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Performance of Newtonian Filters in the Detection of Gravitational Radiation from Coalescing Binaries

By

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Abstract

Post-Newtonian corrections to the gravitational waveform emitted by coalescing binaries have been found to lead to a secular phase accumulation error as compared with the signal calulated in the Newtonian approximation. The matched filtering process which relies on the correlation between the signal and the filter is extremely sensitive to errors in phase. We explore the possibility of compensating for the phase difference caused by the post-Newtonian terms by allowing for a shift in the Newtonian filter parameters. We find that, on the average, we lose by about 30% in the correlation.

Coalescing binaries are the most promising sources of gravitational waves [1,2] for laser interferometric gravitational wave detectors. There are plans to construct such interferometers around the globe and by the end of this century the American LIGO [3] and French/Italian VIRGO [4] will be in operation. Recently it has been shown that post-Newtonian (PN) corrections, spin-orbit (S.O.) and spin-spin (S.S.) couplings, produce in the waveform an accumulating phase error from the one computed from just the Newtonian expression [5]. Therefore, a template constructed from the Newtonian waveform would go out of phase with the signal and the so called "matched filtering" technique for detection would woefully fail. In this letter we show that as long as we are only *searching* for signals a Newtonian filter would perform remarkably well even though the signal contains PN corrections. The key idea here is that we allow the parameters of the Newtonian filter to vary and adjust them so as to produce the maximum possible correlation with the signal. We have found that this flexibility allows for fairly high values of the correlation. In many cases of interest the correlation obtained is 80% of its maximum possible value which would have been obtained had the template been perfectly matched to the signal. On the other hand, a template with the same parameters as those of the signal produces correlations of about 10 to 20%. Also, as a correspondence between the parameters of the filter and the signal could be set up, it might be possible to estimate the parameters of the signal from those of the filter. In other words the filter parameters may be "renormalized".

The signal waveform is constructed using the expression for the rate of change of phase given in [2]. Here we do not take into account the effects due to S.O. and S.S. coupling. The addition of such terms will not alter the thrust of the argument in that, some other Newtonian filter would perform best. We expect that this would change the results only quantitatively, but not qualitatively. Considering only the PN terms, following [2], we write down the equation for the rate of change of frequency,

$$\frac{\dot{f}}{f^2} = \frac{96\pi}{5}\frac{\mu}{M}\frac{x^{2.5}}{F(x)} \tag{1}$$

where,

$$F(x) = \frac{1 - \frac{3}{2}x - \frac{81}{8}x^2 - \frac{675}{16}x^3}{1 - \frac{1247}{336}x + 4\pi x^{1.5} - 4.9x^2 - 38x^{2.5} + 135x^3}$$

and $\dot{f}$ represents the first time derivative of frequency. Here M is the total mass of the binary, μ the reduced mass and $x = (\pi M f)^{2/3}$ the PN expansion parameter. We integrate this expression numerically to obtain the phase as a function of time. For the amplitude we use the Newtonian dependence on the frequency *i.e.* $A(f(t) \approx \text{const} \times f^{2/3}$. Although this is not exact, we do not expect the errors in the amplitude to affect the correlation significantly. The Newtonian expression for the frequency is obtained by setting $F(x) = 1$. We treat here only the white noise case when the filter is just a constant times the signal.

The filters are given by the Newtonian expression and are parameterized by ξ and ϕ. The parameter ξ is the time taken by the binary to coalesce from a certain fiducial frequency f_a, usually taken to be the lower cutoff of the detector bandwidth. Since the frequency monotonically increases with time for the coalescing binary signal f_a is the frequency at the time of arrival of the signal. The quantity ξ is related to the chirp-mass $\mathcal{M} = \mu^{3/5}M^{2/5}$ by the relation,

$$\xi = 3.003\left(\frac{\mathcal{M}}{M_\odot}\right)^{-5/3}\left(\frac{f_a}{100\text{Hz}}\right)^{-8/3} \text{sec}, \tag{2}$$

where $M_\odot$ is the mass of the sun ($M_\odot \simeq 2 \times 10^{33}$gms). The Newtonian filter is constructed so that at $t = 0$ the instantaneous frequency is f_a and phase is ϕ. For white noise the filters are given by,

$$q(t, \xi, \phi) = \mathcal{N} a(t)^{-1/4} cos[\frac{16\pi}{5} f_a \xi [1 - a(t)^{5/8}] + \phi], \tag{3}$$

where,

$$a(t) = 1 - \frac{t}{\xi}$$

and the $\mathcal{N}$ a normalization constant to be determined later.

We now correlate the post-Newtonian signal $s(t, \mu, M, \phi)$ with the Newtonian filter $q(t + \Delta t, \xi + \Delta\xi, \phi + \Delta\phi)$. Here although the signal depends on both the parameters M and μ, we nevertheless define ξ for the signal through the same equation (2), even though it does not represent the coalescence time for the signal. In the Fourier domain the correlation is given by,

$$C = 2\int_{f_a}^{f_u} \tilde{s}(f; \mu, M, \phi)\tilde{q}^*(f; \xi + \Delta\xi, , \phi + \Delta\phi)e^{-2\pi i f \Delta t} df. \tag{4}$$

We consider three cases for the frequency intervals (f_a, f_u) namely:

1. 100 - 400 Hz,
2. 40 - 400 Hz,
3. 10 - 100 Hz.

where f_u is the upper cutoff of the frequency range. In the first case the lower cutoff is taken fairly high and the upper cutoff f_u is taken so that $F(x)$ remains positive. For lower ranges of masses $F(x)$ in fact does not differ significantly from unity. The lower cutoff in case (2) corresponds to the early LIGO stage, while in case (3) it corresponds to the advanced LIGO. The upper cutoffs in case (2) and case (3) are taken 10 times the lower cutoffs. This would give 95% of the correlations if there had been no upper cutoffs and the signal waveform Newtonian.

We now maximise the correlation over the parameters by allowing variations in $\Delta t, \Delta\xi$ and $\Delta\phi$. The normalisations for the signal and the filter in the three cases have been chosen so that the integrals of s^2 and q^2 in the time domain, corresponding to the respective ranges of frequencies are unity. This implies that the values of the correlations are always less than one. The effectiveness of a filter is therefore guaged by the value of correlation as compared with unity (as would be the case if the filters were correctly matched). The results are tabulated in Table I for the three frequency ranges mentioned above and in the range of masses $0.5M_{\odot}$ to $10M_{\odot}$. Although the expression in equation (1) is valid only for the test mass case we nevertheless state the values of correlations for cases with comparable masses. We observe that the correlations on the average are about 0.7 rising to above 0.8 in several cases. The reason for these fairly high correlations is that the phase accumulation due to PN corrections is basically quadratic in time [6] and this is almost compensated by a filter whose chirp-mass slightly differs from that of the signal. The flexibility in the parameters of the filter allows for a fairly good matching. The $\Delta\phi$ is simply an additive correction to the phase; Δt corrects for the difference in frequencies between the filter and the signal, making it in fact close to zero near the middle of the time interval for which the signal lasts; $\Delta\xi$ corrects for the rate of change of frequency. Hence the Newtonian filters are able to correct for the phase upto its second time derivative. Figure 1(a) shows the phase difference between the signal and the filter which have the same value of ξ parameter while figure 1(b) shows the phase difference when the best matching filter is chosen. The above filter, we find, has necessarily different parameters. We notice that in figure 1(b) the phase difference is kept quite small over a long period of time and therefore a high correlation is to be expected.

We observe that for a shorter frequency range the matching is better. For example, in case (1) the correlations are higher than in case (2). Secondly shifting to lower frequencies gives a better correlation because of the relatively mild changes in the frequency since $\dot{f}$ is smaller (see equation (1)) and therefore the correlations in case (3) are on the whole higher than in case (2). We must of course bear in mind the normalizations assumed in the table. In absolute terms the correlations for the 40 - 400 Hz case will be greater than those corresponding to 100 - 400 Hz since the normalizations in the latter case for the filters are chosen larger than in the former. Any comparision should take into account these normalizations. Generally we find that for higher masses the correlations are lower, because of the drastic acceleration in the frequency as the masses fall towards each other. However, in case (3) even for very low masses the correlations are low. This behaviour we attribute to the large integration time in which it is difficult to find any Newtonian filter which matches well with the signal.

What can we say about the shifts in the parameters $\Delta t, \Delta\xi$ and $\Delta\phi$? In place of Δt a better parameter is the number of cycles $\Delta n = f_a \Delta t$. For $M_1 \gtrsim 5M_\odot$ and $M_2 \lesssim 2M_\odot$ and for the frequency ranges 40 - 400 Hz and 100 - 400 Hz, $\Delta t < 0$ and Δn lies between 2 and 3 *i.e.* the filter begins 2 to 3 cycles after the signal has arrived. This number comes down to between 0 and 2 cycles for 10 - 100 Hz. The $\Delta\phi$ adjusts itself to optimize the matching. Also in place of $\Delta\xi$, the chirp-mass $\Delta\mathcal{M} = \mathcal{M}_d - \mathcal{M}_s$ is the more suitable parameter for understanding these shifts. For the 100 - 400 Hz the $\Delta\mathcal{M}$ is larger and can vary from 5% for pairs comprised of lower masses to 17% for a $10M_\odot, 1.4M_\odot$ pair. For the larger ranges of frequency *i.e.* 40 - 400 Hz the $\Delta\mathcal{M}$ falls to about 1% growing upto to 4% for larger masses. For cases (1) and (2) the chirp-mass detected is always larger than the signal chirp-mass. However for the 10 - 100 Hz range $\Delta\mathcal{M} < 0$ *i.e.* the detected chirp-mass is smaller, but the change is less than 1%. For this frequency range, a small value of $\Delta\mathcal{M}$ suffices to adjust the phase optimally. Table II lists the values of $\Delta t, \Delta\mathcal{M}$ and $\Delta\phi$ for a few typical cases. It should be possible to use this type of correspondence to roughly deduce the chirp-mass of the signal from the value of the detected chirp-mass. The detected chirp-mass in general will correspond to several chirp-masses of the signal in a small neighbourhood. Also the arrival times could be estimated to a certain accuracy. Work on these aspects is in progress and will involve the study of the covariance matrix of the parameters of the signal and the filter.

Here we have demonstrated that Newtonian filters work reasonably well and could be used for a first detection with an average loss of 30% on the signal to noise ratios. If we consider higher derivatives of frequency f say $\ddot{f}$ *etc.* as parameters [6] we should get a better match, but the computation is very likely to increase. It should be possible to construct filters which not only enable us to save on the computation time but also span the set of signal waveforms adequately. A deeper analysis of the signal waveforms is in order so that efficient techniques can be developed. This work is now in progress.

References

[1] K. S. Thorne, in *in 300 years of gravitation*, edited by S. W. Hawking and W. Israel (Cambridge Univ. press, Cambridge, 1987) p. 330.

[2] C.Cutler, T. A. Apostolatos, L. Bildsten, L. S. Finn, E. E. Flanagan D. Kennefick, D. M. Markovic, A. Ori, E. Poisson, G. J. Sussman and K. S. Thorne, Phys. Rev. Lett. **70**, 2984 (1993).

[3] A. Abramovici *et. al.*, Science **256** 325 (1992).

[4] C. Bradaschia *et. al.*, Nucl. Instrum. Methods Phys. Res., Sect A 518 (1990)

[5] B. S. Sathyaprakash and S. V. Dhurandhar, Phys. Rev. D **44**, 3819 (1991).

[6] B. F. Schutz, Class. Quantum Grav. **6** 1761 (1993)

Figure Captions

Figure 1(a) & 1(b) :
The figure shows how the phase difference between the signal with post-Newtonian corrections and the Newtonian filter builds up as a function of time. The masses considered for the signals are $M_1 = 8M_\odot$ and $M_2 = 1.4M_\odot$, the time of arrival $t_a = 0$ and initial phase $\phi = 0$. The frequency interval considered is 40 - 400 Hz. In Fig 1(a) we take the parameters of the filter to be same as the ones used for generating the signal *i.e.* the same values for the time of arrival $t_a = 0$, the chirp-mass $\mathcal{M}$ and the initial phase ϕ. In Fig 1(b) we plot the phase difference between the same signal and the Newtonian filter which correlates best with the signal. This filter has slightly shifted values for each of its parameters from those of the signal. The shifts observed in this case are $\Delta t = -0.54$ sec, $\Delta\mathcal{M} = 0.0469M_\odot$ and $\Delta\phi = 1.93$ rad. We observe that this amounts to selecting a filter which maintains a small phase difference with the signal for the largest possible time.

Table Captions

- **Table I:**
  The table presents the correlations obtained for a representative combinations of masses M_1, M_2 for the binaries in the frequency ranges

  1. 100 - 400 Hz,
  2. 40 - 400 Hz,
  3. 10 - 100 Hz.

  All the masses are given in units of $M_\odot$.

- **Table II:**
  The table illustrates the comparision between the signal parameters and the best matched Newtonian filter parameters. All the three parameters, the time of arrival t_a, chirp mass $\mathcal{M}$ and initial phase ϕ show shifts of which, the variations in $\Delta\mathcal{M}$ and Δt are regular. As ϕ is a very sensitive parameter, it exhibits no regular pattern in its shifts. All the masses are given in units of $M_\odot$.

TABLE I

M_2 \ M_1	1.4	5.0	8.0	10.0	$(f_a - f_u)$
0.5	.8350	.7258	.6969	.6858	100 - 400 Hz
	.8006	.6363	.5932	.5759	40 - 400 Hz
	.4860	.8265	.6682	.6090	10 - 100 Hz
1.0	.8452	.8036	.7784	.7657	100 - 400 Hz
	.7913	.6983	.6658	.6510	40 - 400 Hz
	.5697	.8313	.7121	.6666	10 - 100 Hz
1.4	.8632	.8238	.8087	.8136	100 - 400 Hz
	.7821	.7235	.6968	.6866	40 - 400 Hz
	.6175	.8273	.7301	.6920	10 - 100 Hz
2.0	.8598	.8657	.8536	.8409	100 - 400 Hz
	.7683	.7467	.7279	.7198	40 - 400 Hz
	.6848	.8198	.7469	.7158	10 - 100 Hz

TABLE II

M_1, M_2 $\mathcal{M}$	5,1 1.8355	8,1.4 2.7221	8,2 3.3302	10,1.4 2.9943	$(f_a - f_u)$
Δt(in secs) $\Delta\mathcal{M}$(in $M_\odot$) $\Delta\phi$(in rads)	$-.018$ $+.0886$ 5.72	$-.022$ $+.3369$ 2.25	$-.018$ $+.4709$ 5.92	$-.021$ $+.5025$ 1.53	100 - 400 Hz
Δt(in secs) $\Delta\mathcal{M}$(in $M_\odot$) $\Delta\phi$(in rads)	$-.049$ $+.0063$ 0.61	$-.054$ $+.0469$ 1.93	$-.047$ $+.0706$ 0.05	$-.055$ $+.0773$ 2.17	40 - 400 Hz
Δt(in secs) $\Delta\mathcal{M}$(in $M_\odot$) $\Delta\phi$(in rads)	$+.004$ $-.0105$ 6.22	$-.132$ $-.0143$ 2.64	$-.140$ $-.0165$ 3.24	$-.168$ $-.0142$ 5.04	10 - 100 Hz

Figure 1(a)

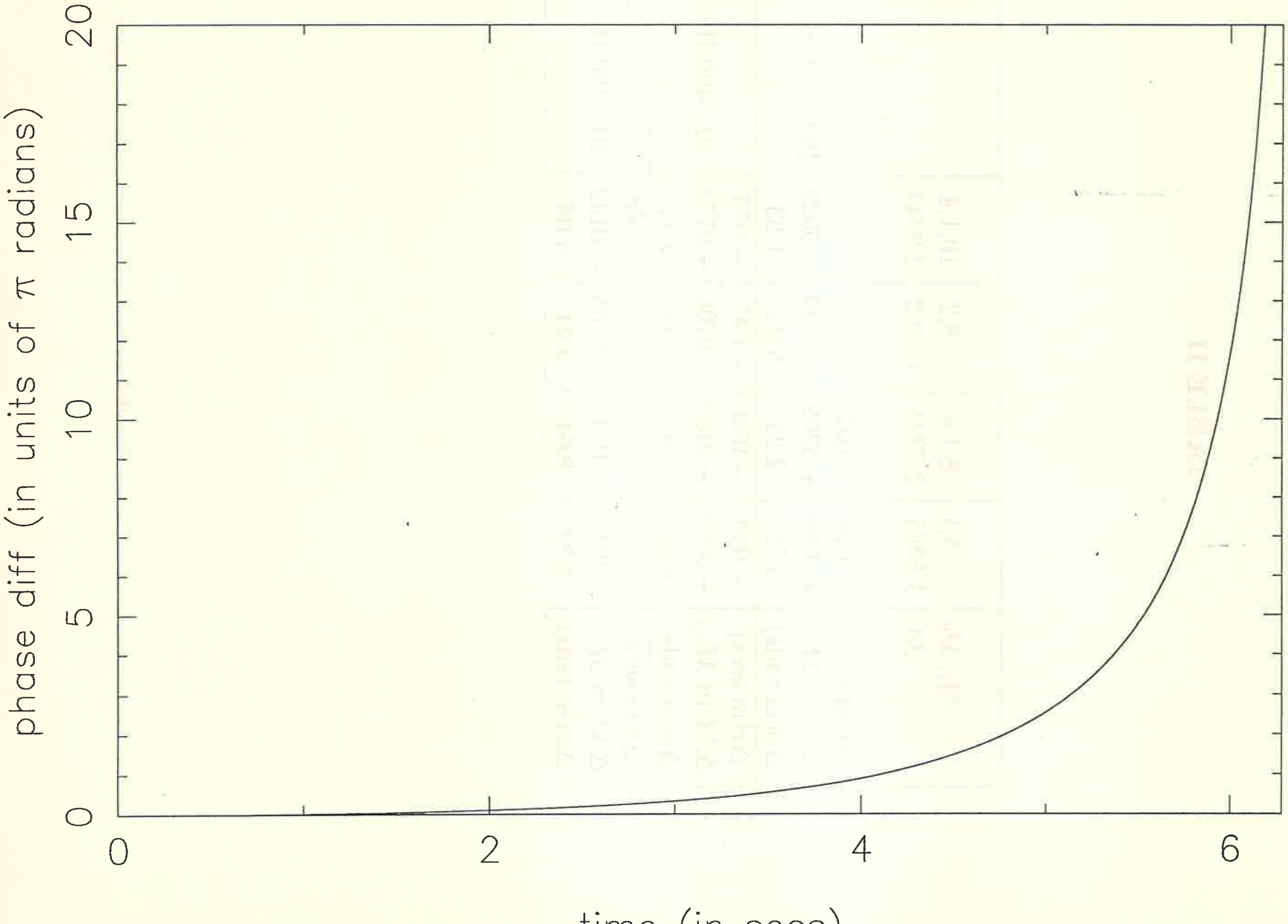

Figure 1(b)