quad m \approx 0 \tag{33}$$

UP TO A FINITE CONSTANT AND WE HAVE TAKEN $\Lambda_{\mathbf{QCD}} \approx \mathbf{1\ GeV}$ AND $\xi_0 \approx \pi/\mathbf{2}$
IN THIS APPROXIMATION,

$$\mathbf{e}^{\phi_0} \approx \cosh\mathbf{\Sigma_0} - \sinh\mathbf{\Sigma_0} \tag{34}$$

IF QCD COUPLING IS TAKEN TO BE REASONABLY SMALL, THEN CONTRIBUTION TO FERMION MASSES DUE TO S-DUALITY BREAKING IS REALLY VERY SMALL. THUS QCD SECTOR IS UNAFFECTED BY THIS PHENOMENA.

⋄ AT THIS STAGE AXION AND DILATON ARE MASSLESS GOLDSTONE BOSONS JUST AS THE CASE WITH PIONS IN CURRENT ALGEBRA TREATMENTS. EXPLICIT SYMMETRY BREAKING TERMS HAVE TO BE INTRODUCED ALONG CERTAIN DIRECTIONS TO GENERATE MASSES.

$$\mathcal{L}_{\mathbf{m}} = -\frac{\mathbf{f_s^2 m_s^2}}{\mathbf{2}} \frac{\mathbf{v_0}}{(\mathbf{v_0} - \mathbf{v_3})} = -\frac{\mathbf{f_s^2 m_s^2}}{\mathbf{2}} (\mathbf{1} + \mathbf{e}^{-\mathbf{2}\phi} + \chi^{\mathbf{2}}) \quad (35)$$

EXPANDING THE FIELDS AROUND VEV WE CAN READ OFF THE MASS TERMS - QUADRATIC IN FIELD FLUCTUATIONS.

$$\mathcal{L}_m = -\frac{f_s^2 m_s^2}{2} \Big[\frac{1}{f_s^2} (2e^{-2\phi_0} \phi^2 + \chi^2) - \frac{2}{f_s} (e^{-2\phi_0} \phi + \chi_0 \chi) + ... \Big]$$

THE SECOND TERM IS TADPOLE, AS USUAL. WE HAVE NOT SHOWN THE CONSTANT TERM AND ..... STAND FOR HIGHER POWERS OF $\frac{\mathbf{1}}{\mathbf{f_s}}$ WE CAN READ OFF THE MASSES:

$$\mathbf{m}_\chi^{\mathbf{2}} = \mathbf{m}_{\mathbf{s}}^{\mathbf{2}}, \quad \mathbf{m}_\phi^{\mathbf{2}} = \mathbf{2e}^{-\mathbf{2}\phi_{\mathbf{0}}} \mathbf{m}_\chi^{\mathbf{2}} \quad (36)$$

THUS AXION AND DILATON MASSES ARE RELATED THROUGH A POWER OF QCD COUPLING CONSTANT, AS WE HAVE IDENTIFIED: $\mathbf{e}^{\phi_{\mathbf{0}}} = \mathbf{g}_{\mathbf{YM}}^{\mathbf{2}}$.

EXCURSION TO σ-MODEL AND PCAC.

LET US HAVE A SHORT EXCURSION INTO σ-

MODEL AND PCAC. $SU(2) \otimes SU(2)$ ALGEBRA THE TRANSFORMATIONS ARE GENERATED BY SIX GENERALIZED CHARGES $Q^A, Q_5^A, \ A = 1, 2, 3$

$$\left[Q^A, Q^B\right] = i^{ABC} Q_C, \ \left[Q_5^A, Q_5^B\right] = i^{ABC} Q_C, \left[Q^A, Q_5^B\right] = i^{ABC} Q_{5C} \quad (37)$$

DEFINE: $Q_\pm^A = \frac{1}{2}(Q^A \pm Q_5^A)$. THEN $\{Q_+^A\}$ GO TO THEMSELVES SAME IS TRUE FOR $\{Q_-^A\}$. REST COMMUTE.

THE ALGEBRA IS LINEARLY REALIZED BY INTRODUCING THREE ISOVECTOR SCALARS IN ADDITION TO THE PIONS.

ALTERNATIVELY ONE MIGHT CONSIDER A SINGLET SCALAR SUCH THAT

UNDER

SUMMARY AND CONCLUSIONS

•WE PROPOSE THAT S-DUALITY IS A FUNDAMENTAL SYMMETRY OF NATURE.

• A PARTNER FOR AXION IS INTRODUCED, ϕ. THEY PARAMETRIZE THE COSET $\frac{SL(2,R)}{(U(1)}$. IT IS ASSUMED THAT VEV OF DILATON IS RELATED TO QCD COUPLING CONSTANT, JUST AS VEV OF AXION IS RELATED TO θ – PARAMTER. SINCE g^2_{YM} AND θ PARAMETRIZE DISCRETE S-DUALITY GROUP $SL(2, Z)$.

• IN MORE GENERAL FRAME WORK LIKE STRING THEORY, $e^{<\phi>}$ IS EXPECTED TO CONTROL ALL COUPLING CONSTANTS AND DILATON COUPLES TO ENTIRE MATTER SECTOR.

• A UNIVERSALITY PROPOSITION:
S-DUALITY IS AN UNIVERSAL SYMMETRY. THUS ALL FERMIONS: QUARKS AND LEPTONS COUPLE TO AXION AD DILATON IN AN S-DUALITY SYMMETRIC MANNER. HOWEVER THE COUPLING CONSTANTS AND SCALES IN QUARK SECTOR AND LEPTON SECTOR CAN BE TAKEN TO BE DIFFERENT.

• SCALES: (i) FOR QUARKS IT IS QCD SCALE AND (ii) LEPTONS E-W SCALE. SIMILARLY THE COUPLING CONSTANT FOR THE FORMER IS NUCLEON-NUCLEON-AXION COUPLING AND FOR THE LATTER IT IS ELECTRON-ELECTRON-AXION COUPLING - TO BE USED AS ESTIMATED IN MODELS.

• WE ARE INFLUENCED BY STRING THEORY. HOWEVER, THE ARGUMENT IS PHENOMENOLOGICAL AND SPECULATIVE HERE.
A SINGLE YUKAWA COUPLING FOR ALL QUARKS AND ANOTHER ONE FOR ALL LEPTONS.
IN LEPTON SECTOR WE TAKE IT TO BE

$$g_{\chi ee} \sim 10^{-15} \tag{38}$$

ASSUME, UP TO A MULTIPLICATIVE CONSTANT

$$\bar{\mathbf{g}}_{\mathbf{1Y}}, \bar{\mathbf{g}}_{\mathbf{2Y}} \sim \mathbf{g}_{\chi \mathbf{ee}} \sim \mathbf{10}^{-\mathbf{15}} \tag{39}$$

• CONSEQUENCES: S-DUALITY BREAKING CONTRIBUTES TO FERMION MASSES (BOTH QUARKS AND LEPTONS). AS IN QUARK SECTOR, LEPTONS ALSO GET SMALL MASS CORRECTIONS.
IN E-W THEORY, QUARKS AND LEPTONS GET THEIR MASSES DUE TO HIGGS VEV.
HOW ABOUT NEUTRINOS?
IF WE USE VALUE OF $\mathbf{g}_{\chi \mathbf{ee}}$ TO ESTIMATE EFFECT OF S-DUALITY BREAKING IN GIVING MASSES TO LEPTONS, WE FIND

$$m_\nu \sim \bar{g}_{1Y} \Lambda_{weak} \mathrm{sinh} \Sigma_0 \approx 10^{-2} eV \tag{40}$$

WE APPROXIMATED $\Lambda_{weak} \sim 1TeV$. OTHER LEPTONS ALSO GET MASS CORRECTIONS OF THIS ORDER.

- **AXION AND DILATON MIGHT BE COPIOUSLY PRODUCED IN ACCELERATORS, HOWEVER, THESE WILL NOT BE DETECTED BY THE DETECTORS SURROUND THE MACHINES SINCE THE PURPOSE IS TO LOOK FOR HIGGS, SUSY PARTICLES .... LIGHT, WEAKLY INTERACTING AXION AND DILATON CAN TRAVEL LONG DISTANCES TO THE DETECTORS DESIGNED TO CATCH THEM. IF TWO LIGHT PARTICLES OF SIMILAR MASS ARE EXPERIMENTALLY OBSERVED IT MIGHT POINTS TOWARDS A NEW SYMMETRY OF NATURE.**