EMERGENT UNIVERSE : THEORY AND OBSERVATIONS

B. C. Paul*
Physics Department
North Bengal University
Siliguri -734 013, India

***IUCAA Resource Centre, NBU**

Outline of the Talk

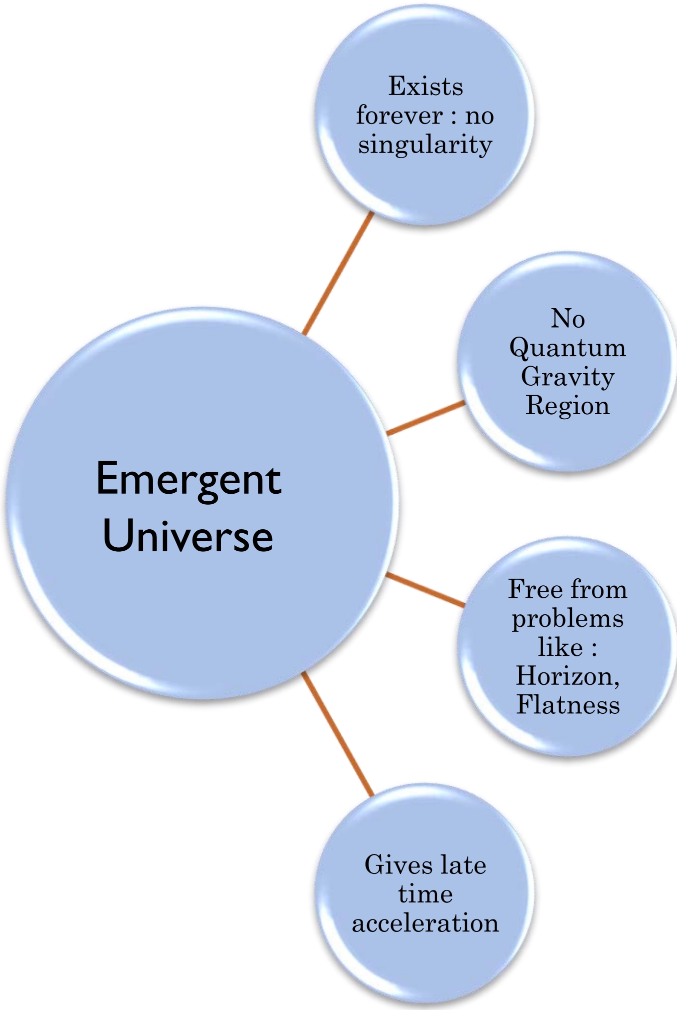
- Introduction on Emergent Universe (EU)
- Cosmological Solution by Harrison
- EU in Non-flat universe & Construction of Potential for EU
- EU Model in flat Universe
- EU with Astronomical Observations
- Brief Discussions

Emergent Universe

- A universe which is ever-existing, large enough so that space-time may be treated as classical entities.
- No time like singularity.
- The Universe in the infinite past is in an almost static state but it eventually evolves into an inflationary stage.

- An emergent universe model, if developed in a consistent way is capable of solving the well known conceptual problems of the Big-Bang model.

Why Emergent Universe ?



INTRODUCTION

$$ds^2 = dt^2 - a(t)^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$$

$$a(t) = a_0 (1 + e^{\alpha t})^{\varpi}$$

$$a(0) = a_0 \neq 0$$

Ever existing universe.
No singularity !

In 1967, Harrison found a cosmological solution in a closed model with radiation and a positive cosmological constant :

$$a(t) = a_i \left[1 + \exp\left(\frac{\sqrt{2}\, t}{a_i} \right) \right]^{\frac{1}{2}}$$

As $t \rightarrow -\infty$, the model goes over asymptotically to an Einstein Static Universe, the expansion is given by a finite number of e-folds :

$$N_o = \ln\left(\frac{a_o}{a_i} \right) \approx \frac{t_o}{\sqrt{2}} a_i$$

- Radius is determined by Λ . However, de Sitter phase is never ending (NO Exit).

- Ellis and Maartens (CQG, 21 (2004) 223) :

Considered a dynamical scalar field to obtain EU in a Closed Universe. In the model, taken up minimally coupled scalar field ϕ in a self-interacting potential V(ϕ).

In this case the initial size a_i of the universe is determined by the field's kinetic energy.

To understand we now consider a model consisting of ordinary matter and minimally coupled homogeneous scalar field in the next section.

- The KG equation for field:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

- The conservation equation for matter is

$$\dot{\rho} + 3(1+\omega)H\rho = 0$$

- The Raychaudhuri equation

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left[\frac{1}{2}(1+3\omega)\rho + \dot{\phi}^2 - V(\phi)\right]$$

- First Integral Friedmann equation :

$$H^2 = \frac{8\pi G}{3}\left[\rho + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right] - \frac{K}{a^2}$$

- It leads to $$\dot{H} = -4\pi G\left[\dot{\phi}^2 + (1+\omega)\rho\right] + \frac{K}{a^2}$$

- Raychaudhuri equation gives condition for inflation,

$$\ddot{a} \rangle 0 \Leftrightarrow \dot{\phi}^2 + \frac{1}{2}(1+3\omega)\rho \langle V(\phi)$$

- For a positive minimum $a_i \equiv a(t_i) \rangle 0$,

$$H_i = 0 \Leftrightarrow \frac{1}{2}\dot{\phi}_i^{\ 2} + V_i + \rho_i = \frac{3K}{8\pi G a_i^{\ 2}}$$

where the time t_i may be infinite.

- The Einstein Static universe is characterized by $K=1$ and $a=a_i=Const.$, we now get

$$\frac{1}{2}(1-\omega_i)\,\rho_i+V_i=\frac{1}{4\pi G\,a_i^{\,2}},$$

$$(1+\omega_i)\,\rho_i+\dot{\phi}_i^{\,2}=\frac{1}{4\pi G\,a_i^{\,2}}$$

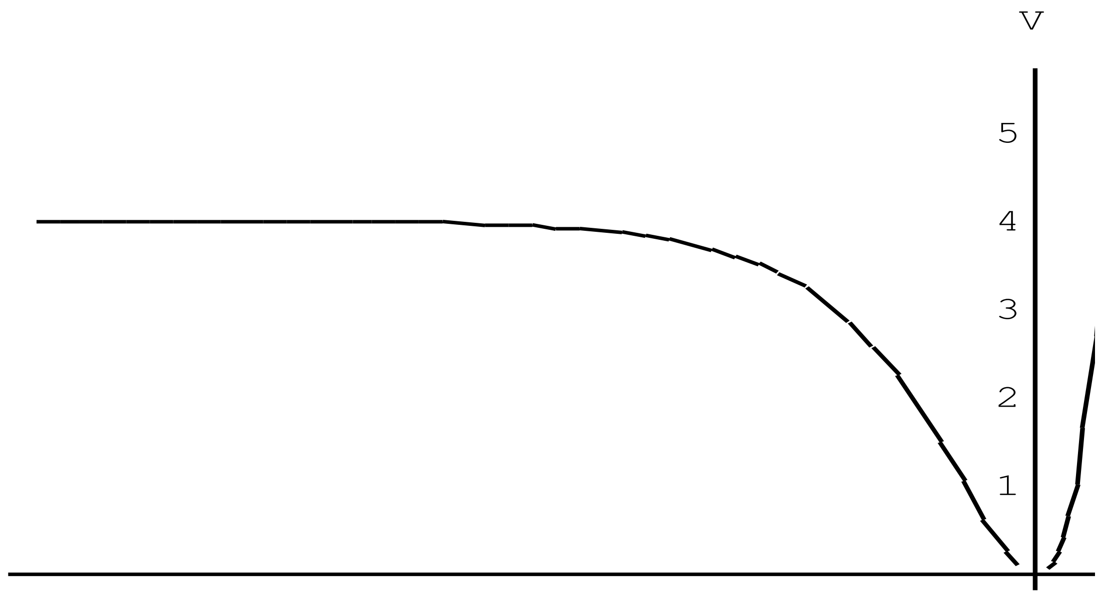
Case I : If the KE of the field vanishes, there must be matter to keep the universe static.

Case II : If only scalar field with non-zero KE, the field rolls at constant speed along the flat potential.

- A simple Potential for Emergent Universe with scalar field only is given by

$$V(\phi) \to V_i \quad as \quad \phi \to -\infty, \quad t \to -\infty$$

- But drops towards a minimum at a finite value ϕ_f



- Potential is given by

$$V - V_f = (V_i - V_f) \left[\exp\left(\frac{\phi - \phi_f}{\alpha} \right) - 1 \right]^2$$

- <u>Determination of the parameters in the potential :</u>

  Considering the space-time filled with a minimally coupled scalar field with potential :

$$V = V(\phi) = \left(C\,e^{G\phi} - D\right)^2 + E$$

- where A, B, C, D are constants to be determined by specific properties of the EU.

$$V' = 2CG\left(C\,e^{G\phi} - D\right)e^{G\phi}$$

$$V'' = 2CG^2\left(2Ce^{G\phi} - D\right)e^{G\phi}$$

- Potential has a minimum at

$$\phi_o = \frac{1}{G}\left(\ln\left(\frac{D}{C}\right)\right) \quad with \quad V_o = V(\phi = \phi_o) = E$$

- Set the minimum of the potential at the origin of the axis, we choose C=D and E = 0

$$V(\phi) = C\left(e^{G\phi} - 1\right)^2$$

- By definition EU corresponds to a past-asymptotic Einstein-static model :

$$V(\phi \to -\infty) = \frac{2}{\kappa a_i^2}$$

where a_i is the radius of the initial static model. To determine an additional parameter recall that an ES universe filled with a single scalar field satisfies

$$V(\phi) = \frac{2}{\kappa a_i^2} = \dot{\phi} \quad \Rightarrow \quad C = \frac{2}{\kappa a_i^2}$$

- Thus we have

$$V(\phi) = \frac{2}{\kappa a_i^2}\left(e^{G\phi} - 1\right)^2$$

- Basic properties of EU have fixed three of the four parameters in the original potential. The parameter B will be fixed as follows :

$$V' = \frac{4G}{\kappa a_i^2}\left(e^{G\phi} - 1\right)e^{G\phi} \langle 0 \quad for \quad -\infty \langle \phi \langle 0$$

- in the higher derivative context $\Rightarrow \quad G \rangle 0 \quad \Rightarrow. \quad \alpha \langle 0$

- Consider R^2 - modified action

(Ellis et al. CQG 21 (233) 2004)

$$I = \int d^4x \sqrt{-g}\,\left[R + \alpha\, R^2\right]$$

Define $\Omega^2 = 1 + 2\alpha R$ and consider the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2\, g_{\mu\nu}$, so that

$$\tilde{R} = \frac{1}{\Omega^2}\left[R - 6g^{\mu\nu}\,\nabla_\mu \nabla_\nu (\ln\Omega) - 6g^{\mu\nu}\,\nabla_\mu (\ln\Omega)\nabla_\nu (\ln\Omega)\right]$$

If we now equate $\phi = \sqrt{3}\,\ln(1 + 2\alpha R)$ we get

$$I = \int d^4x \sqrt{-g}\,\left\{\tilde{R} - \frac{1}{2}\,\tilde{g}_{\mu\nu}\partial_\mu \phi\,\partial_\nu \phi - \frac{1}{4\alpha}\left(e^{-\phi/\sqrt{3}} - 1\right)^2\right\}$$

The corresponding potential in EU is the reflection of the potential that obtained from higher derivative gravity.

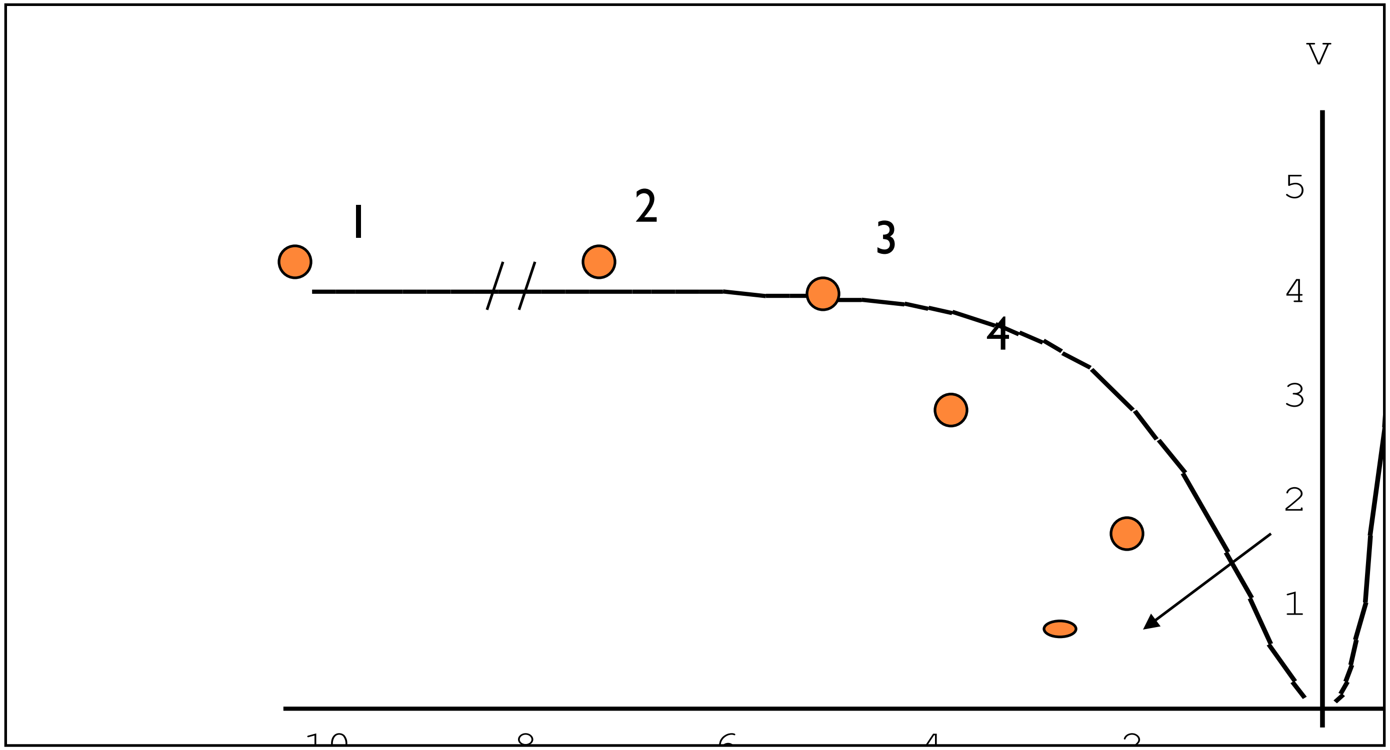
The different parts of the potential are :

(1) Slow-rolling regime or intermediate pre-slow roll phase

(2) Scale factor grows (slow-roll phase)

(3) inflation is followed by a re-heating phase

(4) standard hot Big Bang evolution



OUR MODEL : (SM, BCP, NKD, SD, AB, CQG 23, 6927 (2006))

In looking for a model of emergent universe, We assume the following features for the universe :

1. The universe is isotropic and homogeneous at large scales.

2. Spatially flat (WMAP results) : $\Omega_t = 1.02 \pm 0.02$
3. It is ever existing, No singularity

4. The universe is always large enough so that classical description of space-time is adequate.
5. The universe may contain exotic matter so that energy condition may be violated.

6. The universe is accelerating (Type Ia Supernovae data)

- EOS $$p = A\rho - B\rho^{\frac{1}{2}}$$
- The Einstein equations for a flat universe in RW-metric

$$\rho = 3\frac{\dot{a}^2}{a^2} \qquad p = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}$$

- Making use of the EOS we obtain

$$2\frac{\ddot{a}}{a} + (3A+1)\frac{\dot{a}^2}{a^2} - \sqrt{3}\,B\frac{\dot{a}}{a} = 0$$

- On integration

$$a(t) = \left(\frac{3\kappa(A+1)}{2}\left(\sigma + \frac{2}{\sqrt{3}\,B}e^{\frac{\sqrt{3}}{2}Bt}\right)\right)^{\frac{2}{3(A+1)}}$$

- 1) If $B < 0$ Singularity

  2) If $B > 0$ and $A > -1$ Non-singular (EU)

- Composition of the Emergent Universe :

$$\frac{d\rho}{dt}+3(\rho+p)\frac{\dot{a}}{a}=0$$

- Using EOS we get,

$$\rho(a)=\frac{1}{(A+1)^2}\left[B+\frac{K}{a^{3(A+1)/2}}\right]^2$$

- This provides us with indications about the components of energy density in EU

$$\rho=\frac{B^2}{(A+1)^2}+\frac{2KB}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}+\frac{K^2}{(A+1)^2}\frac{1}{a^{3(A+1)}}=\rho_1+\rho_2+\rho_3$$

$$p=-\frac{B^2}{(A+1)^2}+\frac{KB(A-1)}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}+\frac{AK^2}{(A+1)^2}\frac{1}{a^{3(A+1)}}=p_1+p_2+p_3$$

COMPOSITION OF UNIVERSAL MATTER FOR SOME VALUES OF A

A	$\frac{\rho_1}{\Lambda}$ in unit $\frac{K}{B}$	$\omega_2 = \frac{1}{2}(A-1)$	$\frac{\rho_2}{\Lambda}$ in unit $\left(\frac{K}{B}\right)^2$	$\omega_3 = A$	*Composition*
$\frac{1}{3}$	$\frac{9}{8a^2}$	$-\frac{1}{3}$	$\frac{9}{8a^4}$	$\frac{1}{3}$	Dark Energy Exotic Matter Radiation
$-\frac{1}{3}$	$\frac{9}{2a}$	$-\frac{2}{3}$	$\frac{9}{4a^2}$	$-\frac{1}{3}$	Dark Energy Exotic Matter Cosmic Strings
1	$\frac{1}{2a^3}$	0	$\frac{1}{4a^6}$	1	Dark Energy Exotic Matter Stiff matter
0	$\frac{2}{8a^{3/2}}$	$-\frac{1}{2}$	$\frac{1}{a^3}$	0	Dark Energy Exotic Matter Dust

The suitability of our model studied:

Brane World Scenario

1) A. Banerjee, T. Bandyopadhyay, S. Chakraborty,
   Grav. Cosmo 13, 290 (2007)
2) A. Banerjee, T. Bandyopadhyay, S. Chakraborty,
   GRG 26, 075017 (2008)

Phantom & Tachyon Field

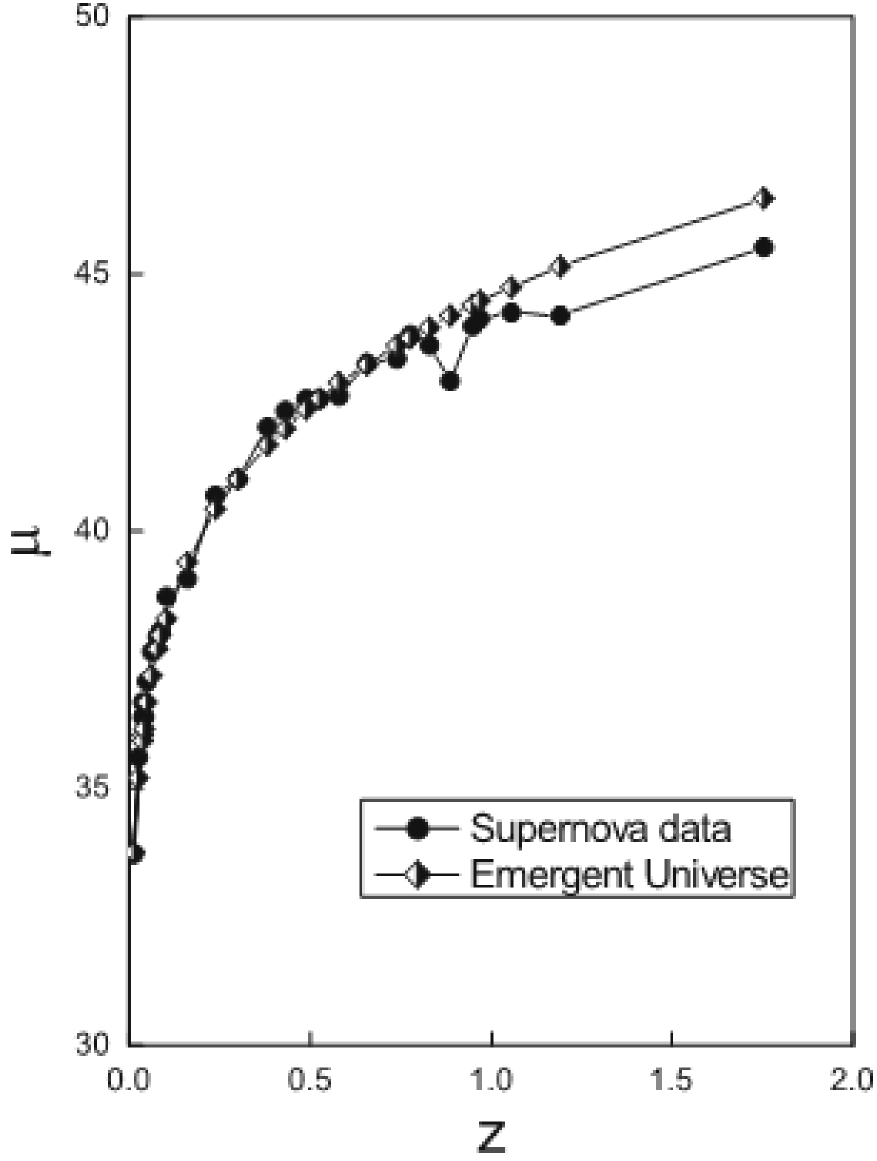
3) U. Debnath, CQG 25, 205019 (2008)

Non-linear Sigma model

4) A. Beesham, S. V. Chervon, S. Maharaj
   CQG 26, 075017 (2009)

EU with GB term :

5) BCP & SG (GRG 42, 795,2010)



50
45
40
35
30
μ
0.0
0.5
1.0
1.5
2.0
z
Supernova data
Emergent Universe

Constraints from Observations
(BCP, P. Thakur, S. Ghose,   MNRAS 2009)

Field Equations

- Hubble Parameter in terms of Redshift **(z)**

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}$$

- Components of matter and dark energy are conserved separately. So we can use energy conservation equation together with EOS to obtain the expression for energy density.

$$\rho_{emu} = \left[\frac{B}{1+A} + \frac{1}{A+1} \frac{K}{a^{\frac{3(A+1)}{2}}} \right]^2$$

where 'K' is a constant of integration and 'K > 0'

- For convenience we re-write the expression for energy density.

$$\rho_{emu} = \rho_{emu_0} \left[A_s + \frac{1-A_s}{a^{\frac{3(A+1)}{2}}} \right]^2$$

$$\rho_{emu_0} = \left(\frac{K+B}{A+1} \right)^2$$

$$A_s = \left(\frac{B}{A+1} \right) \frac{1}{\rho_{emu_0}^{\frac{1}{2}}}$$

- Finally, using Friedmann equation, we can express Hubble Parameter for the model in terms of redshift parameter.

$$H(z) = H_0 \left[\Omega_{b_0} (1+z)^3 + \left(1 - \Omega_{b_0}\right) \left[\frac{B + K(1+z)^{3(A+1)/2}}{B+K} \right]^2 \right]^{1/2}$$

$$where,$$

$$\Omega = \Omega_{b_0} + \Omega_{emu_0} = 1$$

Here Ω_{b_0} represents baryonic energy density and Ω_{emu_0} represents the energy density of the exotic fluid (the suffix '0' signifies present values).

CONSTRAINT FROM H(Z)-Z DATA :

- Hubble parameter, as can be seen, is a function of some variables.

$$H^2(H_0, A, B, K, z) = H_0^{\,2} E^2(A, B, K, z)$$

where,

$$E(A, B, K, z) = \left[\Omega_{b_0}(1+z)^3 + (1-\Omega_{b_0})\left[\frac{B + K(1+z)^{3(A+1)/2}}{B+K}\right]^2\right]^{1/2}$$

- Remember that 'K' enters the theory as an integration constant and we are more concerned about finding suitable range of values for A and B for a physically viable cosmological model. For a fixed value of 'K' these two parameters can be constrained on 'A-B' plane simply minimizing a χ^2_{H-z} function.

$$\chi^2_{H-z}(H_0, A, B, K, z) = \sum \frac{H(H_0, A, B, K, z) - H_{obs}(z)}{\sigma_z^2}$$

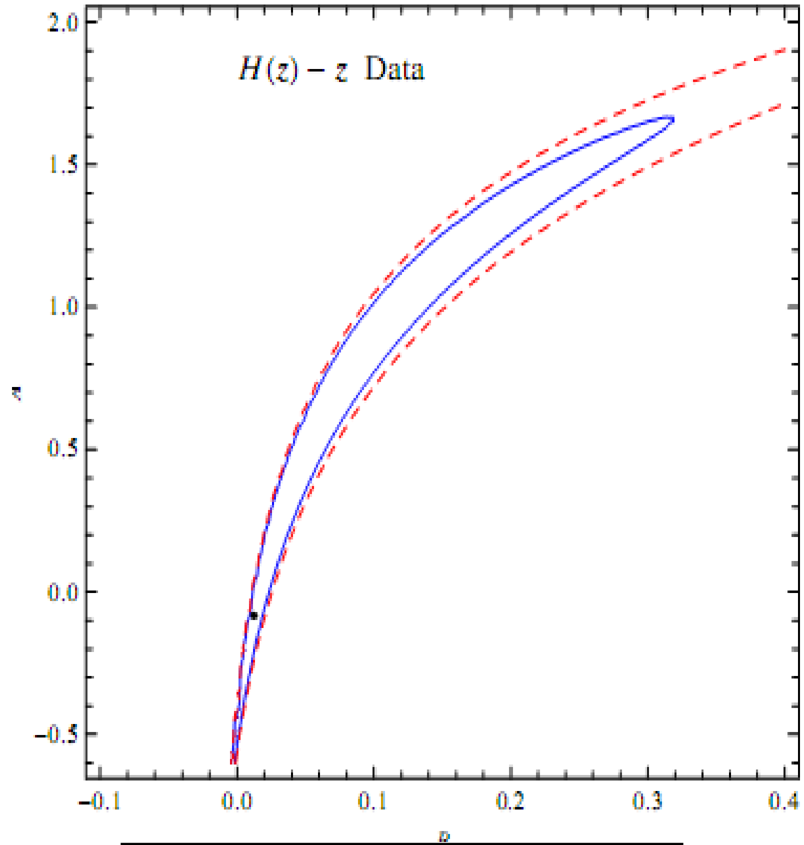
H_o is not important for our analysis so we marginalize over it and obtain probability distribution function in terms of 'A, B and K' only.

$$L(A, B, K) = \int dH_0 P(H_0) \exp\left(\frac{-\chi^2_{H-z}(H_0, A, B, K, z)}{2} \right)$$

Here $P(H_0)$ is the prior distribution function for present value of the Hubble parameter. We consider Gaussian prior here with $H_0 = 72 \pm 8$.

- We use Stern data set here to constrain the parameters.

z Data	$H(z)$	σ
0.00	73	± 8.0
0.10	69	± 12.0
0.17	83	± 8.0
0.27	77	± 14.0
0.40	95	± 17.4
0.48	90	± 60.0
0.88	97	± 40.4
0.90	117	± 23.0
1.30	168	± 17.4
1.43	177	± 18.2
1.53	140	± 14.0
1.75	202	± 40.4



- Blue(solid)-95%
- Red (dotted) -99%

Best fit:
A=0.122
B=-0.0823

Joint Analysis with H(z)-z data and BAO peak parameter :

- The value of the BAO peak parameter (A) is independent of cosmological models and for a flat universe it can be written as :

$$A = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left(\frac{\int_0^{z_1} \frac{dz}{E(z)}}{z_1} \right)^{2/3}$$

where, $z_1 = 0.035$ and $\Omega_m = \Omega_b + (1-\Omega_b)(1 - B/(K+B))^2$

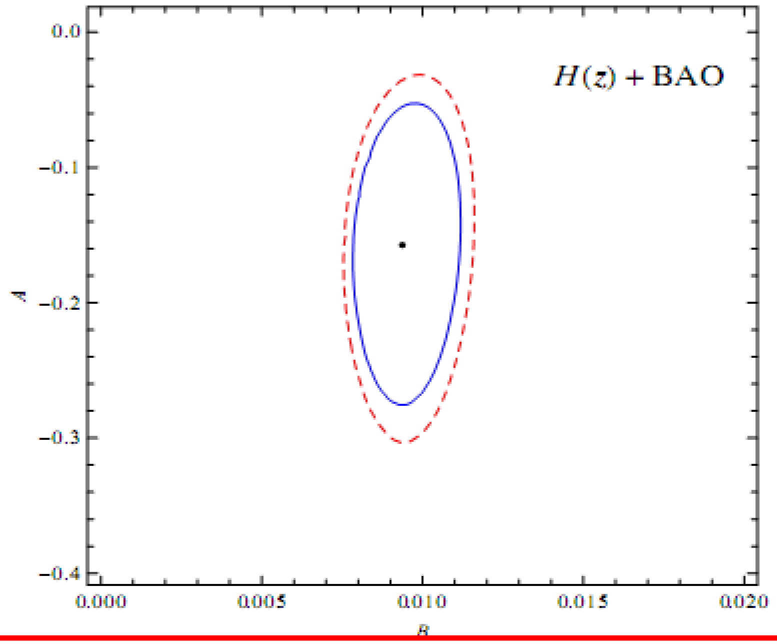
$$A = 0.469 \pm 0.017$$

- We define

$$\chi^2_{BAO} = \frac{(A - 0.469)^2}{(0.017)^2}$$

and for joint analysis we consider

$$\chi^2_{joint} = \chi^2_{H-z} + \chi^2_{BAO}$$



- Blue(solid)-95%
- Red (dotted) -99%

Best fit:
A=0.0094
B=-0.1573

Upto 95 % confidence level : $-0.3053 \leq A \leq -0.0306, 0.0077 \leq B \leq 0.0116$ (*Blue*)

Upto 99% confidence level : $-0.2757 \leq A \leq -0.0500, 0.0078 \leq B \leq 0.0114$ (*red*)

JOINT ANALYSIS WITH H(z)-z, BAO AND CMB SHIFT PARAMETER :

- CMB shift parameter (R) is defined by :

$$R = \sqrt{\Omega_m} \int_0^{z_{ls}} \frac{dz'}{H(z')/H_0}$$

where z_{ls} is the redshift at the surface of last scattering.

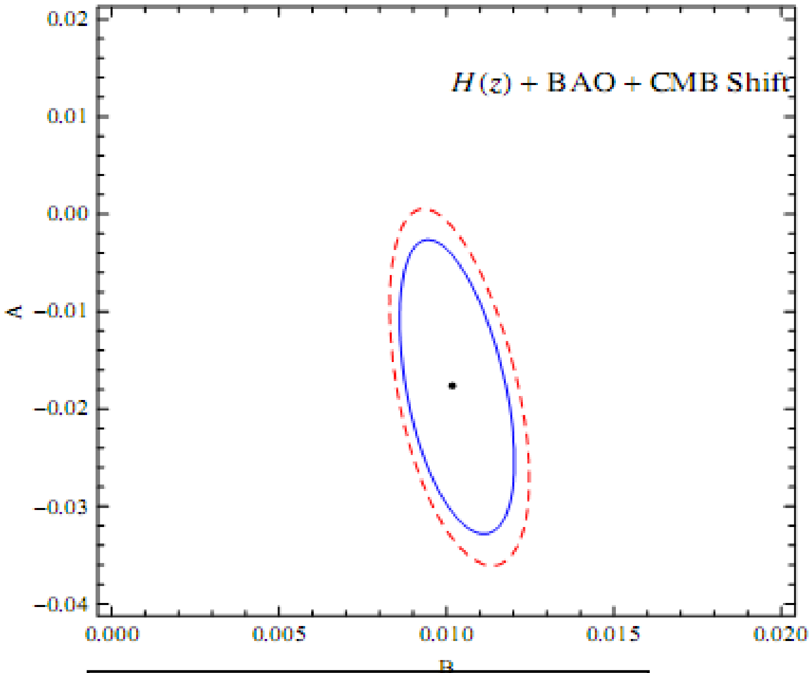
WMAP7 data gives us $R = 1.70 \pm 0.03$

- We consider $$\chi^2_{CMB} = \frac{(R - 1.700)^2}{(0.03)^2}$$

with

$$\chi^2_{Tot} = \chi^2_{H-z} + \chi^2_{BAO} + \chi^2_{CMB}$$

From this analysis we find new constraints :



- Blue(solid)-95%
- Red (dotted) -99%

Best fit:
A=0.0102
B=-0.0176

Unto 99% confidence level:

$$-0.0360 \leq A \leq 0.0005$$

$$0.0083 \leq \mathrm{B} \leq 0.0125$$

Upto 95% confidence level:

$$-0.0328 \leq A \leq -0.0024$$

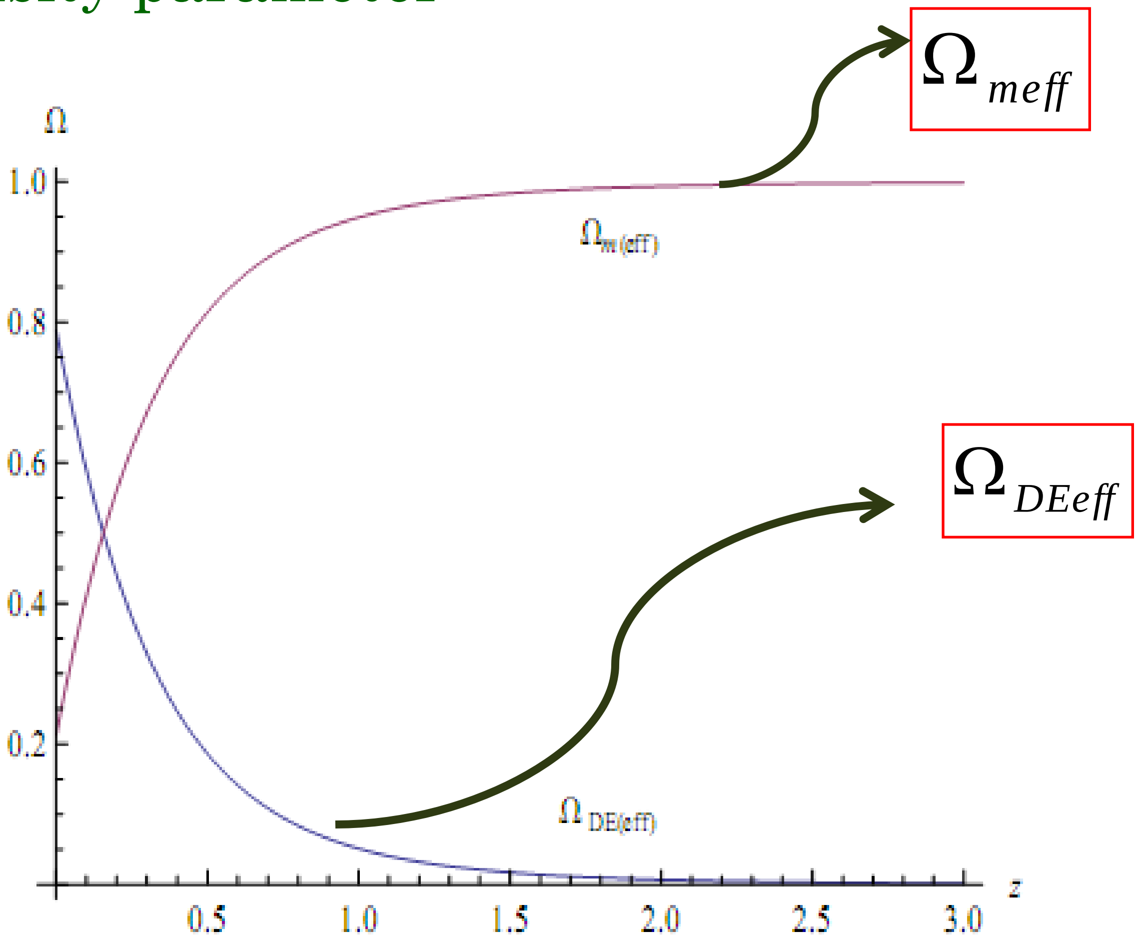
$$0.0086 \leq B \leq 0.0120$$

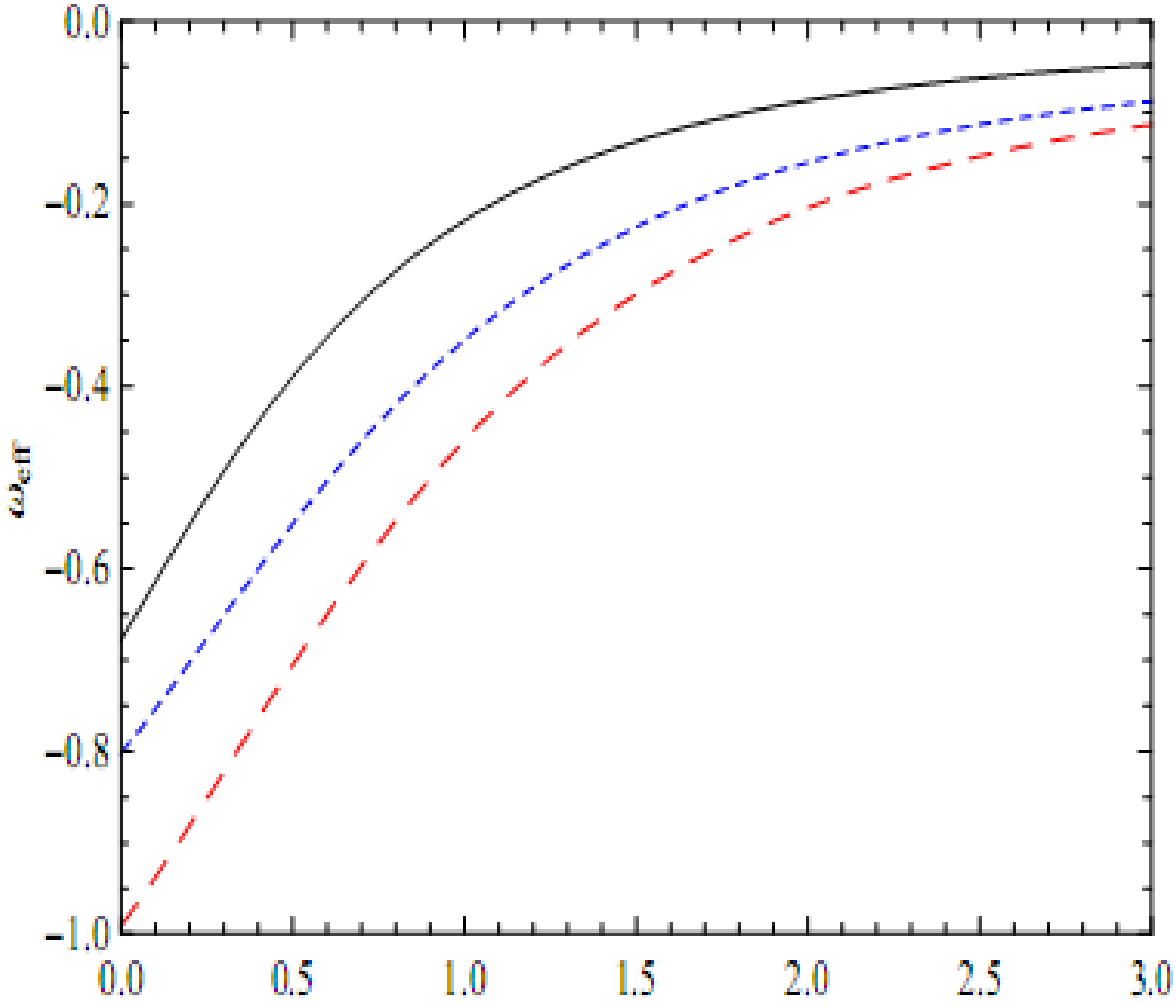
Best Fit value :

$$A = -0.0176, \quad B = 0.0102$$

SOME COSMOLOGICAL PARAMETERS :

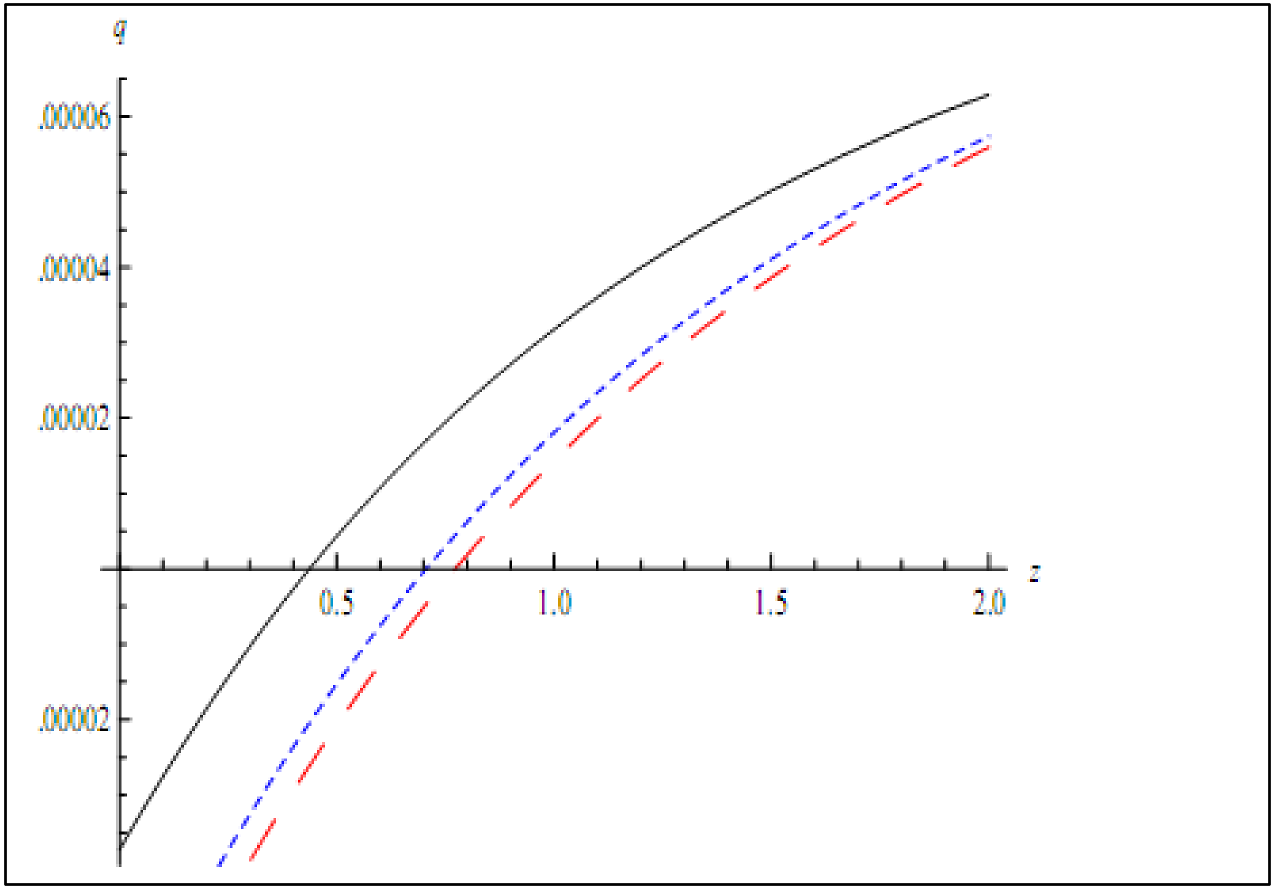
- Density parameter





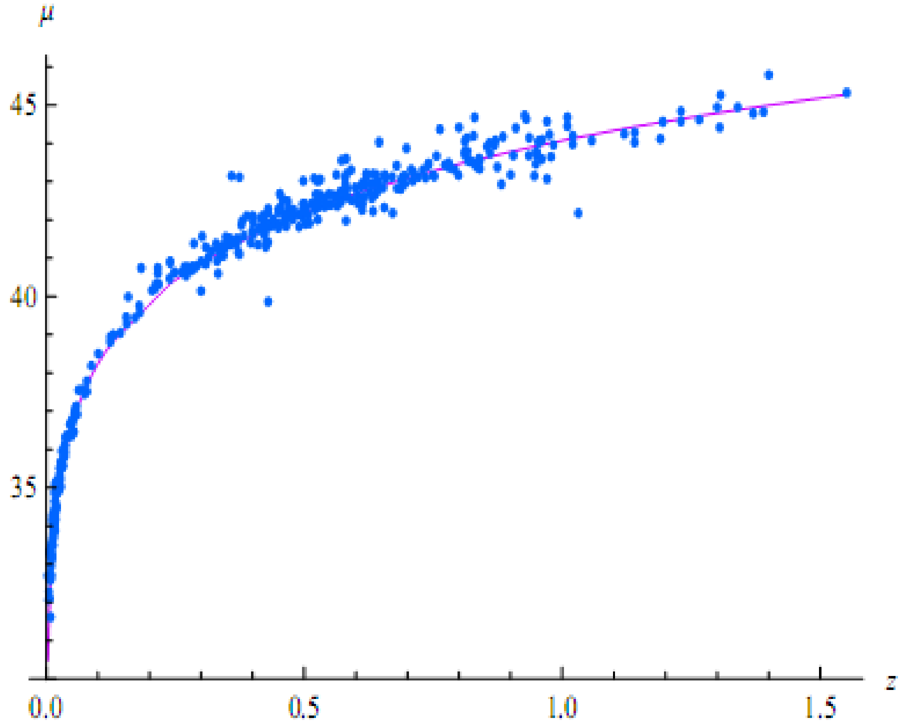
- Black (solid) – Best Fit
- Blue(dotted) – 95 %
- Red (dashed)- 99%

Deceleration parameter :



- Black (solid) – Best Fit
- Blue(dotted) – 95 %
- Red (dashed)- 99%

- Compare with SN Ia data :



Blue Points – Observation
Solid Line - Theoretical

EU with DE, Exotic Matter and Dust :

- Let us get back to the original work by Mukherjee et. al.
- As was seen in Table. Value of the parameter 'A' in the EOS ($p = A\rho - B\rho^{1/2}$) selects a specific composition of matter energy content of the universe.
- For '$A = 0$' the universe contains dark energy, exotic matter and dust.

EU WITH….

- Our aim is to see if we can find a range of values for the model parameters in an observationally viable scenario !
- The Friedmann equation can be written for any 'A' as:

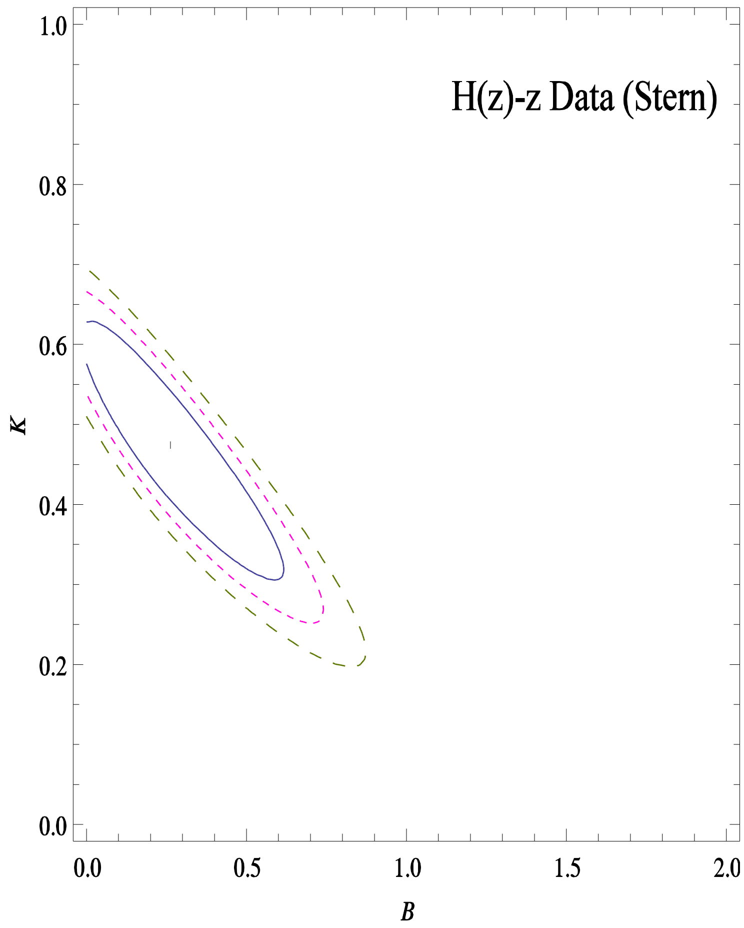
$$H(z)^2 = H_0^2 \left(\Omega_{const} + \Omega_1 (1+z)^{\frac{3}{2(A+1)}} + \Omega_2 (1+z)^{3(A+1)} \right)$$

where $\Omega = \frac{\rho}{\rho_c}$ and $\rho(z) = \rho_{const} + \rho_1 (1+z)^{\frac{3}{2}(A+1)} + \rho_2 (1+z)^{(3(A+1)}$

- The as previously shown we can define for Stern data set

$$\chi^2_{stern}(H_0, A, B, K, z) = \sum \frac{(H(H_0, B, K, z) - H_{obs}(z))^2}{\sigma_z^2}$$

Constraints from Observed Hubble Data (Stern) :



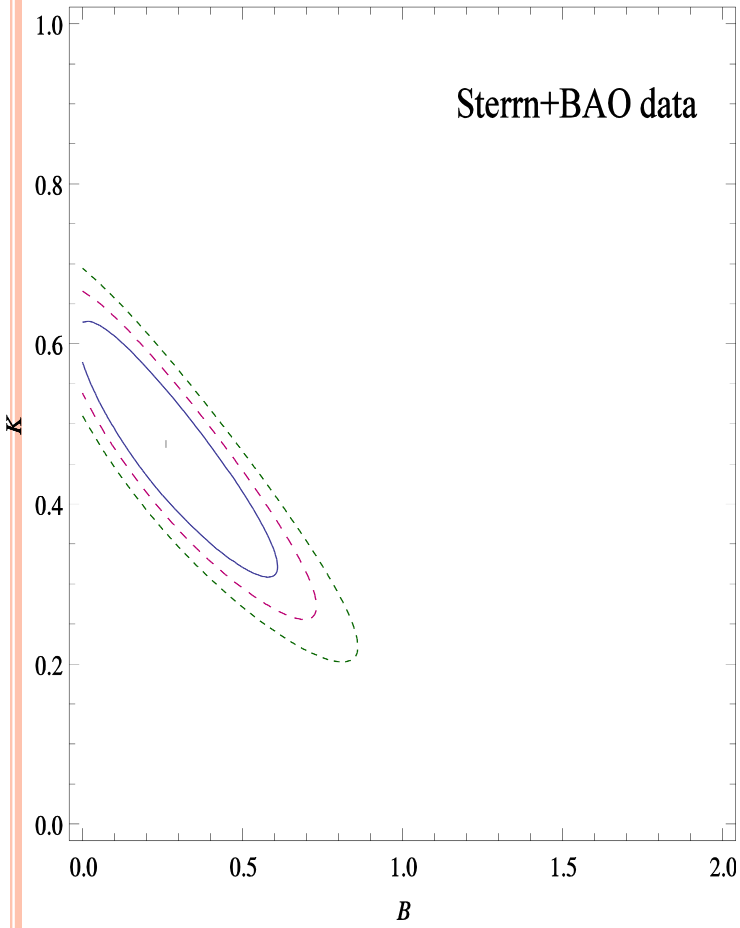
For analysis with Stern data :

$0.003 < B < 0.5996$
$0.303 < K < 0.63$

Within 68.3% confidence

- Blue (Solid) – 68.3%
- Pink (Dotted) – 95 %
- Green (Dashed) – 99%

FROM STERN + BAO DATA :



From joint analysis with Stern and BAO data:

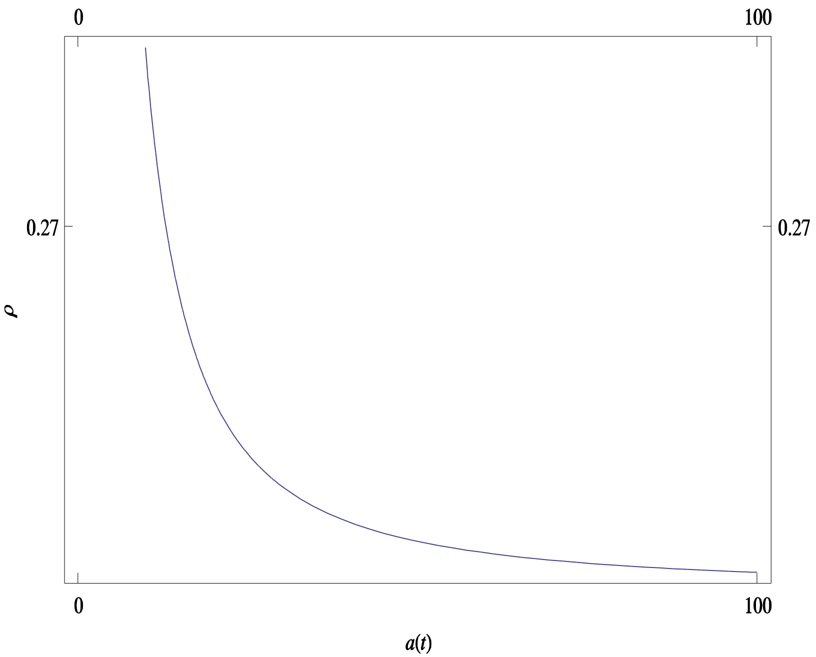
$0.009 < B < 0.606$

$0.3126 < K < 0.6268$

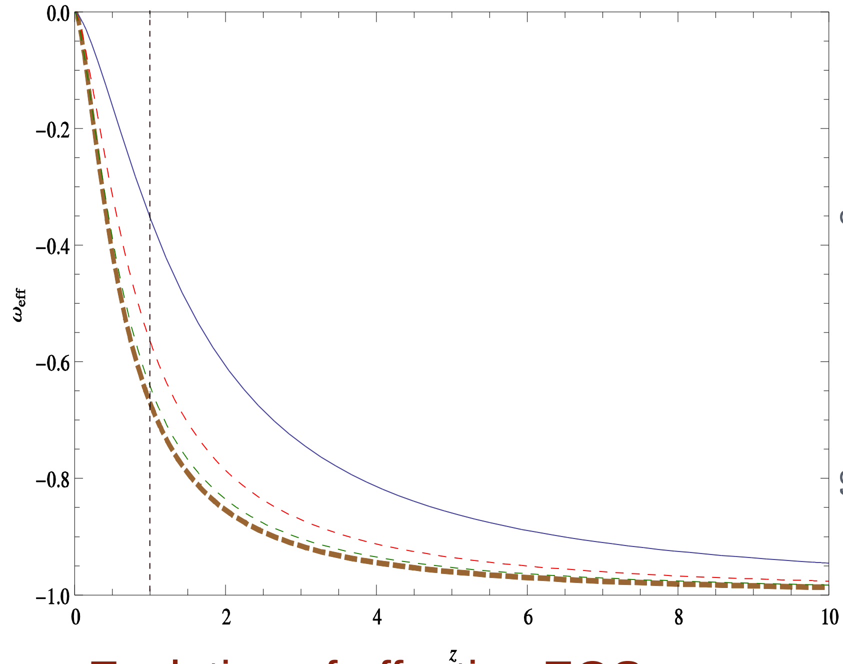
Within 68.3% confidence.

- Blue (Solid) – 68.3%
- Pink (Dotted) – 95 %
- Green (Dashed) – 99%

OTHER COSMOLOGICAL PARAMETERS:



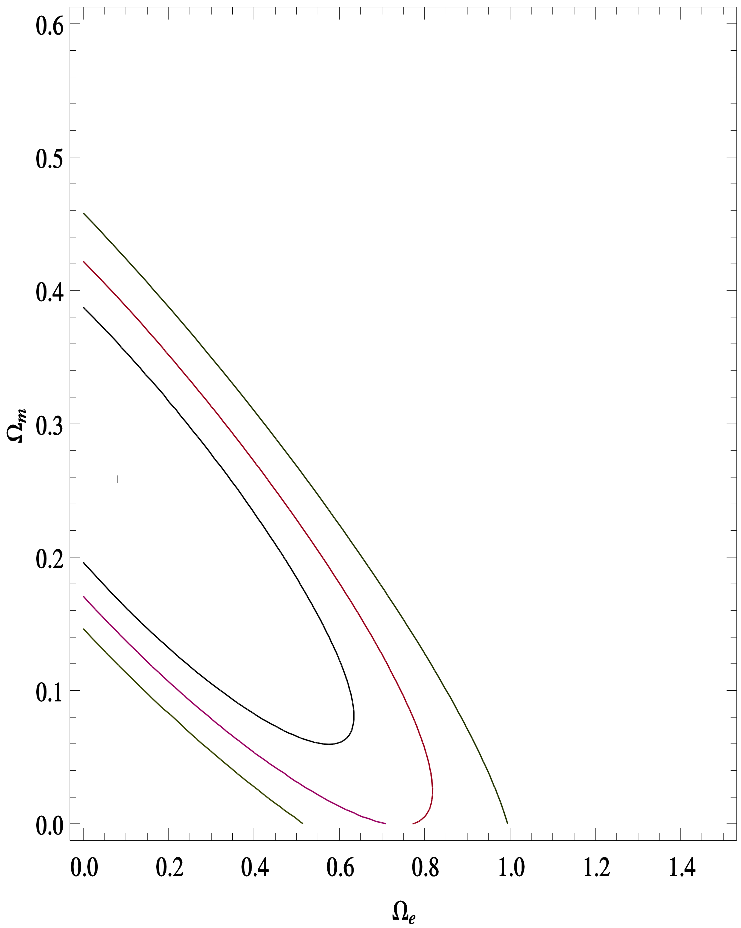
Evolution of density



Evolution of effective EOS parameter :

- Blue- Best fit
- Red-68.3%
- Green-95 %
- Thick-99%

DENSITY PARAMETERS :



Here $\Omega_\Lambda = 1 - \Omega_d - \Omega_e$

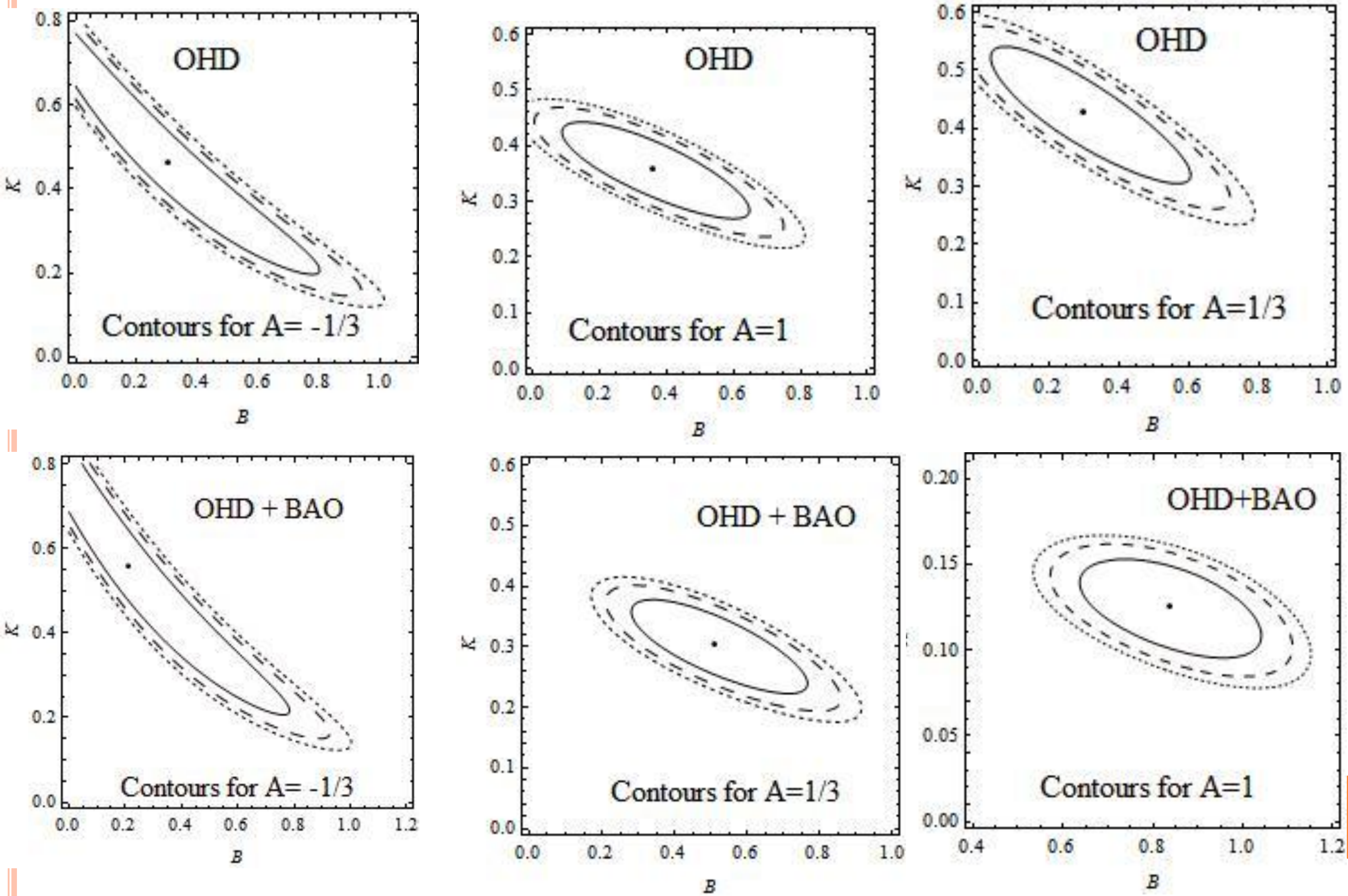
The model can permit $\Omega_\Lambda = 0.72$

Within 68.3% confidence.

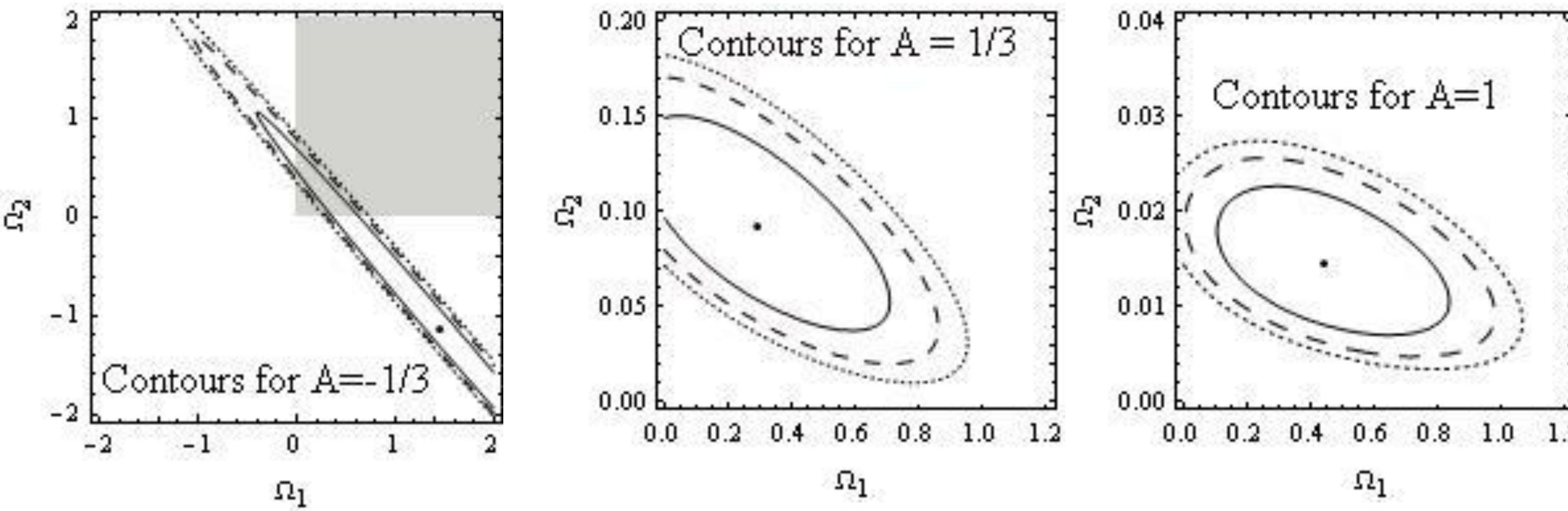
For details :
Paul, Ghose & Thakur, *MNRAS*, **413**, 686 (2011)

- Blue– 68.3%
- Pink– 95.4 %
- Green– 99.9%

Other compositions :



DENSITY PARAMETERS FOR OTHER CASES:



Discussion :

- The emergent universe scenario are not isolated solutions they may occur for different combination of matter and radiation.
- In the Ellis et al model the universe spends a period in the Einstein Static state before an early inflation takes over. In our model we obtained solution of the Einstein equations, which evolves from a static state in the infinite past to be eventually dominated by cosmological constant.
- With GB-term we determine the coupling of the dilaton field which admits EU scenario.

Contd. :

- Matter required to realize an EU is determined.
- In EU inflationary stage as well as late acceleration can be accommodated
- Particle creation may occur at a phase when Hubble parameter varies slowly (early inflationary phase).
- Einstein's static universe permitted in this theory is unstable
- We determine the allowed range of values of the EOS parameter using different observational data. Using H(z) vrs. Z data (Stern et al. Data), BAO (Ries et al). We also took the shift parameter predicted from CMBR data. To determine the EOS parameters.

- We have compared magnitude (μ) vs redshift (z) curves obtained in our model with that obtained from observational data (SNeIa- Wu et al..).

- We found that an accelerating universe can be accommodated in EU which is permitted in the presence of exotic matter (decided by EOS parameter A).

THANK YOU