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Primordial Magnetic Fields in the Post-recombination Era and Early Reionization

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ABSTRACT

We explore the ways in which primordial magnetic fields influence the thermal and ionization history of the post-recombination universe. After recombination the universe becomes mostly neutral resulting also in a sharp drop in the radiative viscosity. Primordial magnetic fields can then dissipate their energy into the intergalactic medium (IGM) via ambipolar diffusion and, for small enough scales, by generating decaying MHD turbulence. These processes can significantly modify the thermal and ionization history of the post-recombination universe. We show that the dissipation effects of magnetic fields which redshifts to a present value $B_0 = 3 \times 10^{-9}$ Gauss smoothed on the magnetic Jeans scale and below, can give rise to Thomson scattering optical depths $\tau \gtrsim 0.1$, although not in the range of redshifts needed to explain the recent WMAP polarization observations. We also study the possibility that primordial fields could induce the formation of subgalactic structures for $z \gtrsim 15$. We show that early structure formation induced by nano-Gauss magnetic fields is potentially capable of producing the early re-ionization implied by the WMAP data. Future CMB observations will be very useful to probe the modified ionization histories produced by primordial magnetic field evolution and constrain their strength.

1 INTRODUCTION

Magnetic fields play an important role in understanding most structures in the universe (Parker 1979; Zeldovich, Ruzmaikin & Sokoloff 1983). Their origin however is not yet clearly understood. Observed galactic magnetic fields could have arisen from dynamo amplification of a seed magnetic fields $\simeq 10^{-20}$ G (cf. Ruzmaikin, Shukurov & Sokoloff 1988; Beck et al 1996; Shukurov 2004; Brandenburg & Subramanian 2004); with the seed field itself originating in the early universe or from astrophysical processes (e.g. Harrison 1970; Subramanian, Narasimha and Chitre 1995; Kulsrud et al 1997; Grasso & Rubenstein 2001 and Widrow 2002 for reviews). However there are potential difficulties for the dynamo theory to overcome, due to the constraints implied by helicity conservation and the more rapid growth of small-scale magnetic fields (Cattaneo & Vainshtein 1991; Kulsrud & Anderson 1992; Gruzinov & Diamond 1994; Subramanian 1998, 1999, 2002; Blackman & Field 2000; Kleoorin *et al* 2000; Brandenburg 2001; Brandenburg & Subramanian 2000; Blackman & Brandenburg 2002; and Brandenburg & Subramanian 2004 for a recent review). Magnetic fields with larger coherence scales may also be present in clusters of galaxies (Clarke, Kronberg & Bohringer 2001; Carilli and Taylor 2002; Vogt and Ensslin 2003) and at high redshifts (Oren & Wolfe 1995). Such large-scale coherent fields potentially present further problems for the dynamo paradigm.

Alternatively, large-scale magnetic fields could also have arisen from primordial magnetic fields $\simeq 10^{-9}$ G, generated in the early universe, for instance during inflation (cf. Turner & Widrow 1988; Ratra 1992; cf. Grasso & Rubenstein 2001; Giovannini 2004 for reviews). Such a primordial tangled magnetic field will also influence the formation of structures in the universe, like galaxies (Rees & Reinhardt 1972; Wasserman 1978; Kim, Olinto & Rosner 1996; Subramanian & Barrow 1998a, SB98a hereafter, Sethi 2003) and give rise to CMBR temperature and polarization anisotropies (Barrow, Ferreira & Silk 1997; Subramanian & Barrow 1998b, 2002; Durrer, Ferreira, Kahniashvili, 2000; Seshadri & Subramanian 2001; Mack, Kashniashvili, Kosowsky, 2002; Subramanian, Seshadri & Barrow 2003). These considerations can be used to constrain the magnetic field amplitude and the shape of its power spectrum. In this paper we consider the possible role magnetic fields could have played in determining the thermal and ionization history of the universe in the post-recombination epoch.

For redshifts $z \lesssim 1100$, primeval plasma begins to recombine to form neutral hydrogen. The ionized fraction decreases by nearly an order of magnitude by $z \simeq 1000$, finally reaching a value of $\simeq 10^{-4}$ for $z \lesssim 100$ (for details see Peebles 1993). The matter temperature continues to follow the CMBR temperature, both falling as $\propto 1/a$ for $z \gtrsim 100$. (here a is the expansion factor of the universe). At smaller redshifts, matter 'thermally' decouples from the radiation and the matter temperature falls as $\propto 1/a^2$. In the standard picture, this thermal and ionization history holds up to $z \simeq 10$–20, when the formation of first structure can lead to reionization and reheating. Recent WMAP observations of CMBR anisotropies suggests that the universe reionized as early as $z \simeq 17$ (Kogut et al. 2003). On the other hand, observations of high redshift quasars suggest that the universe is fully ionized for $z \lesssim 5$ but the neutral fraction reaches values $\simeq 10\%$ between $5.2 \lesssim z \lesssim 6$ (Fan et al. 2002, Becker et al. 2001, Djorgovski et al. 2001). Such early reionization, as implied by WMAP observations, presents a challenge, and is not yet fully understood (cf. Ricotti & Ostriker 2003). One of our aims will also be to explore the possibilities offerred by primordial magnetic fields in this respect.

In the pre-recombination epoch, magnetic fields evolve as $B \propto 1/a^2$ on sufficiently large scales, larger than the magnetic Jeans length λ_J (see below). In the post-recombination epoch there is a sharp drop in the electron density as the universe becomes mostly neutral. This can lead to dissipation of magnetic field energy into the medium from ambipolar diffusion (Cowling 1956, for details see Shu 1992 and references therein). There is also a sharp drop in radiative viscosity after recombination (Jedamzik, Katalinić, & Olinto 1998; SB98a). For scales smaller than λ_J, non-linear effects can then lead to the field generating decaying MHD turbulence, and the dissipation of magnetic field energy on such scales, into the intergalactic medium. These processes will affect the thermal and ionization history of the universe. In this paper we explore how the standard ionization and thermal history might get modified due to such dissipation of magnetic field energy in the post-recombination epoch, from ambipolar diffusion and decaying turbulence.

In addition tangled primordial magnetic fields can also induce early formation of structures in the universe (Kim et al. 1996; SB98, Gopal & Sethi 2003). We study in detail this process and find the range of redshifts at which the first structures might collapse. Such early structure formation can also lead to changes in the thermal and ionization history of the universe, and be a potential source of the early re-ionization inferred from WMAP.

In the next section we set the notation for describing primordial magnetic fields and summarize briefly some of the relevant results from earlier work on their evolution. The processes which dissipate magnetic field energy into the IGM are considered in §3. After briefly describing the equations that govern the ionization and thermal history of the universe in §4, we apply them to compute the effect of magnetic energy dissipation on the IGM in the subsequent section. The formation of the first nonlinear structures induced by tangled primordial magnetic fields and their possible effects are considered in §6, The last section summarizes our conclusions. Throughout this paper we use, unless specified otherwise, the currently-favoured FRW model: spatially flat with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ (Spergel et al. 2003, Reiss et al. 2004, Tonry et al. 2003, Perlmutter et al. 1999, Riess et al. 1998) with $\Omega_b h^2 = 0.02$ (Spergel et al. 2003, Tytler et al. 2000) and $h = 0.7$ (Freedman et al. 2001).

2 MAGNETIC FIELDS IN THE EARLY UNIVERSE

Let us suppose that some process in the early universe generated tangled magnetic fields. Firstly, on large-enough scales, the velocity induced by tangled primordial fields is generally so small that it does not lead to any appreciable distortion of the initial field (Jedamzik, Katalinic and Olinto 1998, SB98a). In this 'linear' regime, to a very good approximation, the magnetic field simply redshifts as $\mathbf{B}(\mathbf{x},t) = \tilde{\mathbf{B}}(\mathbf{x})/a^2$. Here $\mathbf{x}$ is the comoving coordinate. We assume the early universe tangled magnetic field, $\tilde{\mathbf{B}}$ to be initially an isotropic and homogeneous random process. This allows one to write, in Fourier space (see .i.e. Landau & Lifshitz 1959):

$$\langle \tilde{B}_i(\mathbf{q})\tilde{B}_j^*(\mathbf{k})\rangle = \delta_D^3(\mathbf{q}-\mathbf{k})\left(\delta_{ij} - k_i k_j/k^2\right) M(k) \tag{1}$$

Here $M(k)$ is the magnetic field power spectrum and $k = |\mathbf{k}|$ is the comoving wavenumber. This gives $\langle \tilde{\mathbf{B}}^2(\mathbf{x})\rangle = \int (dk/k)\Delta_B^2(k)$, where $\Delta_B^2(k) = 8\pi k^3 M(k)$ is the power per logarithmic interval in k space residing in magnetic tangles,

We shall often consider the following two forms of the power spectrum $M(k)$. For several applications, the results are dominated by the smallest scales of the tangled magnetic field. It then suffices to consider a simple, single scale power spectrum:

$$M(k) = B_0^2 \delta(k - k_\star)/(8\pi k^2) \tag{2}$$

In this normalization B_0 coincides with the RMS of the magnetic field, redshifted to the present epoch i.e. $B_0^2 = \langle \tilde{\mathbf{B}}^2(\mathbf{x})\rangle$. We shall also consider the effects of taking a more complicated spectrum, like a power law spectrum, with $M(k) = Ak^n$ cut off at $k = k_{max}$; with k_{max} determined by the effects of damping by radiative viscosity before recombination. One can fix A by demanding that the smoothed field strength over a scale, k_G, (using a sharp k-space filter) is B_G. We will assume $n > -3$. Using the same filter, the RMS value of the field smoothed over a wavenumber k, is given by

$$\tilde{B}^2(k) = B_G^2 (k/k_G)^{3+n}. \tag{3}$$

As we shall see below the energy dissipation from ambipolar diffusion depends on $\langle((\nabla_{\mathbf{r}}\times\mathbf{B})\times\mathbf{B})^2\rangle = (1/a^{10})\langle((\nabla_{\mathbf{x}}\times\tilde{\mathbf{B}})\times\tilde{\mathbf{B}})^2\rangle$, where now $\mathbf{r} = a\mathbf{x}$ is the proper co-ordinate. Using Eq. (1) and assuming the magnetic fields to be a Gaussian process:

$$\langle((\nabla_{\mathbf{x}}\times\tilde{\mathbf{B}})\times\tilde{\mathbf{B}})^2\rangle = \frac{7}{3}\int dk_1\int dk_2 M(k_1)M(k_2)k_1^2k_2^4 \tag{4}$$

The effects of the distortion of the initial magnetic field, and the nonlinear processing of the magnetic spectrum begins to be felt at all scales l which satisfy the inequality $v(l)/l \gtrsim H(t)$. Here $v(l)$ is the velocity induced by the magnetic field on a proper scale, $l = a/k$, and $H = \dot{a}/a$ is the Hubble expansion rate at cosmic time t (cf. Banerjee and Jedamzik 2003; SB98a). For large fluid Reynolds number (see below), the fluid velocity induced by the tangled magnetic field is of order the Alfvén velocity; that is $v(l) \approx V_A(k = a/l, t)$ where in the post-recombination epochs, $V_A(k,t)$ is given by

$$V_A(k,t) = \frac{B(k,t)}{(4\pi\rho_b(t))^{1/2}} \approx 1.5\times10^{-5}\left(\frac{B(k,t)a^2}{10^{-9}\mathrm{G}}\right) c\, a^{-1/2}. \tag{5}$$

Here $\rho_b(t)$ is the density of baryons, and $B(k,t)$ is the magnetic field smoothed on a scale $l = a/k$ at time t. For scales which are not yet affected by nonlinear processing, one can assume the field evolves approximately with a constant $B(k,t)a^2(t) = \tilde{B}(k) = B_G(k/k_G)^{(n+3)/2}$, where for the second equality we have adopted the power law spectrum of Eq. 3.

Note that the scale below which nonlinear effects become important is also approximately equal to the magnetic Jeans length, below which the distortion of the field can lead to magnetic pressure gradients which can counteract the gravitational collapse (cf. SB98a). In fact in a linear analysis this condition allows us to define the proper magnetic Jeans' wave number, say K_J, from equating the two terms: $4\pi G\rho_m = K_J^2B^2/(8\pi\rho_b)$, giving

$$K_J = \frac{4\pi\sqrt{2\rho_m\rho_b G}}{B}. \tag{6}$$

Defining the comoving Jeans scale by $k_J = aK_J$, and noting that $H^2(t) = 8\pi G\rho_m/3$, the above condition is equivalent to the condition $[k_J/a(t)]V_A(k_J,t) = \sqrt{3}H(t)$. This can be explicitly seen to be very similar to the condition discussed above on the length scales below which nonlinear evolution is important. The comoving Jeans' length $\lambda_J = 2\pi/k_J$, or the comoving Jeans' wavenumber k_J do not depend on time, at early epochs where the universe is matter dominated; and assuming that the field even at the scale k_J just redshifts as $\propto 1/a^2$, without significant distortion (cf. SB98a). This is because in this case $V_A \propto a^{-1/2}$ and $H(t) \propto t^{-1} \propto a^{-3/2}$ and hence $k_J \propto a(t)H(t)/V_A$ is constant with time. So any scale which is linear/nonlinear just after recombination, is approximately linear/nonlinear at all epochs (till the vacuum energy starts dominating).

Putting in numerical values we get

$$k_J \simeq 14.8\,\mathrm{Mpc}^{-1}\left(\frac{\Omega_m}{0.3}\right)^{1/2}\left(\frac{h}{0.7}\right)\left(\frac{B_J}{10^{-9}\mathrm{G}}\right)^{-1} \tag{7}$$

where $B_J = B(k_J,t)a^2(t)$ is the redshifted value of the field smoothed on the scale k_J. Again for the power law spectrum given by Eq. 3, we will have $B_J = B_G(k_J/k_G)^{(n+3)/2}$, giving an implicit equation for k_J. For a nearly scale invariant spectrum with say $n = -2.9$, $k_G = 1h\,\mathrm{Mpc}^{-1}$, one gets for the above cosmological parameters $k_J \sim 13\,\mathrm{Mpc}^{-1}(B_G/10^{-9}\mathrm{G})^{-0.95}$.

There is another scale which plays an important role in what follows. This is the Alfvén-wave damping length-scale $k_{\max}$, below which tangled magnetic fields are strongly damped by radiative viscosity in the pre-recombination universe (Jedamzik, Katalinic and Olinto 1998, SB98a). We have

$$k_{\max} \simeq 235\,\mathrm{Mpc}^{-1}\left(\frac{B_m}{10^{-9}\mathrm{G}}\right)^{-1}\left(\frac{\Omega_m}{0.3}\right)^{1/4}\left(\frac{\Omega_b h^2}{0.02}\right)^{1/2}\left(\frac{h}{0.7}\right)^{1/4} \tag{8}$$

Here B_m is the field smoothed over scales larger than the cut-off scale, (and redshifted to the present epoch), which act as the effective large scale field for the cut-off scale perturbations (cf. Jedamzik, Katalinic & Olinto 1998; SB98a; Seshadri & Subramanian 2001).

The magnetic field smoothed on comoving wave numbers $k \lesssim k_J$ evolve in a 'linear' fashion in the post-recombination epoch. As we discuss below the magnetic field at these scale is mainly dissipated by ambipolar diffusion. For $k_{\max} \gtrsim k \gtrsim k_J$, non-linear effects can also lead to decaying MHD turbulence and consequent dissipation of the magnetic field energy. In the post-recombination era density perturbations seeded by the primordial magnetic field can grow on scales with $k \lesssim k_J$ (Wasserman 1978; Kim et al 1996; SB98a; Sethi 2003; Gopal & Sethi 2003). This can lead to early collapse of structure which might have important implication for the ionization of the universe. We discuss this scenario in more detail in §6. Before this we first consider the magnetic field energy dissipation into the IGM due to ambipolar diffusion and decaying turbulence.

3 ENERGY INPUT INTO THE IGM FROM PRIMORDIAL MAGNETIC FIELD DISSIPATION

3.1 Ambipolar diffusion

Ambipolar diffusion is important for magnetic field energy decay in a mostly neutral medium. The post-recombination universe satisfies this criterion as the ionized fraction of hydrogen $n_e/n_{\rm B} \equiv x_e \simeq 10^{-4}$ at $z \simeq 100$ (Peebles 1968, Zel'dovich, Kurt, & Sunyaev 1969, Peebles 1993). In the presence of a tangled magnetic field, the Lorentz force acts only on the small fraction of ionized component, thereby generating a velocity difference between the ionized and the neutral components. This relative velocity between charged and neutral particles is damped by ion-neutral collisions, which leads to dissipation of magnetic field energy (see e.g. Cowling 1956, Mestel & Spitzer 1956). This energy dissipation process known as ambipolar diffusion is important in mostly neutral molecular clouds (for details see Shu 1992).

The neutral particles in the early universe are neutral hydrogen and helium. Throughout this paper we neglect the effect of energy dissipation on the small fraction of neutral helium atoms. The volume rate of energy dissipation due to ambipolar diffusion is then (Cowling 1956) [1]:

$$\Gamma_{\rm in} = \frac{\rho_n}{16\pi^2\gamma\rho_b^2\rho_i}|(\nabla{\bf x}{\bf B}){\bf x}{\bf B}|^2 \qquad (9)$$

Here ρ_n, ρ_i, and ρ_b are the densities of neutral hydrogen, ionized hydrogen, and total baryon density, respectively. Also $\gamma = \langle w\sigma_{\rm in}\rangle/(m_n + m_i)$ (Shu 1992); where w is the ion-neutral relative velocity and $\sigma_{\rm in}$ is the cross section for the collision between ions and neutrals. For $w \lesssim 10\,{\rm km\,sec^{-1}}$, $\langle w\sigma_{\rm in}\rangle \simeq 3\times 10^{-9}$ independent of the relative velocity of ions and neutrals. This approximation holds for the parameter space we consider in this paper (for a detailed discussion and references see Shu 1992). This energy is deposited into the neutral component of the medium. However owing to collisions between electrons, protons, and neutrals the energy is rapidly thermalized at rates much higher than the expansion rate of the universe (see e.g. Madau, Meiksin & Rees 1997 and references therein). The volume rate of energy deposition in electrons, (required for Eq. (17) below), is $\Gamma_e = x_e\Gamma_{\rm in}$. Ambipolar diffusion is the main process of magnetic field energy dissipation at comoving length scales at which velocities are linear .i.e. $k \lesssim k_J$. However, even smaller length scales, upto $k^{-1} \lesssim k_{\rm max}^{-1}$ can contribute to the energy input into the IGM due to ambipolar diffusion (if the decay due to the the MHD turbulence discussed below is not efficient enough).

3.2 Decaying Turbulence

For magnetic fields which vary on length scales smaller than the magnetic Jeans scale, or for $k > k_J$, an additional mode of decay is possible. Such small scale tangled magnetic fields can induce decaying MHD turbulence.

Firstly, as we pointed out in §2, for magnetic fields at comoving scales below the comoving magnetic Jeans' length i.e. $k > k_J$, the rate of energy transfer due to the nonlinear interaction between modes, $\sim (k/a)V_A(k,t)$ becomes smaller than the Hubble rate $H(t)$. However, just prior to recombination, velocity perturbations at these scales, are over-damped owing to large radiative viscosity of the medium (Jedamzik, Katalinić, & Olinto 1998, SB98a) and therefore cannot lead to turbulence. After recombination, the radiative viscosity dramatically decreases; the viscous force per unit mass ${\bf F}_V/\rho_b$ becomes that due to the free streaming photons, with ${\bf F}_V/\rho_b = -\alpha{\bf v}$, where $\alpha = (4/3)(\rho_\gamma/\rho_b)n_e\sigma_t c$. Here σ_t is the Thomson cross section and ρ_γ the photon density. The corresponding Reynolds number for fluid motions (The dimensionless ratio of the nonlinear term ${\bf v}\cdot\nabla{\bf v}$ to the viscous term in the fluid momentum equation), which is given by $R = kv/(a\alpha)$ becomes very large. In fact writing $R = (k/k_J)(v/V_A)R_J$, we have

$$R_J = \frac{3\sqrt{3}\rho_b}{4\rho_\gamma}\frac{L_\gamma}{D_H} \qquad (10)$$

where $L_\gamma = (n_e\sigma_t a)^{-1}$ is the comoving photon mean free path and $D_H = c/(Ha)$ is the comoving Hubble radius. We have used also the fact that $[k_J/a(t)]V_A(k_J,t) = \sqrt{3}H(t)$ in deriving Eq. 10. Just after recombination, $L_\gamma \sim 10^4$ Mpc (for an ionization fraction of 10^{-4}) while $D_H \sim 100$ Mpc. So magnetic tangles with $k > k_J$, for which nonlinear interactions are important, the Reynolds numbers are also large enough that decaying MHD turbulence can be induced. This becomes increasingly the case at smaller redshifts, because $R_J \propto a^{5/2}$ increases rapidly as the universe expands.

It is also important to check whether the ambipolar diffusion discussed above allows for such turbulence. For this one needs to estimate the magnetic Reynolds number associated with ambipolar diffusion, that is the dimensionless ratio of the advection term in the induction equation for ${\bf B}$, to the ambipolar diffusion term. This is given by $R_{\rm ambi} \sim KvB/[K^2B^2/(4\pi\rho_i\rho_n\gamma)]$, where $K = k/a$ is the proper wavenumber and v the velocity induced by the magnetic field. We have numerically,

[1] Note that the expression below doesn't agree with Shu (1992) Eq. (27.19), though the formulae agree for a mostly neutral medium. The formula given here is correct for an arbitrary ionized medium (Cowling (1956), Eq. (27))

$$R_{\rm ambi} \approx 8.5 \times 10^3 \frac{v}{V_A} \left(\frac{1+z}{10^3}\right)^{3/2} \left(\frac{x_e}{10^{-3}}\right) \left(\frac{k}{k_J}\right)^{-1} \tag{11}$$

So initially we have $R < R_{\rm ambi}$ and radiative viscosity is more important; we have already seen that this itself is weak enough to allow decaying turbulence to be induced. As the universe expands and with decreasing redshift, R_J increases as $a^{5/2}$, while $R_{\rm ambi}$ decreases as $a^{-3/2} \propto (1+z)^{3/2}$ and so eventually ambipolar diffusion is more important. But even for $k = k_{max} = 235$, $R_{\rm ambi}$ is of order unity only at redshifts $z \sim 20$. So for modes with $k > k_J$ decaying turbulence is important from recombination to fairly low $z < 20$ (for $k < k_{max}$), after which any remaining energy will be drained much faster by ambipolar diffusion. For decaying turbulence, nonlinear interactions between different modes causes the magnetic field to decay by cascading energies to smaller and smaller scales and subsequent dissipation, independent of the exact physical mechanism of dissipation [2].

Simulations of such decaying MHD turbulence in flat space suggests that the magnetic energy decays as a power law at large times (see e.g. Banerjee & Jedamzik 2003, Christensson, Hindmarsh & Brandenburg 2001, Biskamp & Müller 2000), the exact power law depending on the magnetic spectral index on large scales. One can model this decay as,

$$\mathcal{E}_{\rm B} = \frac{\mathcal{E}_{\rm B0}}{(1+\tilde{t}/\tilde{t}_d)^m} \tag{12}$$

where $\tilde{t}$ is the time in 'flat-space', $\tilde{t}_d$ is the relevant dynamical time for decay, and $\mathcal{E}_{\rm B}$ is the magnetic energy in flat space, with $\mathcal{E}_{\rm B0}$ its initial value. Also simple scaling arguments suggest $m = 2(n+3)/(n+5)$, for an initial power spectrum with $n > -3$ (cf. Olesen 1997, Shiromizu 1998, Christensson, Hindmarsh & Brandenburg 2001, Banerjee & Jedamzik 2003). One can take over this result into the expanding universe if one can map the flat space MHD turbulence decay problem into the FRW universe. It is well known (Brandenburg, Enqvist & Olsen 1996, SB98a) that viscous MHD in the radiation dominated universe can be conformally mapped to flat space viscous MHD. In the matter dominated era the following transformation of variables can be used to approximately map the expanding universe viscous MHD to flat space (Banerjee 2003):

$$\tilde{\mathbf{B}} = \mathbf{B}a^2, \quad d\tilde{t} = dt/a^{3/2}, \quad \tilde{v} = a^{1/2}v, \quad \tilde{\rho} = \rho a^3, \quad \tilde{p} = pa^4, \quad \tilde{\epsilon} = a^4\epsilon \tag{13}$$

The validity of this mapping requires $p \ll \rho$. Integrating $d\tilde{t} = dt/a^{3/2}(t)$, with $a^{-3/2}(t) = c_1/t$, as appropriate for a matter dominated universe (which obtains till the vacuum energy dominates), one gets $\tilde{t} = c_1 \ln(t/t_i)$; here we have also used the condition that $\tilde{t} = 0$ when $t = t_i$ the initial epoch when the decay starts, Similarly $\tilde{t}_d = c_1 \ln((t_i + t_d)/t_i)$, where t_d is the physical decay timescale for the turbulence, which we may approximate as the Alfvén time scale for the mode k_{max}. So $\tilde{t}/\tilde{t}_d = \ln(t/t_i)/\ln(1 + (t_d/t_i))$ independent of the constant c_1. Further, using the above transformation for the magnetic field, $\tilde{\mathbf{B}} = \mathbf{B}a^2$, the magnetic energy will scale as $E_B(t) = \mathcal{E}_B/a^4$, where $\mathcal{E}_B$ itself, in our phenomenological model, decays as in Eq. (12). The transformation between $\tilde{t}$ and t then implies that a power-law decrease in magnetic field energy in flat space will correspond to a logarithmic decrease of the energy (on scales $k_J < k < k_{max}$), in the matter dominated era, due to decaying turbulence, over and above the decay due to expansion. Also using this transformation we can model the rate of dissipation of magnetic field energy, on these scales, in a matter dominated expanding universe as:

$$\frac{dE_{\rm B}}{dt} = -4H(t)E_B - \frac{3mH(t)E_B}{2[\ln(1+t_d/t_i) + \ln(t/t_i)]} \tag{14}$$

Here the first term corresponds the redshifting due to expansion and the second due the energy lost from the field due to the decaying turbulence. Using $E_B(t) = \mathcal{E}_B/a^4$ and substituting for $\mathcal{E}_B$ from Eq. 12, we have for the rate of energy input due to decaying turbulence,

$$\Gamma_{\rm decay} = \frac{B_0(t)^2}{8\pi}\frac{3m}{2}\frac{[\ln(1+t_d/t_i)]^m}{[\ln(1+t_d/t_i)+\ln(t/t_i)]^{m+1}} \times H(t), \tag{15}$$

where we have defined $B_0^2(t)/8\pi = \mathcal{E}_{B0}/a^4$. For $(t - t_i) \ll t_d$, the second term in the denominator, $\ln(t/t_i)$, can be dropped. On the other hand, for $t - t_i \gg t_d$, the first term, $\ln(1 + t_d/t_i)$ in the denominator of Eq. 15 can be dropped, to get

$$\Gamma_{\rm decay} = \frac{B_0(t)^2}{8\pi}\frac{3m}{2}\frac{[\ln(1+t_d/t_i)]^m}{\ln^{m+1}[((1+z_i)/(1+z))^{3/2}]} \times H(t) \tag{16}$$

Here we have used the relation $t \propto a^{3/2} \propto (1+z)^{-3/2}$ valid in the matter dominated universe at high redshift. We are now in a position to study the effects of the energy input into the IGM due to ambipolar diffusion and decaying turbulence; first we recall briefly the relevant equations involved in calculating the thermal and ionization history of the IGM.

[2] Note that for smaller scales, R due to the free-streaming radiative viscosity will in fact increase. However for smaller scales the diffusive fluid viscous force and ambipolar damping becomes increasingly more important. The corresponding Reynolds number due to fluid viscosity is $R_f = av/k\nu$, where ν is the kinematic viscosity of the hydrogen gas. And that due to ambipolar damping is $R_{\rm ambi}$ given in the text. Just after recombination, both $R_f(k_J) \gg R_{\rm ambi} \gg R(k_J)$. But since R_f and $R_{\rm ambi}$ decrease with increasing k, while R increases with k, for a large enough k we will have $R_{\rm ambi}(k) < R(k)$. Below this scale ambipolar damping will dominate the radiative viscosity, and drain the energy from the turbulence. The energy lost from the magnetic field thus eventually goes to heat the gas.

4 THERMAL AND IONIZATION STATE OF IGM

In the post-recombination universe, the IGM is mostly neutral reaching an ionization fraction $x_e \simeq 10^{-4}$ by $z \simeq 100$ (Peebles 1993). At smaller redshifts the ionization fraction decreases very slowly because the recombination rate of hydrogen is much smaller than the expansion rate of the universe with such small ionization levels. The temperature of matter (which refers to the temperature of either electrons, protons, or the neutral particles which remain in equilibrium with each other) continues to be nearly equal to the CMBR temperature for $z \gtrsim 100$ because of inverse Compton scattering (see e.g. Peebles 1993). At smaller redshifts the matter temperature $T_e \propto 1/a^2$, i.e. it falls more rapidly as compared to CMBR temperature $T_{\rm cbr}$ which evolves as $\propto 1/a$. The dissipation of magnetic field energy into the IGM alters its thermal and ionization evolution. It heats the system which in turn changes the ionization state by collisional ionization. Thermal and ionization evolution in the presence of magnetic field dissipation is given by (see e.g. Peebles 1993):

$$\begin{aligned} \dot{T}_e &= -2\frac{\dot{a}}{a}T_e + \frac{x_e}{1+x_e}\frac{8\rho_\gamma\sigma_t}{3m_e c}(T_\gamma - T_e) + \frac{\Gamma_e}{(1.5k_B n_e)} \\ \dot{x}_e &= \left[\beta_e(1-x_e)\exp\left(-h\nu_\alpha/(k_B T_{\rm cbr})\right) - \alpha_e n_b x_e^2\right] C + \gamma_e n_b (1-x_e) x_e \end{aligned} \tag{17}$$

Here Γ_e is the volume rate of injection of energy into the electrons. $\Gamma_e = x_e\Gamma_{\rm in}$ for energy dissipation from ambipolar diffusion (Eq. (9)) and $\Gamma_e = x_e\Gamma_{\rm decay}$ for energy injection from MHD decaying turbulence (Eq. (15)). (Also k_B is the Boltzmann constant). In the ionization evolution equation the first two terms are the usual terms for the recombination of the primeval plasma (for details and notation see Peebles 1968, Peebles 1993). For $z \lesssim 1000$, $C \simeq 1$ and the first term on the right hand of the equation rapidly decreases. This means that after the recombination is completed the only important term is the recombination term (the second term on the right hand side) which gives a slow decrease in the ionization fraction. The third term on the right hand side corresponds to the collisional ionization of the IGM. This term is usually very small because collisional ionization coefficient γ_e is exponentially suppressed at temperatures in the post-recombination universe $T_e \simeq 0.25((1+z)/1000)\,$eV, which are much smaller than the hydrogen ionization potential, $E_i = 13.6\,$eV. Owing to magnetic field energy dissipation, the matter temperature might increase sufficiently to make the collisional ionization term important and thereby cause a substantial change in the ionization state of IGM. We show that this mechanism can be responsible for partially ionizing the universe at high redshifts.

5 THERMAL AND IONIZATION HISTORY CHANGES DUE TO PRIMORDIAL MAGNETIC FIELD DISSIPATION

5.1 The effects of ambipolar diffusion

As already discussed, ambipolar diffusion is the principle mechanism of the dissipation of magnetic field energy for magnetic field at scales $k \lesssim k_J$. At smaller scales decaying MHD turbulence can also dissipate energy, but ambipolar diffusion might continue to be an important source of energy dissipation for $k \simeq k_{\rm max}$. (Indeed, we find that less than 40 percent of the energy is dissipated by decaying turbulence for n=-2.9 index even by $z = 10$; so for nearly scale invariant spectra it would be appropriate also to consider ambipolar diffusion effects for scales up to k_{max}).

One can get a rough estimate of the values of B_0 which might cause significant change in ionization and thermal history of the universe by comparing the magnetic field energy density with the thermal energy density of the universe. If at a certain epoch a fraction f of magnetic field energy is dissipated into the IGM then it will typically raise it to a temperature: $T = fB_0^2/(8\pi)/n_b k_B$; with $n_b = n_b(t_0)(1+z)^3$ and $B_0 = B_0(t_0)(1+z)^2$. Taking $f = 0.1$, this give $T \simeq 10^4\,{\rm K}[(1+{\rm z})/100]$ for $B_0 \simeq 10^{-9}\,$G. For $z \gtrsim 100$, this is an overestimate because owing to inverse Compton scattering off CMBR photons matter temperature cannot increase much above CMBR temperature (Eqs. (17)). However it does give a rough estimate of the magnitude of B_0 that are of interest. The fraction of the energy dissipated f will depend on the magnetic field power spectrum. From Eq. (4) and Eq. (9), it can be seen that the rate of dissipation is dominated by the smallest scale (largest k) for the scale-free magnetic field power spectrum (Eq. (3); This will correspond to the large-k cut-off, $k_{\rm max}$.

In Figure 1 and 2 we show ionization and thermal history of the universe for some interesting values of B_0 for both delta function and power law power spectra. For delta function power spectrum, we take $k_\star = k_{\rm max}$ (Eq. 8) and for the power law power spectrum B_0 is defined as RMS value smoothed at $k = k_{\rm max}$. For integrating Eqs. (17) we start with initial conditions $T_e = T_{\rm cbr}$ and $x_e = 1$ at $z = 2000$. For ambipolar diffusion, we do not take into account the back-reaction of the energy dissipation on the magnetic field energy and use $B \propto 1/a^2$. We are justified in neglecting it as for values of B_0 that are of interest less than 10% of the magnetic field energy is dissipated into the IGM by ambipolar diffusion. We integrate Eqs. (17) up to $z = 5$ as the intergalactic medium below that redshift is known to be highly ionized from Gunn-Peterson test (see e.g. Becker et al. 2001 and references therein).

The modified ionization and thermal history of IGM is quantified in terms of two observationally measurable parameters:

$\tau_{\rm ion}$, y.

$$\tau_{\rm ion} = \int_0^{t_0} n_e \Theta(n_e - n_{es}) \sigma_t c dt$$
$$y = \int_0^{t_0} n_e \sigma_t \frac{k_B(T_e - T_{\rm cbr})}{m_e c^2} c dt \tag{18}$$

Here $\tau_{\rm ion}$ measures the optical depth for Thomson scattering in excess of the standard recombination scenario; n_{es} is the number density of electrons for the same cosmological model in the absence of magnetic field, $\Theta(n_e - n_{es}) = 1$ if $n_e \geqslant n_{es}$ and zero otherwise. We assume the universe to be fully ionized between the present and $z = 5$, and add the $\tau_{\rm ion}$ in this range to the results from magnetic field decay. For late ionization models, $\tau_{\rm ion}$ reduces to the optical depth to the last scattering surface defined in the literature (see e.g. Bond 1996). Our reason for defining $\tau_{\rm ion}$ differently is that for the class of models to be discussed here, ionization history begins to differ from the standard case very close to the standard recombination epoch. The definition of $\tau_{\rm ion}$ in Eq. (18) correctly takes that into account. The y parameter measures the fractional energy lost/gained by the CMBR which induces spectral distortion of CMBR (Zeldovich & Sunyaev 1969, Peebles 1993). Observations from COBE-FIRAS put stringent upper limit of this parameter: $y \lesssim 1.5 \times 10^{-5}$ (Fixsen et al. 1996). A non-zero value of $\tau_{\rm ion}$ can be constrained from the measurement of CMBR temperature and polarization anisotropies (see e.g. Bond 1996).

The quantity of direct interest for studying the effect of re-ionization on the CMBR anisotropies is the visibility function $V(z) = (d\tau/d\eta)\exp(-\tau(z))$; here $d\eta = dt/a$ is the conformal time and $\tau = \int_0^\infty n_e \sigma_t c dt$ corresponds to the total optical depth not just owing to reionization after the recombination. The visibility functions measures the normalized probability that a photon last scattered between η and $\eta + d\eta$. We show in Figure 3 visibility functions for the some representative models in comparison with the standard visibility function. We also show visibility function for a late re-ionization model in which the universe ionizes at $z = 17$ and remains ionized till the present.

Recent WMAP observation of temperature-polarization cross-correlation anisotropies suggest that $\tau \simeq 0.17 \pm 0.08$ (Spergel et al. 2003, Kogut et al. 2003), which means that between 10 to 20 % percent photons are re-scattered after the epoch of recombination. In view of this observation, we consider parameter range that can give $\tau \gtrsim 0.1$ as interesting. For the delta function power spectrum $B_0 \gtrsim 2 \times 10^{-9}$ G and $k_\star = k_{\rm max}$ give $\tau_{\rm ion} \gtrsim 0.1$. The class of power law models we study are nearly scale free ($n \simeq -3$) (for detailed discussion see § 6). For these models, $B_0 \gtrsim 3 \times 10^{-9}$ gives $\tau_{\rm ion} \gtrsim 0.1$. The y-parameter from ambipolar diffusion for the class of models of interest is $\lesssim 2 \times 10^{-7}$, which is much smaller as compared to upper limit from COBE-FIRAS observation.

It should however be borne in mind that CMBR observations are not only sensitive to the optical depth but also to the shape of visibility function (Zaldarriaga 1997). Therefore our results should not be used directly to interpret CMBR observations because, as seen in Figure 3, visibility function in our case gets contribution from a much wider range of redshifts than in the case of usual reionization models which have been compared with observations. A genuine comparison is possible only after theoretical predications of the CMBR anisotropies are computed for our models. A preliminary study shows that our models cannot explain the WMAP results, which require a higher degree of ionization for $z \lesssim 30$ than our models, but generically give new anisotropies at smaller scales (larger ℓ) which are distinct from the late reionization models. We plan to pursue this issue in more detail in a future study.

5.2 The effects of decaying Turbulence

We now consider the effect of energy input into the IGM due to decaying MHD turbulence. We calculate this energy input using Eq. 15 given above. For our computations we consider a few cases with $n \simeq -3$ (see § 6 for detailed discussion). For instance $n = -2.9$ implies a decay index $m = 0.2/2.1 \sim 0.095$. In all cases we adopt $z_i = 1000$. We also have to give the ratio t_d/t_i. For this we assume that the decay time-scale is comparable to the Alfvén crossing time associated with the smallest surviving scales in the magnetic spectrum, $k_{\rm max}$. Noting that the time of recombination t_i corresponds also roughly to the Alfvén crossing time associated with tangles on the magnetic Jeans scale, we can take $t_d/t_i \simeq (k_J V_A(k_J, t_i)/(k_{\rm max} V_A(k_{\rm max}, t_i)) \simeq (k_J/k_{\rm max})^{(n+5)/2}$.

Magnetic field dissipation from decaying turbulence is a faster dissipation process close to the recombination epoch as compared to the ambipolar diffusion. This can be the dominant mechanism of magnetic field energy dissipation for $k_J < k < k_{\rm max}$. In figures 4 and 5 we show the typical ionization and thermal histories for this case. The visibility function for this decay process is shown in Figure 3. The models shown in the figures give $\tau_{\rm ion} \gtrsim 0.1$ and a negligibly small y parameter as comparable to observational bounds on y. For the magnetic field strengths $B_0 \gtrsim 3 \times 10^{-9}$ G, decaying turbulence gives $\tau_{\rm ion} \gtrsim 0.1$.

It should be noted that much of the contribution to $\tau_{\rm ion}$ comes from close to the epoch of recombination in this case as opposed to the magnetic field decay from ambipolar diffusion. And therefore these two processes will lead to different changes in CMBR anisotropies; this can also be seen from from the plots for the visibility functions shown in Figure 3.

In reality both the dissipation mechanisms can act in conjunction. (As discussed in Section 3.2, for $k > k_J$ modes, decaying turbulence is a faster dissipation process close to recombination, whereas at redshifts below $z \sim 20$, ambipolar diffusion dominates). For a field strength $B_0 \simeq 3 \times 10^{-9}$ G for nearly scale invariant model $n = -2.8$, $\tau_{\rm ion} \simeq 0.15$ when both the dissipation mechanisms are included. As is seen in Figure 3, these two dissipation mechanisms affect the visibility function at different epochs; their combined effect on the visibility function is a function enveloping the curves corresponding to both these mechanisms.

It is of interest to note that both the dissipation mechanisms, and especially dissipation due to decaying turbulence, lead to changes in the initial visibility surface close to the standard recombination. This can have interesting consequences over and above the changes that occur due to late reionization. For instance, the usual late reionization models lead to decrement in temperature anisotropies for $\ell \gtrsim 40$ by a factor $\exp(-2\tau)$ and an enhancement of the anisotropies for smaller ℓ by a similar factor (see e.g. Bond 1996). This effect is very difficult to discern from just temperature anisotropies owing to cosmic variance (information about τ from WMAP results comes solely from the temperature-polarization cross-correlation (Kogut et al. 2003)). However a change in the initial visibility surface can change the diffusion length from Silk damping (see e.g. Hu & Sugiyama 1995) which has consequences different from the usual late reionization models. We hope to study these aspects in detail in the future.

Recent observations of high redshift objects have shown that the universe has detectable amount of neutral hydrogen for $z \geqslant 5.2$ (Becker et al. 2001, Djorgovski et al. 2001, Fan et al. 2002). In particular these observations put an upper bound on the ionized fraction $x_e \lesssim 0.9$ .i.e. at least 10% universe becomes neutral in the redshift range $5.2 \lesssim z \lesssim 6$. This in conjunction with the the WMAP result suggests that there might have been two epochs of reionization–one occurring as early as $z \simeq 17$ and the other proceeding for $z \lesssim 5$ (Kogut et al. 2003). The ionization histories from magnetic field dissipation (Figures 1 and 5) are compatible with the requirement that more than 10% of the universe was neutral for $z \geqslant 5.2$.

6 PRIMORDIAL MAGNETIC FIELDS AND EARLY STRUCTURE FORMATION

In the previous two sections we considered the effect of dissipation of magnetic fields on the intergalactic medium. Primordial magnetic fields also generate density perturbations in the post-recombination universe, which can gravitationally collapse to form structures (Wasserman 1978). These structures might form at high enough redshifts to affect the state of intergalactic medium (Kim et al. 1996, SB98a). We consider this scenario in this section.

6.1 Density evolution and power spectrum

Density evolution in the presence of magnetic field, for scales larger than the Jeans scale, is governed by equations (Wasserman 1978, Kim et al. 1996, SB98a, Gopal & Sethi 2003):

$$\begin{aligned} \frac{\partial^2 \delta_b}{\partial t^2} &= -2\frac{\dot a}{a}\frac{\partial \delta_b}{\partial t} + 4\pi G(\rho_{\rm DM}\delta_{\rm DM} + \rho_b\delta_b) + S(t,\mathbf{x}) \\ \frac{\partial^2 \delta_{\rm DM}}{\partial t^2} &= -2\frac{\dot a}{a}\frac{\partial \delta_{\rm DM}}{\partial t} + 4\pi G(\rho_{\rm DM}\delta_{\rm DM} + \rho_b\delta_b) \end{aligned} \tag{19}$$

Here $S(t,\mathbf{x}) = \nabla.(\nabla\mathbf{x}\mathbf{B}(t_0,\mathbf{x}))\mathbf{x}\mathbf{B}(t_0,\mathbf{x})/(4\pi\rho_b(t_0)a^3)$ is the source term from magnetic fields; subscript 'DM' refers to dark matter. As seen in Eqs. (19), dark matter perturbations are not directly affected by the magnetic field, but are generated by baryonic perturbations.

To solve these equations, we define $\delta_{\rm m} = (\rho_{\rm DM}\delta_{\rm DM} + \rho_{\rm b}\delta_{\rm b})/\rho_{\rm m}$ with $\rho_{\rm m} = (\rho_{\rm DM} + \rho_{\rm b})$. This transforms Eqs. (19) to:

$$\begin{aligned} \frac{\partial^2 \delta_b}{\partial t^2} &= -2\frac{\dot a}{a}\frac{\partial \delta_b}{\partial t} + 4\pi G\rho_{\rm m}\delta_{\rm m} + S(t,x) \\ \frac{\partial^2 \delta_{\rm m}}{\partial t^2} &= -2\frac{\dot a}{a}\frac{\partial \delta_{\rm m}}{\partial t} + 4\pi G\rho_{\rm m}\delta_{\rm m} + \frac{\rho_b}{\rho_m}S(t,x) \end{aligned} \tag{20}$$

The second of Eqs. (20) can be solved by Green's function method. Its solution is:

$$\delta_{\rm m}(x,t) = A(x)D_1(t) + B(x)D_2(t) - D_1(t)\int_{t_i}^{t} dt' \frac{S(t',x)D_2(t')}{W(t')} + D_2(t)\int_{t_i}^{t} dt' \frac{S(t',x)D_1(t')}{W(t')} \tag{21}$$

Here $W(t) = D_1(t)\dot D_2(t) - D_2(t)\dot D_1(t)$ is the Wronskian and $D_1(t)$ and $D_2(t)$ are the solutions of the homogeneous part of the $\delta_{\rm m}$ equation (see e.g. Peebles 1980). Here t_i corresponds to the epoch of recombination as the perturbations cannot grow before this epoch. The homogeneous solutions correspond to perturbations generated by sources before recombination, e.g. during inflationary epoch. We neglect these in our analysis. Neglecting homogeneous solutions, the solution to Eq. (21) for $z \gg 1$ is:

$$\delta_{\rm m}(\mathbf{x},t) \simeq \frac{3\Omega_b}{5\Omega_m^2}\left(\frac{3}{2}\left(\frac{t}{t_i}\right)^{2/3} + \left(\frac{t_i}{t}\right) - \frac{5}{2}\right)S(\mathbf{x},t_i)t_i^2 \tag{22}$$

Using this solution, the evolution of $\delta_{\rm b}$ can be solved from:

$$\frac{1}{a^2}\frac{\partial}{\partial t}\left(a^2\frac{\partial\delta_{\rm b}}{\partial t}\right) = \frac{3}{2}H^2\delta_{\rm m} + S(t,\mathbf{x}) \tag{23}$$

Here we have used $H^2 = (8\pi G/3)\rho_m$. The fastest growing solution of Eq. (23) for $z \gg 1$ is $\propto t^{2/3}$. This shows that the fastest growing modes of both the baryonic and the dark matter perturbations grow at the same rate and as can be readily checked from Eqs. (23) and Eqs. (22) are also equal in magnitude.

The power spectrum of matter perturbations can written as:

$$P(k,t) = \langle\delta_{\rm m}(k,t)\delta^*_{\rm m}(k,t)\rangle \equiv D^2(t)P(k) \tag{24}$$

Here k is the comoving wave vector and $D(t)$ is dimensionless function that gives the time dependence of the solution to Eq. (23); dependences on density etc. are absorbed in the definition of $P(k)$. For magnetic field power spectrum given in Eq. (2) and using Eq. (1), the matter power spectrum can be evaluated (Kim et al. 1996):

$$P(k) = \frac{B_0^4 k^3 H_0^{-4}}{(8\pi)^3\Omega_m^4\rho_c^2 k_\star^2} \text{ for } k \leqslant 2k_\star \tag{25}$$

For the power law magnetic field power spectrum, the matter power spectrum can be computed numerically (Gopal & Sethi 2003). The matter power spectrum cannot grow below the magnetic Jean's length as the magnetic pressure will prevent the structures from collapsing below this scale (Kim et al. 1996, SB98a). This can roughly be taken into account by imposing a sharp cut-off in the power spectrum below this scale. We adopt the more refined analysis of Kim et al. (1996). Kim et al. (1996) computed the power spectrum close to the Jeans' length by calculating the back-reaction on the velocity field to the first order, this results in the modification of the growth factor $D(t)$, which becomes a function of k for $k \simeq k_J$ and grows more slowly than the growing mode of $D(t)$ (Eq. (22)) close to the Jeans' scale. For $k > k_J$, the power spectrum doesn't grow .i.e. $D(t) = 0$ for these scales and for scales $k \ll k_J$ the power spectrum grows as the usual growing mode (Eq. (22)). Therefore, the first structures to collapse will have scales close to the magnetic Jeans' scale λ_J. The mass dispersion in a given radius R can then be written as:

$$\sigma^2(R,t) = 4\pi\int_0^{k_J} dk k^2 P(k) D^2(k,t) W^2(kR) \tag{26}$$

Here $W(kR)$ is the window function. We use the Gaussian window for our analysis $W(kR) = \exp(-k^2R^2/2)$. Following discussion preceding Eq. (26), we have introduced a cut-off for $k > k_J$. As seen in Figures 2 and 6, the universe can get heated to temperatures $\simeq 10^4$ K, which increases the sound speed and therefore the Jeans' length of the medium from thermal pressure. At $z \simeq 15$, the Jeans' scale from thermal pressure corresponds to $k_J^{\rm th} \simeq 200\,{\rm Mpc}^{-1}$. This corresponds to scales smaller than the magnetic Jeans' length for the values of magnetic field we consider here. Therefore, the effect of thermal Jeans' length is not important in our analysis.

6.2 Early structure formation

Tangled magnetic fields can induce early collapse of structures in the universe. This could also result in early re-ionization of the universe. The first structures to collapse would correspond to the length scales close to magnetic Jeans' length. One can get some information about the generic behaviour of $\sigma(R)$ from Eq. (26). Firstly $\sigma \propto R^{-3}$ for $R \gtrsim \lambda_J$ for the delta function magnetic power spectrum, which shows that even though first structures might form early, the formation of larger structures is suppressed. For $R = \alpha\lambda_J$ for where α is some number typically close to one and $k_\star = k_J$ the mass dispersion is nearly independent of B_0, as can be established from Eqs. (25) and (26). (Since for $k_\star = k_J$, $\sigma^2(R,t) \propto B_0^4/(k_J^2\alpha^2\lambda_J^6) \propto B_0^4 k_J^4/\alpha^2$, and since $k_J \propto B_0^{-1}$, σ becomes independent of B_0).

The main dependence of $\sigma(R,t)$ is on the total matter content of the universe: $\sigma(R,t) \propto 1/\Omega_m^2$. Therefore the redshift at which the first structures collapse becomes nearly independent to the value of B_0, although the mass contained in these structures, which depends on the scale $k_\star = \alpha k_J$, does depend on B_0, through the k_J dependence.

This behaviour is generic to most power law magnetic power spectra $M(k) = Ak^n$ (Kim et al. 1996, Gopal & Sethi 2003). Note that the spectral indices which are of interest from demanding that gravity waves are not over produced in the early universe by the anisotropic stress of the magnetic fields are $n \simeq -3$, for $B_G \simeq 10^{-9}$ G (cf. Caprini & Durrer 2002). Further, a spectral index n which is not much larger than -3 is is also required for not over producing CMB anisotropy on small angular scales (cf. Subramanian and Barrow 2002). For $n \leqslant -1.5$, the matter power spectrum is $\propto B_0^4 k^{2n+7}/k_c^{2n+6}$ apart from factors proportional to n (Gopal & Sethi 2003), where B_0 now is the rms value filtered at scale k_c. This gives $\sigma^2(R) \propto B_0^4/(k_c^{2n+6}R^{2n+10})$. For $k_c \simeq k_J$, $\sigma^2(R) \propto B_0^4 k_J^4$ again, and so does not depend on the value of B_0, which agrees with the analysis for the delta function power spectrum. For $n \simeq -3$, $\sigma(R) \propto 1/R^2$, and again even though first structures might form early, the formation of larger structures is suppressed.

In Figure 6 and 7 we show the evolution of $\sigma(R,t)$ for the delta function magnetic power spectrum and power law spectra with $n \simeq -3$, respectively, for $\Omega_m = 0.3$. For a spectrum $P(k)$ with a small scale (large k) cut off, like we have in the present context, the scale of the first collapsed structures, $R = R_f$, would roughly correspond to the wavenumber where $k^3 P(k)$ is maximum. This maximum occurs at $k \sim k_J$, but there is always an uncertainty as to whether this corresponds to $R_f \sim 1/k_J = \lambda_J/(2\pi)$ or $R_f \sim 2\pi/k_J = \lambda_J$. On the other hand if one uses a Press-Schechter type prescription to compute the abundance of objects (cf. Padmanabhan 1993), then this abundance will peak for objects where $d\sigma(R)/dR$ is maximum. This criterion gives roughly $R_f \simeq 0.3 - 0.4\lambda_J$ for all the models shown in Figure 6 and 7 (in between the above two estimates based on k_J). It is in view of this uncertainty in determining the exact value of R_f, that we have given $\sigma(R,t)$ in Figure 6 and 7 for a range around $R = \lambda_J$.

For spherically symmetric perturbations, collapse of a structure corresponds to $\sigma(R,t) = 1.68$ (see e.g. Peebles 1980). Of course for a power spectrum with a cut-off the first collapses will be more pancake like. And there is the added complication of taking account of magnetic pressure effects. Nevertheless it seems reasonable to demand that $\sigma(R_f, z) \sim 1$ for the formation of structures. For delta function power spectrum, the collapse of first structures then occur for $z \simeq 50$ to 60. As the collapse of first structures will result in early ionization of the universe, this model can probably be ruled out from WMAP observation which suggests that the ionization redshift is between 20 and 15. On the other hand for the power law models with nearly scale invariant spectrum, with say $n \sim -2.8$, we see from Figure 7, that the collapse redshift of the first structures can be in the range suggested by WMAP results. Also, from Eq. (26), for the power law model $\sigma(R) \propto 1/R^2$ and therefore only a small fraction of length scales close to the Jeans' length can collapse early with collapse of larger structures suppressed and occurring at a later redshift (see below). This is in contrast to the CDM class of models in which the collapse of first structures leads to collapse of a much wider range of length scales (see e.g. Peebles 1993, Padmanabhan 1993).

Even though the collapse redshift doesn't depend on the value of B_0, the mass of the collapsed object does depend on B_0 through its dependence on R_f. Typical magnetic fields of interest for CMBR anisotropies are B_0 of order a few nano gauss (see e.g. Subramanian & Barrow 2002). For $B_0 \sim 3 \times 10^{-9}$ G the total mass enclosed within a radius R_f is $M_f \simeq 1 - 3 \times 10^{10}\,\mathrm{M}_\odot$, which is much smaller than a typical $L_\star$ galaxy. For a field $B_0 \sim 10^{-9}$ G, the mass M_f of the first collapsed objects will be smaller, by a factor ~ 30. Therefore the first structures to collapse would be sub-galactic. As we emphasised above, galactic structures, owing to $1/R^2$ suppression of $\sigma(R)$, form at a much later redshift. For instance, if the first structures collapse at $z = z_f \simeq 15$, the galactic sized objects would only collapse by $z = z_G$ where $(1 + z_G) \sim (R_f/R_G)^2(1 + z_f) = (M_f/M_G)^{2/3}(1 + z_f) \ll (1 + z_f)$; a luminous galaxy of mass $M_G \sim 10^{12}\,\mathrm{M}_\odot$ would not have collapsed by the present epoch. This means that even though magnetic fields can induce the formation of first structures, it would have little impact on the formation of galactic and larger scale structures at the present epoch (see also Gopal & Sethi 2003).

From our discussion above we can conclude that (i) collapse of first structures could have commenced for $z \simeq 15$–30, (ii) only a small fraction of mass range close to the magnetic Jeans' scale collapse (iii) the collapse redshift is nearly independent of the strength of the magnetic field, if the magnetic field is specified as rms filtered at the Jeans scale and (iv) the mass of the first collapsed objects will be sub-galactic their exact value depending on B_0. These conclusions hold for magnetic field strengths for which the magnetic Jeans' length exceeds the thermal Jeans' length.

In light of the recent WMAP observations it is of interest to ask if this early collapse can lead to early reionization of the universe. This is generally a difficult question to address given the uncertainties in understanding astrophysical processes that determine the hydrogen ionizing flux from a collapsed structure (see e.g. Kogut et al. 2003 and references therein). A quantity of interest for this analysis is the fraction of collapsed mass in the universe. In CDM class of models the density field is assumed to be Gaussian, which allows the collapsed fraction to be computed from Press-Schechter method (see e.g. Padmanabhan 1993). The collapsed fraction of mass at scales which go non-linear generally approaches a high fraction of unity in the Gaussian case. In the magnetic field induced structure formation, it is the magnetic field that is assumed to have Gaussian statistics. The density field is $\propto B^2$ and satisfies modified χ^2 statistics (cf. Subramanian and Barrow 1998). In this case the high density regions get more weight than a Gaussian, and therefore it is natural to expect that the formation of first structures would result in the collapse of a large fraction of the mass in the universe. It is then possible that the magnetically induced collapse of early structures have interesting consequences for the reionization of the universe. We hope to return to this issue in a later work.

7 CONCLUSIONS

We have studied here some of the consequences of primordial tangled magnetic fields for the post-recombination universe. In the post-recombination epoch the magnetic field energy can dissipate into the intergalactic medium by ambipolar diffusion and by generating decaying MHD turbulence. An important issue we focussed upon, is the impact of such dissipation on the thermal and ionization history of the universe. We showed that magnetic field dissipation can change the ionization history of the universe sufficiently to have a bearing on recent and future observations of CMBR anisotropy. We have also shown that primordial magnetic fields generically induce the early formation of sub-galactic structures, which could be responsible for the reionization of the universe at $z \simeq 15$, as indicated by recent WMAP results (Kogut et al. 2003).

More specifically, our results are as follows:

1. Primordial magnetic field dissipation can result in ionization histories that give the Thompson scattering optical depth $\tau_{\rm ion} \gtrsim 0.1$. This requires magnetic field strengths $B_0 \gtrsim 2 \times 10^{-9}$ G, with $k_\star = k_{max}$, for the delta function power spectrum. For nearly scale invariant power law power spectra magnetic field strengths (smoothed over k_{max}), $B_0 \gtrsim 3 \times 10^{-9}$ G gives $\tau_{\rm ion} \gtrsim 0.1$ from either of the dissipation processes (ambipolar diffusion and decaying MHD turbulence). Adding the effect of both dissipation mechanisms give $\tau_{\rm ion} \gtrsim 0.15$, for the same field strength. (Note that these two effects can indeed add since they are important at different epochs; the major effect of decaying turbulence occurring at high z after recombination, while ambipolar diffusion dominates at redshifts below $z \simeq 20$.)
2. To infer the impact of these dissipation processes, on CMBR measurements, we have computed the visibility function for the resulting ionization histories. Our preliminary analysis show that the recent WMAP observations are unlikely to have been much affected by the magnetic field dissipation, even though the Thomson scattering optical depth is comparable to the value inferred by WMAP. This is because the visibility function in the case of magnetic field dissipation receives contribution from a much wider range of redshifts than is required to explain the WMAP observations. Future CMBR probes like Planck can potentially detect the modified CMBR anisotropy signal from such partial re-ionization (Kaplinghat et al. 2003). This can be used to detect or further constrain small scale primordial magnetic fields.
3. Primordial magnetic fields can induce the collapse of first subgalactic structures in the universe at high redshifts. We show that for the nearly scale free power law models the collapse redshift is in the range between 10 to 20. The masses of these objects depend on the magnetic field strength smoothed on the Jeans scale, and lie in the range $3 \times 10^9 M_\odot$ to $3 \times 10^{10} M_\odot$, for $B_0 \sim 10^{-9}$ G to $B_0 \gtrsim 3 \times 10^{-9}$ G. As the formation of first structures is likely to ionize the universe, it seems plausible that the formation of these objects could explain the reionization features implied by WMAP results (Kogut et al. 2003) .

In summary, the presence of tangled magnetic fields in the post-recombination epoch can result in several interesting signals. They can induce early collapse of structures, which might explain the recent WMAP polarization results. In addition, the slow dissipation of magnetic field in the post-recombination era can alter the ionization history sufficiently to have interesting observational consequences especially for the future CMBR probes like Planck.

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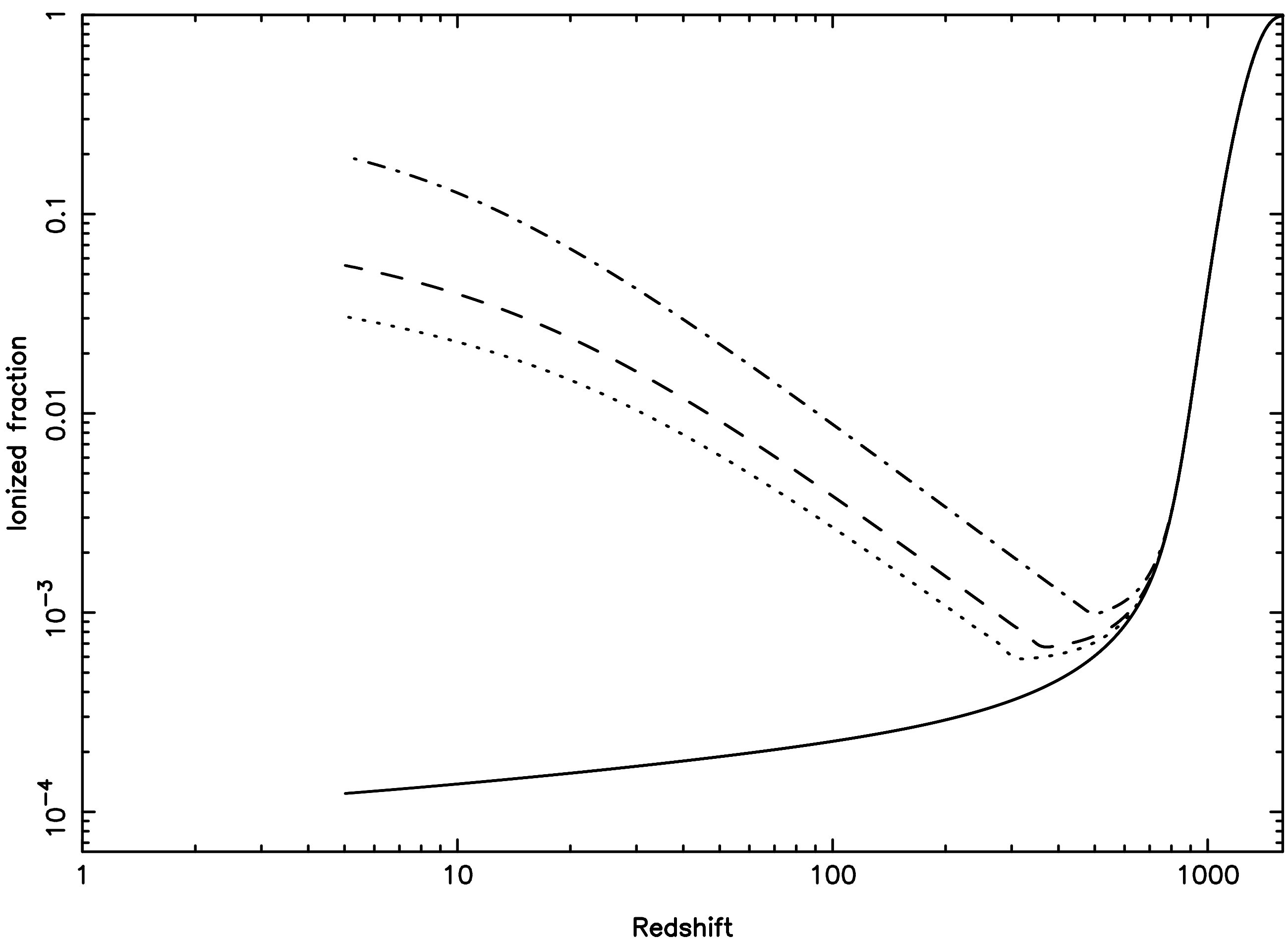

Figure 1. Evolution of the ionization state of the universe is shown for ambipolar dissipation. Different curves are: standard recombination (solid curve); the dotted and dashed curves correspond to nearly scale free magnetic field power spectra with $n = -2.9$ and $n = -2.8$ with $B_0 = 3 \times 10^{-9}$ G; the dot-dashed curves correspond to the delta function magnetic field power spectrum with $B_0 = 3 \times 10^{-9}$ G and $k_\star = k_{\max}$.

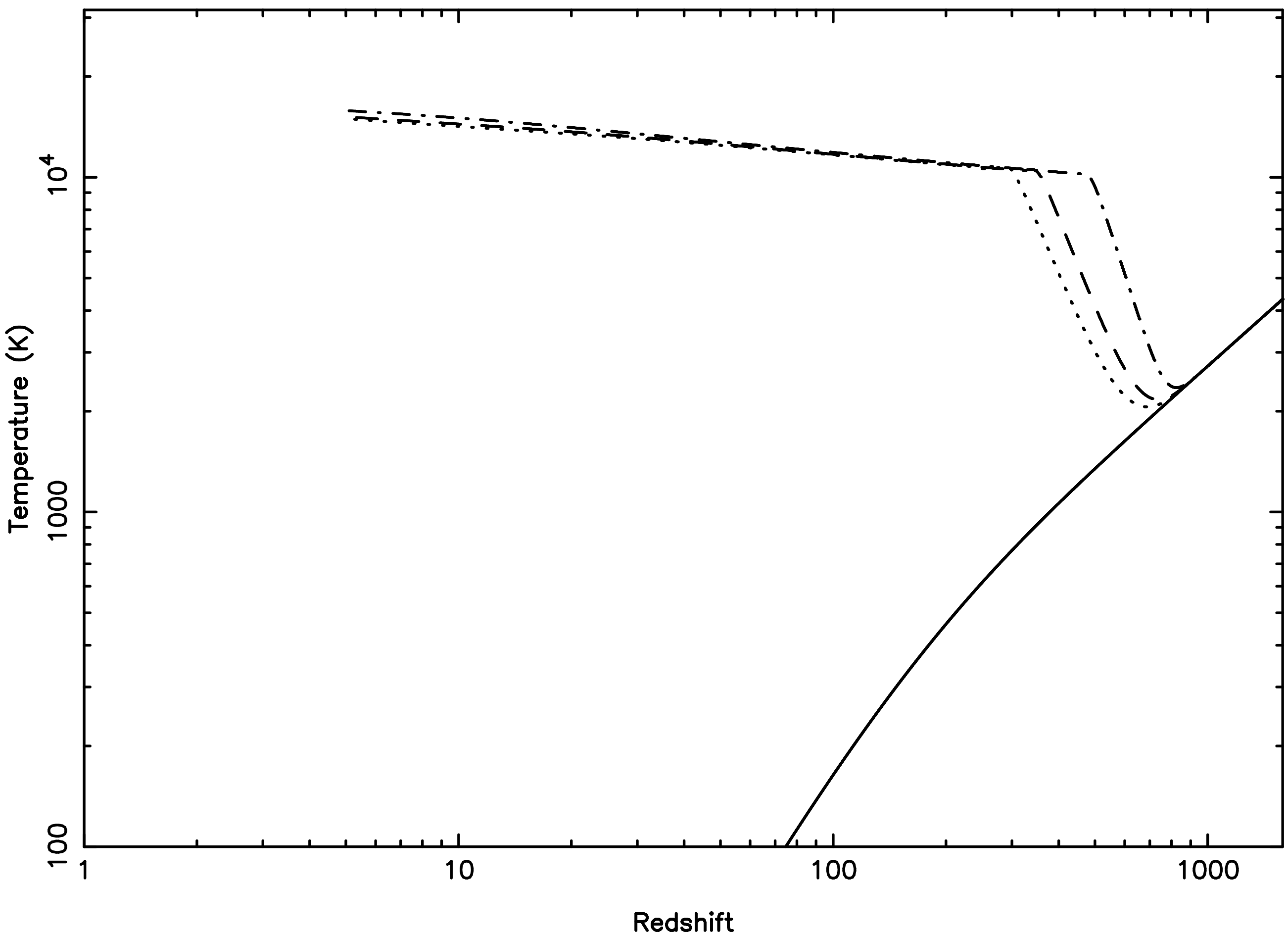

Figure 2. Evolution of the thermal state of the universe is shown for ambipolar dissipation. Curves are for the same parameters as in Figure 1

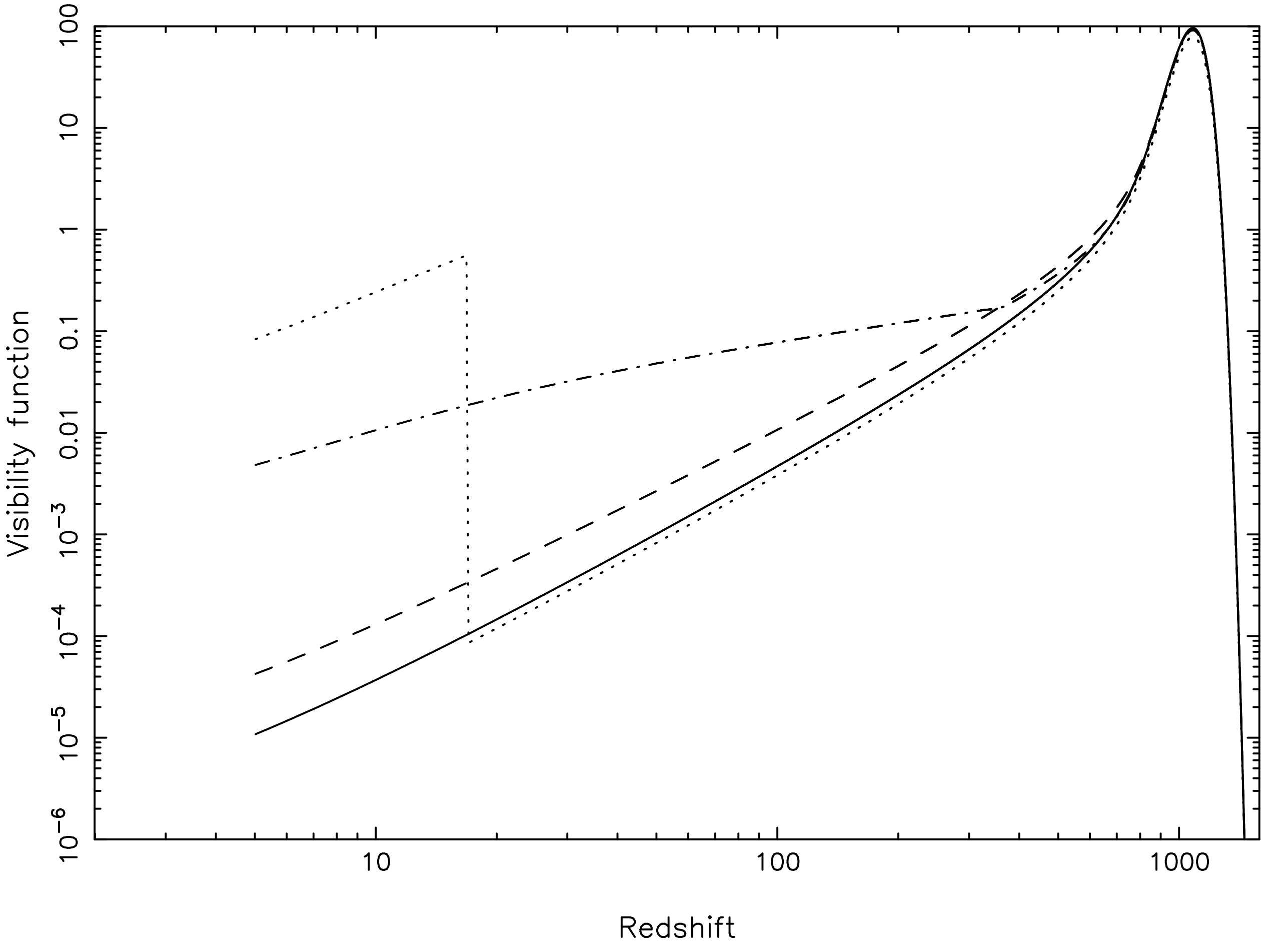

Figure 3. Visibility function, defined as $d\tau/d\eta \exp(-\tau) H_0^{-1}$, is plotted for different models. The solid and the dotted curves are for the standard recombination and a model in which the universe reionizes at $z = 17$, respectively. Dashed curve corresponds to a decaying turbulence model with $B_0 = 3 \times 10^{-9}$ G and $m = 0.2$ (Eq. (15)). Dot-dashed curve corresponds to the ambipolar diffusion case with $B_0 = 3 \times 10^{-9}$ G and $n = -2.8$.

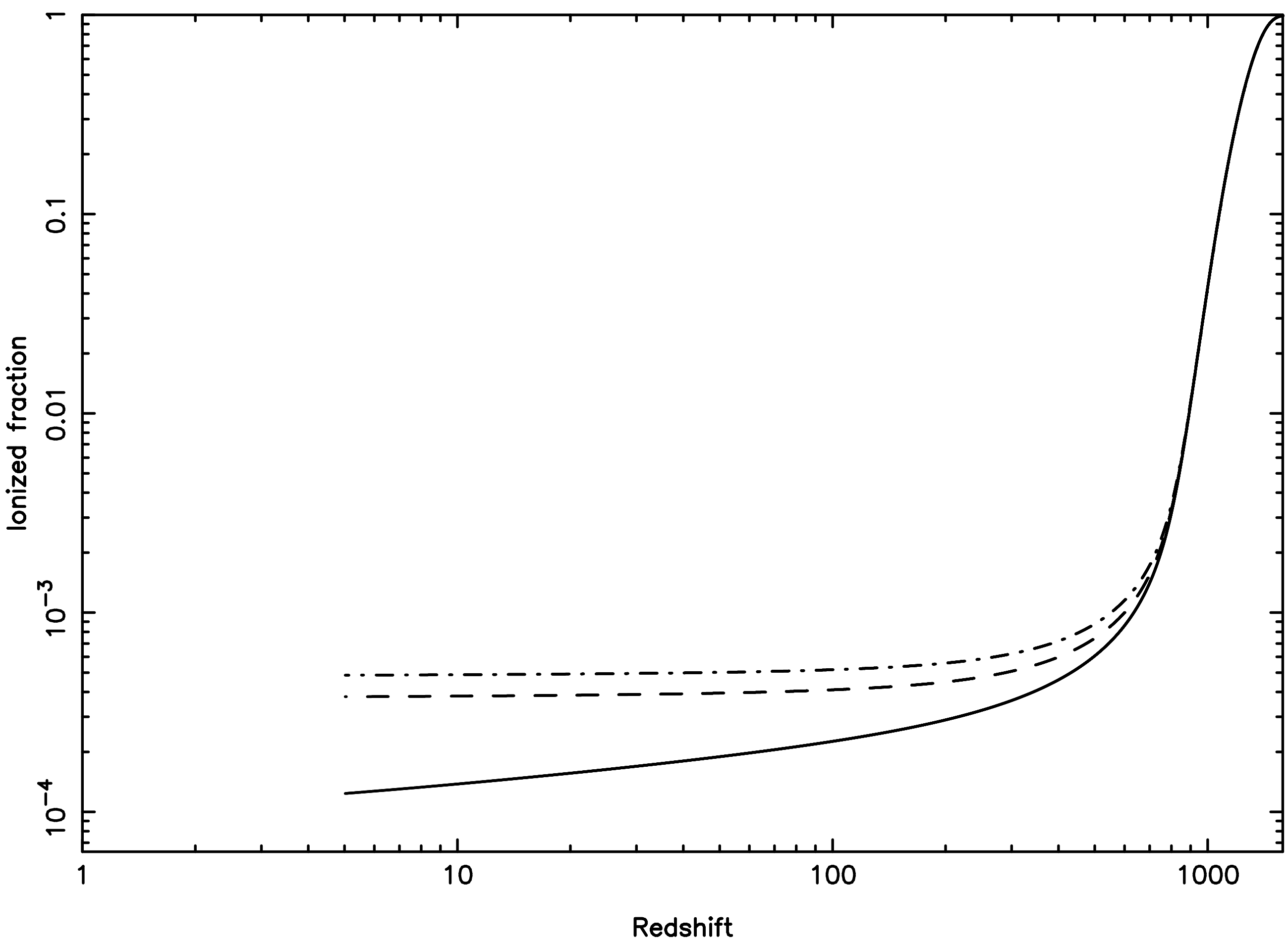

Figure 4. Evolution of the ionization state of the universe is shown for dissipation of magnetic field energy due to decaying turbulence. Different curves are: standard recombination (solid curve), the dot-dashed, and dashed curves correspond to $B_0 = 3 \times 10^{-9}\,\mathrm{G}$, $m = \{0.2, 0.1\}$, respectively (Eq. (15)).

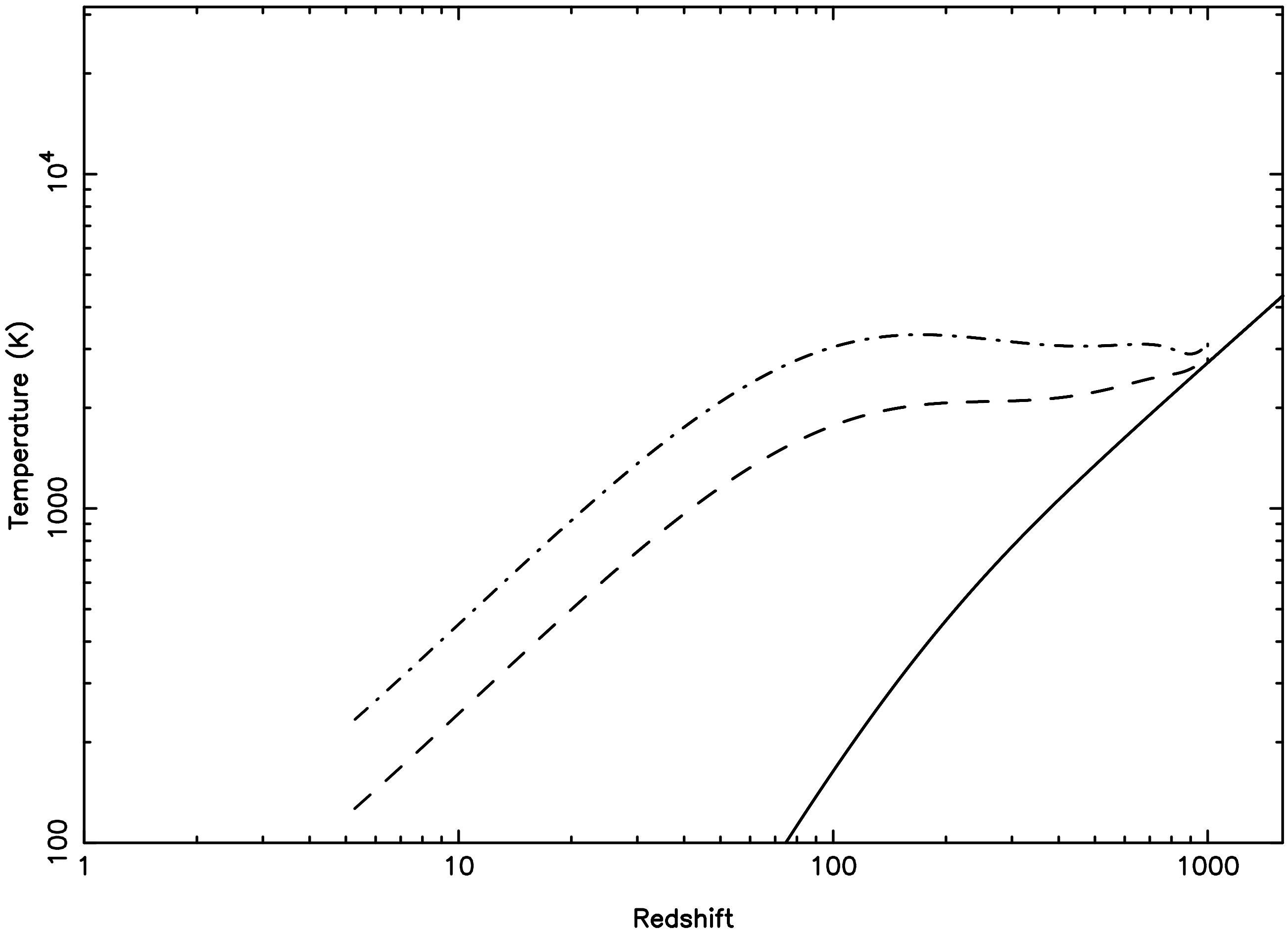

Figure 5. Evolution of the thermal state of the universe is shown for decaying turbulence. Curves are for the same parameters as in Figure 4

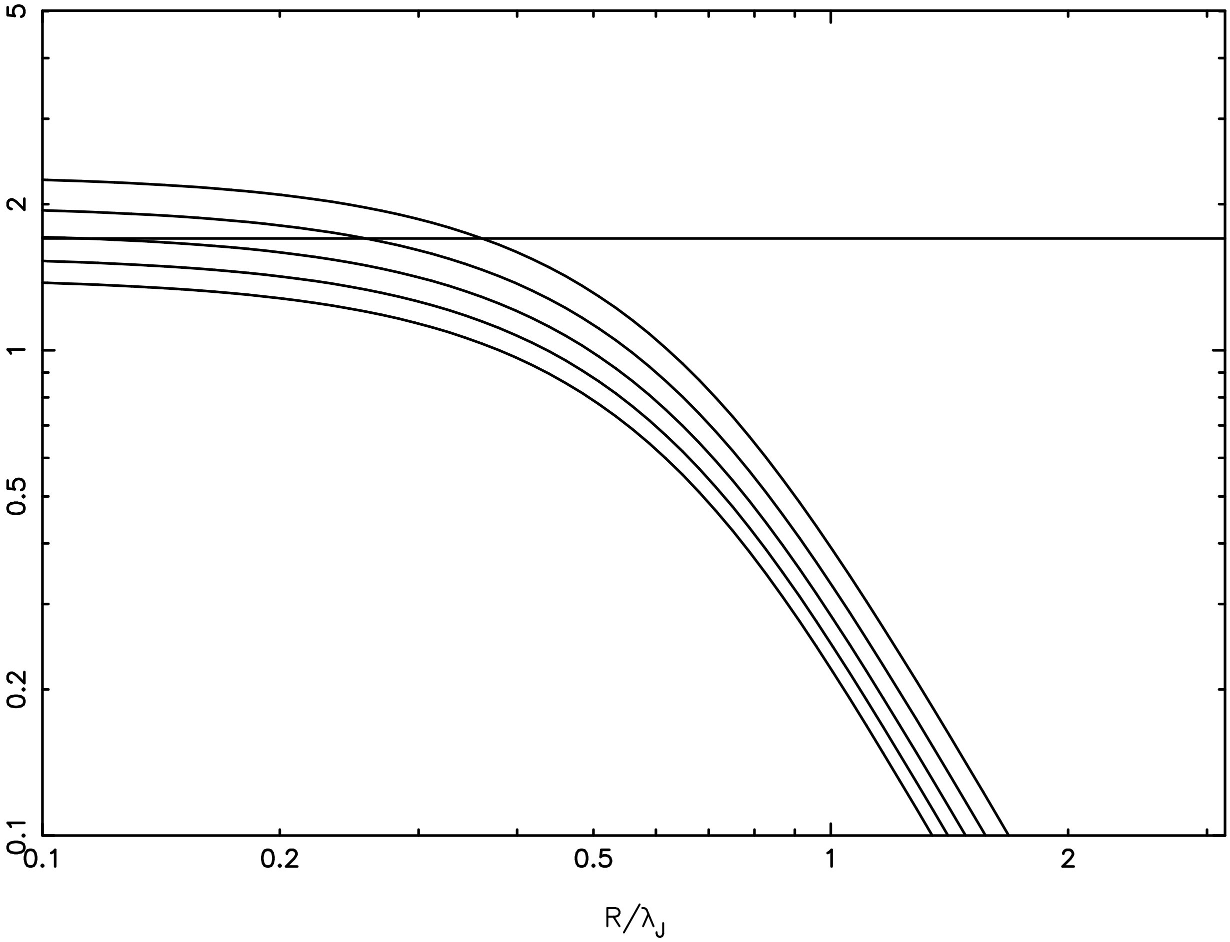

Figure 6. The mass dispersion $\sigma(R, z)$ is shown for the delta function magnetic field power spectrum. Different curves, from top to bottom, correspond to redshifts $z = \{50, 60, 70, 80, 90\}$, respectively. The horizontal line corresponds to $\sigma = 1.68$.

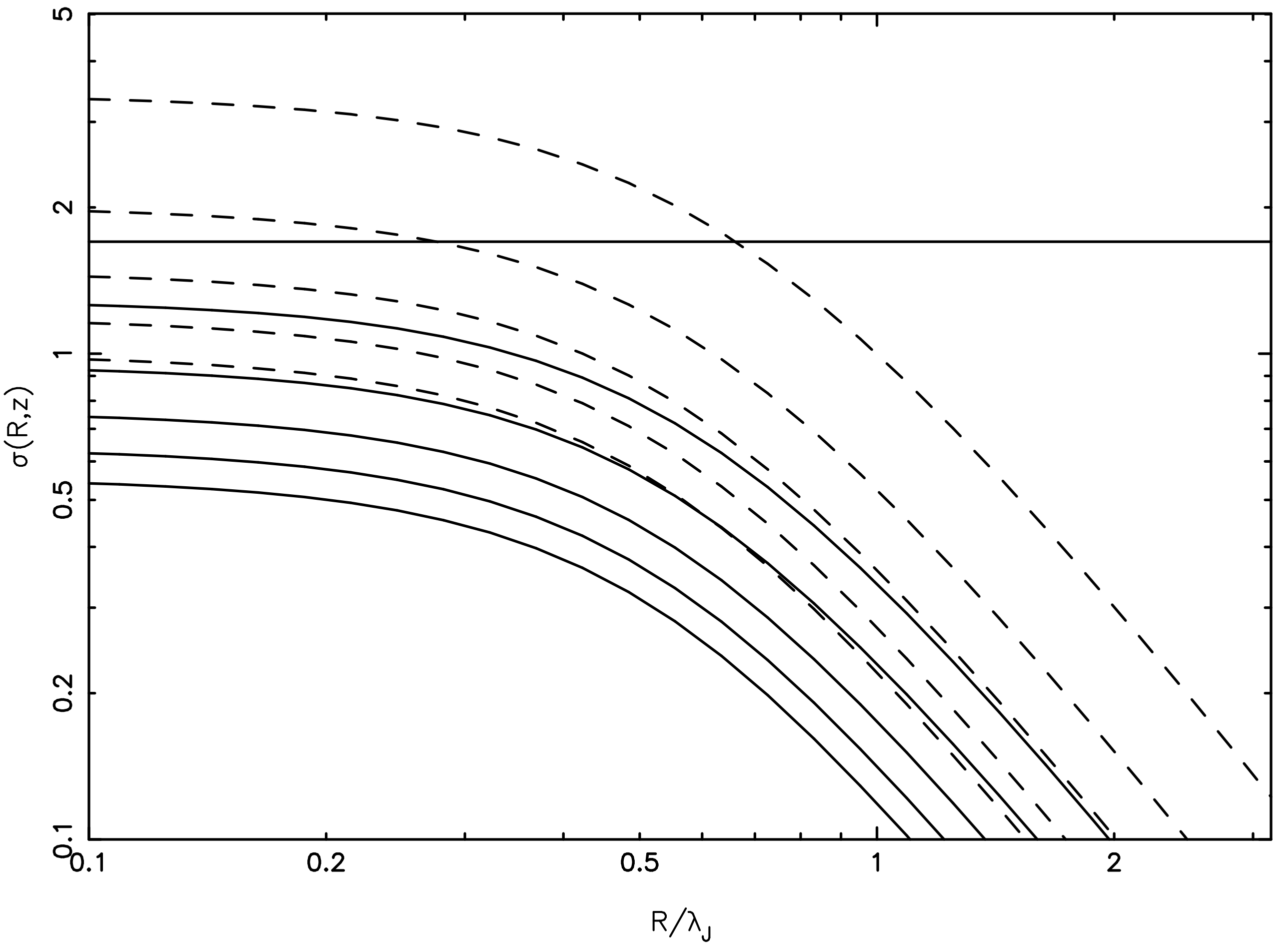

Figure 7. The mass dispersion $\sigma(R, z)$ is shown for two models with nearly scale free magnetic field power spectra. The solid and dashed curves correspond to $n = -2.9$ and $n = -2.8$, respectively. Different curves, from top to bottom, correspond to redshifts $z = \{10, 15, 20, 25, 30\}$, respectively. The horizontal line corresponds to $\sigma = 1.68$.