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# Combining general relativity and quantum theory: points of conflict and contact

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## Abstract

Issues related to bringing together the principles of general relativity and quantum theory are discussed. After briefly summarizing the points of conflict between the two formalisms I focus on four specific themes in which some contact has been established in the past between general relativity and quantum field theory: (i) the role of the Planck length in the microstructure of spacetime, (ii) the role of quantum effects in cosmology and the origin of the universe, (iii) the thermodynamics of spacetimes with horizons, and especially the concept of entropy related to spacetime geometry, (iv) the problem of the cosmological constant.

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## 1. Introduction

The question of bringing together the principles of quantum theory and gravity deserves to be called *the* problem of theoretical physics today. In this review, I shall highlight the points of conflict *and* contact between these two theoretical structures focusing on four major themes which run through all the work in quantum gravity over decades. (i) What is the role played by the length scale  $L_P \equiv (G\hbar/c^3)^{1/2}$  in determining the spacetime microstructure (see, e.g., [1–5])? (ii) What have we learnt regarding the role of quantum gravity in quantum cosmology and in the origin of the universe (see, e.g., [6–12])? (iii) To what extent do we understand the thermodynamics of spacetimes with horizons (see, e.g., [13–18])? (iv) What is the role of quantum gravity *vis-à-vis* the cosmological constant (see, e.g., [19, 20])? Even among these themes, I will concentrate more on the latter two. Before I discuss the concrete issues, it is probably worth comparing some general aspects of quantum theory and general relativity.

## 2. The miracle of quantum field theory

The key feature of quantum field theory is that *it has no right to be as successful as it is!* In proceeding from classical mechanics (with a finite number of degrees of freedom) to quantum mechanics, one attributes operator status to various dynamical variables and imposes the commutation relations among them. Often, it is convenient to provide a representation for the operators in terms of normal differential operators so that the problem can be mapped to solving a partial differential equation—say, the time-dependent Schrödinger equation—with specific boundary conditions. Such problems are mathematically well defined and tractable, allowing us to construct a well-defined (though, in general, not unique) quantum theory for a classical system with a finite number of degrees of freedom.

The generalization of such a procedure to a *field* with an infinite number of degrees of freedom is *not* straightforward. Given a classical field with some dynamical variables, one can attempt to quantize the system by elevating the status of dynamical variables to operators and imposing the commutation rules. But finding a well-defined and meaningful representation for this commutator algebra is a nontrivial task. Further, if one tries to extend the approach of quantum mechanics (based on the Schrödinger picture) to a field, one obtains a *functional* differential equation instead of a partial differential equation. The properties—let alone solutions!—of this equation are not well understood for any field with nontrivial interactions. A somewhat simpler (and better) approach will be to use the Heisenberg picture and try to solve for the operator-valued distributions representing the various dynamical variables. Even in this case, one does not have a systematic mathematical machinery to solve these equations for an interacting field theory. The evolution equations for operators in QED (in (3+1)-dimensions), for example, cannot be solved exactly; however, it is possible to set up a perturbation expansion for the relevant variables in powers of the coupling constant ( $e^2/\hbar c$ )  $\approx 10^{-2}$ . The lowest order of the perturbation series, in which all interactions are switched off, defines the *free-field* theory which can be mapped to a model describing an infinite number of noninteracting harmonic oscillators. The perturbation expansion can then be used to obtain the ‘corrections’ to this free-field theory. Several nontrivial conceptual issues crop up when such an attempt is made.

- (a) To begin with, the decomposition of the field in terms of the harmonic oscillators is not unique and there exists an infinite number of inequivalent representations of the basic commutator algebra for the system. This shows that ‘physical’ quantities such as the ground state, particle number, etc will depend on the specific representation chosen and will not be unique.
- (b) Since the system has an infinite number of degrees of freedom, quantities like the total energy can diverge. The actual form of the divergence depends on the representation chosen for the algebra and the differences between infinite quantities may retain a representation-dependent (finite) value, unless one is careful in regularizing such expressions. In some cases, one may be forced to choose a particular set of harmonic oscillators because of the boundary conditions. Then, the difference between two infinite quantities could be physically relevant (and even observable as in the case of, for example, the Casimir effect).
- (c) The situation becomes worse when the perturbation is switched on. In general, the perturbation series will not converge and has to be interpreted as an asymptotic expansion. Further, the individual terms in the perturbation series will not, in general, be finite, creating a far more serious problem. This is related to the fact that virtual quanta of *arbitrarily* high energy are allowed to exist in the theory (needed for incorporating Lorentz invariance at arbitrarily small length scales) and still propagate as free fields.

- (d) Perturbation theory completely misses all effects which are non-analytic in the coupling constant. In QED, for example, perturbation theory cannot lead to the result that an external electromagnetic field can produce  $e^+e^-$  pairs [21] since this effect has a non-analytic dependency on  $e$  through a factor  $\exp[-(\pi m_e^2/|eE|)]$ .

How does one cope up with these difficulties? Issue (a) is handled by choosing one particular representation for the free-field theory by fiat, and working with it—and ignoring all other representations. This also dodges the issue (b) through normal ordering, once a representation for the harmonic oscillators is chosen. Issue (d) is accepted as a failure of the method and then ignored. Most of the successful effort was concentrated on handling the problem of infinities *in the individual terms* of the perturbation series, that is, on issue (c). The paradigm for handling these infinities can be stated in terms of the concept of renormalization which—though it has nothing to do with divergences, *a priori*—does allow one to cure the divergences, if *all* the divergent terms of a perturbation expansion can be eliminated by redefining the coupling constants in the theory. For an arbitrary field theory, we have no assurance that all the divergences can be so eliminated; in fact, it is quite easy to construct well-defined classical field theories for which divergences cannot be eliminated by this process.

The unexplained miracle of quantum field theory lies in the fact that several physically relevant field theories—describing quantum electrodynamics, electro-weak interactions and QCD—belong to this special class of *perturbatively renormalizable* theories. Nobody knows why this mathematically non-rigorous, conceptually ill-defined, formalism of perturbative quantum field theory works. The miracle becomes even more curious when we note that the bag of tricks fails miserably in the case of gravity.

### 3. Gravity: the thorn in the flesh

Until the seventies, most hardcore particle physicists used to ignore general relativity and gravitation, and the first concrete attempts in putting together principles of quantum theory and gravity were led by general relativists (see, e.g., [6]). It was clear, right from the beginning, that this was going to be a formidable task since the two ‘theories of principle’ differed drastically in many aspects.

- (a) The Lagrangian describing classical gravity, treated as a function of  $h_{ik} = g_{ik} - \eta_{ik}$ , is *not* perturbatively renormalizable; in fact, there does not exist any simple redefinition of the field variables which will lead to a perturbatively renormalizable theory. So the most straightforward approach, based on the belief that nature will continue to be kind to us, is blocked. The miracle fails.
- (b) The principle of equivalence implies that any reasonable description of gravity will have a geometrical structure and that the gravitational field will affect the spacetime intervals in a specific manner, thereby making the spacetime itself dynamical. For a general gravitational field, there will be no way of choosing a special class of spacelike hypersurfaces or a time coordinate.
- (c) Gravity affects light signals and hence determines the causal structure of spacetime. In particular, gravity is capable of generating regions of spacetime from which no information can reach the outside world through the classical propagation of signals. This feature, which may be loosely called ‘the existence of trapped surfaces’, has no parallel in any other interaction. When gravity makes certain regions inaccessible, the data regarding quantum fields in these regions can ‘get lost’. This requires reformulation of the equations of quantum field theory, possibly by tracing over the information which resides in

the inaccessible regions—something which is not easy to do either mathematically or conceptually.

- (d) Since all matter gravitates, the gravitational field becomes more and more dominant at larger and larger scales. In the limit, the asymptotic structure of spacetime is determined by the global, smoothed out distribution of matter in the cosmological context. Hence, the spacetime will not be asymptotically flat in the spatial variables at any given time. The behaviour of the spacetime for  $t \rightarrow \pm\infty$  will also be highly nontrivial and could be dominated by very strong gravitational fields.
- (e) All energies gravitate, thereby removing the ambiguity in the zero level for the energy, which exists in non-gravitational interactions. This feature also suggests that there is no such thing as a free, non-interacting field. Any nontrivial classical field configuration will possess a certain amount of energy which will curve the spacetime, thereby coupling the field to itself indirectly. The gravitational field is not only nonlinear in its own coupling, but also makes *all matter fields* self-interacting.

These features create problems even when one tries to develop a quantum field theory in an external gravitational field. Conventional quantum field theory works best when a static causal structure, global Lorentz frame, asymptotic in–out states, bounded Hamiltonians and the language of vacuum state, particle excitations, etc, are supplied. The gravitational field removes all these features, strongly hinting that we may be working with an inadequate language. Any perturbative language which, at best, gives an algorithm to calculate S-matrix elements, is not going to be of much use in understanding the quantum structure of the gravitational field. Most of the interesting questions—possibly *all* the interesting questions—in quantum gravity are non-perturbative in character; whether a theory is perturbatively renormalizable or not is totally irrelevant in this context. The gradual paradigm shift in the particle physics community from perturbative renormalizability (in 1970s) through perturbative finiteness of supergravity (in early 1980s) to non-perturbative description of superstrings (in late 1990s) represents a grudging acceptance of the lessons from gravity.

Finally, one may ask—given these difficulties—is it really necessary to quantize gravity? The answer is ‘yes’ and can be proved in two steps: (i) one can easily prove that if the Casimir energy does not gravitate, it is possible to construct a perpetual motion machine using two Casimir plates and a set of weights and pulleys (see, e.g., [22]); (ii) if the source for gravity is quantum mechanical (like the Casimir energy), but the field is classical then it is possible to violate the uncertainty principle by a suitable set-up. Thus at least some minimal amount of quantum structure needs to be imposed on gravity.

#### 4. Role of the Planck length in the microstructure of spacetime

Having summarized the points of conflict, let me now turn to the points of contact, beginning with the first of the four themes I mentioned in section 1. The fact that all matter gravitates stresses the need to abandon a description based on free-field theory in order to handle virtual excitations with arbitrarily high energies. An excitation with energy  $E$  will probe length scales of the order of  $(1/E)$  and when  $E \rightarrow E_p$ , the nonlinearity due to self-gravity cannot be ignored for any field. The same conclusion is applicable even to vacuum fluctuations of any field, including gravity. If we attempt to treat the ground state of the gravitational field as a flat spacetime, we must conclude that the spacetime structure at  $L < L_p$  will be dominated by quantum fluctuations of gravity and the smooth macroscopic spacetime can only emerge when the fluctuations are averaged over larger length scales. Hence, the description of a continuum spacetime in terms of (classical) Einstein equations should be thought of as similar to the

description of a solid by elastic constants. While a knowledge of the microscopic quantum theory of atoms and molecules will allow us, in principle, to construct the description in terms of elastic constants, the reverse process is unlikely to be unique. What one could hope for is to take clues from well-designed thought experiments, thereby identifying some key generic features of the microscopic theory.

To begin with, one can prove—using well-chosen thought experiments—that it is not possible to measure intervals smaller than  $L_P = (G\hbar/c^3)^{1/2}$ . (This is demonstrated in [23]; a clear statement in this direction, based on a toy model, is in [2] and a more ‘modern’ approach to the same result is in [24].) More formally, one can prove that quantum fluctuations in the metric lead to the following limit (see [23, 25]):

$$\lim_{x \rightarrow y} \langle l^2(x, y) \rangle \approx (x - y)^2 + L_P^2 \quad (1)$$

where  $l$  is the geodesic distance between  $x^i$  and  $y^i$  and the averaging is over all metric fluctuations. This suggests that Planck length should be thought of as the ‘zero-point length’ of the spacetime, and any correct theory of quantum gravity must incorporate this feature in a suitable form.

One specific consequence of this result is in the case of a Friedmann model for the universe. It can be shown that [26] the lower bound at Planck length leads to an effective metric of the form

$$ds^2 = dt^2 - L_P^2 \left(n + \frac{1}{2}\right) [d\chi^2 + \sin^2 \chi (d\Omega^2)] \quad (2)$$

leading to the *areas* of spherical surfaces being quantized in units of  $L_P^2$ .

It is *not* possible to reconcile the existence of a zero-point length in (1) with a Lorentz invariant, local, QFT description. Both string theory and loop gravity (the two approaches which have been worked out to a fair degree of detail) incorporate this lower bound in different ways. In string theory, nonlocality is built in and hence it is possible to obtain Lorentz invariance at the long-wavelength limit. The situation is less clear in loop gravity but it has the most direct implementation of this principle in, for example, area quantization. In loop gravity the area operator is quantized but the eigenvalues scale differently [3] compared with (2). More recently, there have been attempts to construct quantum cosmological models based on loop gravity (see, e.g., [12]) and it appears that results such as (2) might arise as an asymptotic limit.

I have attempted [5] to use the interpretation of  $L_P$  as a zero-point length to provide a working description of quantum field theory that is free of ultraviolet divergences. The starting point of this analysis is to modify the path integral for the Euclidean Green function  $G_F(x, y)$  in such a way that the path integral amplitude is invariant under the ‘duality’ transformation  $l \rightarrow (L_P^2/l)$  where  $l^2 = (x - y)^2$ . This demands the replacement

$$G(x, y) = \sum e^{-l(x,y)} \rightarrow G_{\text{modified}} \equiv \sum \exp \left[ - \left( l + \frac{L_P^2}{l} \right) \right]. \quad (3)$$

Remarkably enough, it turns out that the path integral sum in (3) can be evaluated rigorously. The final result is quite simple: the modified Green function is related to the original one by the replacement  $(x - y)^2 \rightarrow [(x - y)^2 + L_P^2]$ ! That is,  $G_{\text{modified}}(x, y) = G_{\text{usual}}[l^2 \rightarrow (l^2 + L_P^2)]$ . In other words, the postulate of duality (as defined above) implies the existence of a zero-point length. It is known that string theories—which have a zero-point length built in—do lead to dualities of different kinds. I would like to stress that there is no simple reason to expect, *a priori*, a connection between (3) and the zero-point length.

Once the postulate of path integral duality is accepted, it is possible to obtain several interesting consequences. (i) To begin with, it is clear that gravity acts as a *non-perturbative*

regulator. For example, the modified Feynmann propagator for a massless scalar field has the structure (see [2, 5])

$$\frac{1}{x^2} \rightarrow \frac{1}{(x^2 + L_p^2)} = \frac{1}{x^2} - \frac{L_p^2}{x^4} + \dots \quad (4)$$

Each term on the right-hand side diverges as  $x \rightarrow 0$  and only the sum remains finite in the coincidence limit, as  $x \rightarrow 0$ . (ii) One can compute the corrections to the bare cosmological constant  $\Lambda_{\text{bare}}$  and the Newtonian gravitational constant  $G_{\text{bare}}$  when other fields are integrated out in a path integral. One finds that

$$\Lambda_{\text{ren}} = \Lambda_{\text{bare}} - \frac{1}{4\pi\eta^4 G} \quad G_{\text{ren}}^{-1} = G_{\text{bare}}^{-1} \left[ 1 + \frac{1}{12\pi\eta^2} \right] \quad (5)$$

where  $\eta$  is a pure number related to the number of scalar fields integrated out. The result shows that the value  $\Lambda = 0$  is *unprotected* against large quantum gravitational corrections. (iii) Any form of area quantization implies that the density of BH states on the horizon is of the order of  $(A/L_p^2)$  with clear implications for the entropy of black holes. (iv) The zero-point length also suggests that there will be exponential suppression of modes shorter than the Planck length. This, for example, will allow inflation at quantum gravitational scales [27] since the production of gravitational waves will be suppressed.

## 5. Why did the universe become classical?

Let me now turn to the second theme, namely quantum cosmology. Considering the fact that quantum cosmology was one of the earliest points of contact between QT and GR, it is rather disappointing that it has not produced any concrete results. Fundamental questions regarding the origin of the universe remain unanswered in all approaches, and even the descriptive language requires semiclassical crutches. Serious technical questions (e.g., the exact validity of minisuperspace, canonical versus path integral approaches, Euclidean versus Lorentzian path integral, topology change, etc) are still controversial.

Most of the early work in quantum cosmology was on the question of singularities (and the ‘creation’ of the universe) and these models (see, e.g., [7, 8, 26, 28]) were the precursors of the currently more fashionable (though hardly better justified) pre-big bang models. It is fairly easy to construct toy quantum cosmological models without a singularity or horizon. One example I worked out a long way back [28] has the effective metric

$$ds^2 \cong (\alpha L_p^2 + \tau^2) [d\tau^2 - dx^2 - dy^2 - dz^2] \quad \alpha = \mathcal{O}(1). \quad (6)$$

The difficulty with such pre-big bang models is that they are hopelessly diverse and do not give any more insight than the original assumptions of the model.

Somewhat more concrete results exist as regards the semiclassical limit of quantum cosmology, especially since we cannot invoke an ‘observer’ in this context. It is possible to show that decoherence provides an answer and leads to the density matrix of 3-geometries becoming effectively diagonal. One can define a ‘distance’ in the superspace of 3-geometries  $l^2(g_{\alpha\beta}, g'_{\alpha\beta})$  in terms of which one can illustrate [9] the suppression of off-diagonal components of the density matrix explicitly:

$$\rho_{\text{off-diagonal}} \approx \rho_{\text{diagonal}} \exp \left[ -\frac{l^2(g_{\alpha\beta}, g'_{\alpha\beta})}{L_p^2} \right]. \quad (7)$$

There is another interesting aspect [30] to this analysis: among all systems dominated by gravity, the universe possess a very peculiar feature. If the conventional cosmological

models are reasonable, it then follows that *our universe proceeded from quantum mechanical behaviour to classical behaviour in the course of dynamical evolution defined by some intrinsic time variable*. In terms of the Wigner functional  $W(g, p)$ , this transition can be stated as evolution leading to,

$$W(g_{\alpha\beta}, p^{\alpha\beta}) = A(g)B(p) \rightarrow W(g_{\alpha\beta}, p^{\alpha\beta}) = F[p - p_{\text{class}}(g)] \quad (8)$$

where  $A, B$  are arbitrary functionals and  $F$  is a functional sharply peaked on its argument. This transition is *not* possible for systems with bounded Hamiltonians arising in a low-energy effective theory with a finite number of fields integrated out. It follows that the quantum cosmological description of our universe, as a Hamiltonian system, should contain at least one unbounded degree of freedom. In simple quantum cosmological models, one can write  $\mathcal{H} = \mathcal{H}_{\text{unbound}}(a) + \mathcal{H}_{\text{bound}}(a, q)$ , where  $a$  is the expansion factor and  $q$  denotes all other degrees of freedom. It can also be shown that the unbounded mode—which, in the case of a FRW universe, corresponds to the expansion factor  $a(t)$ —will become classical first, as is experienced in the evolution of the universe.

One might assume that the microscopic description of spacetime is in terms of certain (as yet unknown) variables  $q_i$  and that the conventional spacetime metric is obtained from these variables in some suitable limit. Such a process will necessarily involve coarse-graining over a class of microscopic descriptors of geometry. If one starts with a bounded Hamiltonian for a system with a *finite* number of quantum fields and integrates out a subset of them, the resulting Hamiltonian for the low-energy theory cannot be unbounded. Assuming that the original theory is describable in terms of a bounded Hamiltonian for some suitable variables, it follows that an infinite number of fields have to be involved in its description and an infinite subset of them have to be integrated out in order to give the standard low-energy gravity. This feature is indeed present in one form or the other in the descriptions of quantum gravity based on strings or loop variables.

These arguments can be extended further. Starting from an (unknown) quantum gravitational model, one can invoke a sequence of approximations to progressively arrive at quantum field theory (QFT) in curved spacetime, QFT in flat spacetime, nonrelativistic quantum mechanics and Newtonian mechanics. The more exact theory can put restrictions on the range of possibilities allowed for the approximate theory which are *not derivable from the latter*—an example being the symmetry restrictions on the wavefunction for a pair of electrons at point QM which has its origin in QFT. The choice of vacuum state at low energies could be such a ‘relic’ arising from combining the principles of quantum theory and general relativity [29]. The detailed analysis suggests that the wavefunction of the universe, when describing the large volume limit of the universe, dynamically selects a vacuum state for matter fields—which, in turn, defines the concept of particle in the low-energy limit. The result also has the potential for providing a concrete quantum mechanical version of Mach’s principle.

## 6. Thermodynamics and/of geometry: can the cosmological constant evaporate?

One of the remarkable features of classical gravity is that it can wrap up regions of spacetime thereby producing surfaces which act as one-way membranes. The classic example is that of a Schwarzschild black hole which has a compact surface acting as an observer-independent event horizon. Another example is the de Sitter universe which also has a one-way membrane; but the location of the horizon depends on the observer and hence is coordinate dependent. In fact, the existence of one-way membranes is not necessarily a feature of curved spacetime; it is possible to introduce coordinate charts even in the flat Minkowski spacetime, such that regions

are separated by horizons. The familiar example is the Rindler coordinate frame which has a noncompact surface acting as a coordinate-dependent horizon.

All the three spacetimes mentioned above (Schwarzschild, de Sitter, Rindler) as well as a host of other spacetimes with horizons can be described in a general manner as follows. Consider a  $(D + 1)$ -dimensional *flat* Lorentzian manifold with the line element

$$ds^2 = (dZ^0)^2 - (dZ^1)^2 \dots (dZ^D)^2 \equiv \eta_{AB} dZ^A dZ^B. \quad (9)$$

Spacetimes of relevance to us can all be thought of as four-dimensional sub-manifolds of this  $(D + 1)$ -dimensional manifold, defined in a suitable manner. I will first introduce two new coordinates  $(t, r)$  in place of  $(Z^0, Z^1)$  through the definitions

$$Z^0 = \pm lf(r)^{1/2} \sinh gt \quad Z^1 = \pm lf(r)^{1/2} \cosh gt \quad (10)$$

where  $(l, g)$  are constants introduced for dimensional reasons and we will usually take  $l \propto (1/g)$ . Clearly, the pair of points  $(Z^0, Z^1)$  and  $(-Z^0, -Z^1)$  are mapped to the same  $(t, r)$  making this a 2-to-1 mapping. (The transformations in (2) cover only the two quadrants with  $|Z^1| > |Z^0|$  with the positive sign for the right quadrant and negative sign for the left but can easily be extended to other quadrants with  $\sinh$  and  $\cosh$  interchanged.) Note that if one introduces the Euclidean continuation of the time coordinates with  $iZ^0 \equiv T$ ;  $it = \tau$ , the transformations in (2) continue to be valid but with a periodicity of  $(2\pi/g)$  in  $\tau$ . With the transformations in (2), the metric in (9) becomes

$$ds^2 = f(r)(lg)^2 dt^2 - \frac{l^2}{4} \left( \frac{f'^2}{f} \right) dr^2 - dL_{D-1}^2 \quad (11)$$

in all four quadrants. The choice of  $D$  and the definition of the four-dimensional subspace depend on the spacetime we are interested in: (a) in the simplest case of Rindler spacetime we can take  $f = (1 + 2gr)$ ,  $l = g^{-1}$  with all the  $(D - 2)$  transverse dimensions going for a ride. In fact, we can treat this case as just a redefinition of coordinates, involving a mapping from  $(D + 1) = 4$  to  $(D + 1) = 4$ . (b) If we take  $(D + 1) = 5$  and use—in addition to the mapping given by (2)—a transformation of Cartesian  $(Z^2, Z^3, Z^4)$  to the standard spherical polar coordinates:  $(Z^2, Z^3, Z^4) \rightarrow (r, \theta, \varphi)$ , and choose  $lg = 1$ ;  $f(r) = [1 - (r^2/l^2)]$ , we get the de Sitter spacetime in static coordinates. (c) To obtain the Schwarzschild spacetime<sup>1</sup> we start with a  $(D + 1) = 6$ -dimensional spacetime  $(Z^0, Z^1, Z^2, Z^3, Z^4, Z^5)$  and consider a mapping to four-dimensional subspace in which: (i) the  $(Z^0, Z^1)$  are mapped to  $(t, r)$  as before; (ii) the  $(Z^3, Z^4, Z^5)$  are mapped to standard spherical polar coordinates:  $(r, \theta, \varphi)$  and (iii) we take  $Z^2$  to be an arbitrary function of  $r$ :  $Z^2 = q(r)$ . This leads to the metric

$$ds^2 = A(r) dt^2 - B(r) dr^2 - r^2 d\Omega_{2\text{-sphere}}^2 \quad (12)$$

with

$$A(r) = (lg)^2 f \quad B(r) = 1 + q'^2 + \frac{l^2}{4} \frac{f'^2}{f}. \quad (13)$$

This choice will allow us to obtain *any* spherically symmetric, static, four-dimensional spacetime. For the Schwarzschild solution we will take  $2lg = 1$ ,  $f = 4[1 - (l/r)]$ ; and

$$q(r) = \int^r \left[ \left( \frac{l}{r} \right)^3 + \left( \frac{l}{r} \right)^2 + \frac{l}{r} \right]^{1/2} dr. \quad (14)$$

Though the integral cannot be expressed in terms of elementary functions, it is obvious that  $q(r)$  is well behaved everywhere including at  $r = l$ .

<sup>1</sup> This was first obtained in [33] but the analysis in this reference hides the simplicity and generality of the result.

In the examples of spacetimes with horizons,  $f(r)$  vanishes at some  $r = l$  so that  $g_{00} \approx |(r/l - 1)|$  near  $r = l$ ; such spacetimes have a horizon at  $r = l$ . There exists a natural definition of QFT in the original  $(D + 1)$ -dimensional space and we can define a vacuum state for the quantum field on the  $Z^0 = 0$  surface, which coincides with the  $t = 0$  surface. It is straightforward to show that this vacuum state appears as a thermal state with temperature  $T = (g/2\pi)$  in the four-dimensional subspace. The most important conclusion which follows from this analysis [31] is that the existence of the temperature is a purely kinematic effect arising from the coordinate system we have used—which should also be obvious from the fact that (10) implies periodicity in the imaginary time coordinate.

The QFT based on such a state will be manifestly time symmetric and will describe an isolated system in thermal equilibrium in the subregion  $\mathcal{R}$ . No time asymmetric phenomena such as evaporation, outgoing radiation, irreversible changes, etc, can take place in this situation. This is gratifying since one may be hard pressed to interpret an evaporating Minkowski spacetime or even an observer-dependent evaporation of a de Sitter spacetime; by choosing to work with the quantum state which is time symmetric we can bypass such conceptual issues.

It is also possible to show that 4-dim QFT gets mapped to 2-dim conformal field theory (CFT) in all these spacetimes. Consider, for example, a QFT for a self-interacting scalar field in a spacetime with the metric of the form in equation (11):

$$ds^2 = f(r)(lg)^2 dt^2 - \frac{l^2}{4} \left( \frac{f'^2}{f} \right) dr^2 - g_{\alpha\beta} dx^\alpha dx^\beta \quad g_{\alpha\beta} = g_{\alpha\beta}(r, \mathbf{x}_\perp) \quad (15)$$

where the line element  $g_{\alpha\beta} dx^\alpha dx^\beta$  denotes the irrelevant transverse part corresponding to the transverse coordinates  $\mathbf{x}_\perp$ , as well as any regular part of the metric corresponding to  $dr^2$ . The field equation for a scalar field in this metric can be reduced to the form

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial \xi^2} = \left( \frac{2g}{f'} \right)^2 \frac{f^2}{Q} \left( \frac{\partial \phi}{\partial r} \right) \left( \frac{\partial Q}{\partial r} \right) + (lg)^2 f \left[ (\nabla_\perp^2 \phi) + \frac{\partial V}{\partial \phi} \right] \quad (16)$$

where  $2g\xi = \ln f$ . The right-hand side vanishes as  $r \rightarrow l$  because  $f$  vanishes faster than all other terms. It follows that near the horizon we are dealing with a  $(1 + 1)$ -dimensional CFT governed by

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial \xi^2} \approx 0 \quad (17)$$

which has an extra symmetry of conformal invariance.

If we take  $f(r) \propto (r - l)$  near  $r = l$  and separate out the time dependence by  $\phi = \phi_\omega e^{-i\omega t}$ , it is easy to see that, near  $r = l$ , the fundamental wave modes are  $\phi = e^{-i\omega t \pm i\omega \xi} = e^{-i\omega(t \pm \xi)} = (e^{-i\omega z} e^{-i\omega \bar{z}})$  where  $\tau = it$  and  $z \equiv (\xi + i\tau)$  is the standard complex coordinate of conformal field theory. The boundary condition on the horizon can be expressed most naturally in terms of  $z$  and  $\bar{z}$ . For example, purely ingoing modes are characterized by  $(\partial f / \partial \bar{z}) = 0$ ;  $(\partial f / \partial z) \neq 0$ . Since the system is periodic in  $\tau$ , the coordinate  $z$  is on a cylinder ( $R^1 \times S^1$ ) with  $\tau$  being the angular coordinate ( $S^1$ ) and  $\xi$  being the  $R^1$  coordinate. The periodicity in  $\tau$  is clearer if we introduce the related complex variable  $\rho$  by the definition  $\rho = \exp g(\xi + i\tau) = \exp(gz)$ . The coordinate  $\rho$  respects the periodicity in  $\tau$  and is essentially a mapping from a cylinder to a plane. It follows that the modes  $(e^{-i\omega z}, e^{-i\omega \bar{z}})$  become  $(\rho^{-i\omega/g}, \bar{\rho}^{-i\omega/g})$  in terms of  $\rho$ .

The situation is simpler in the case of a free-field with  $V = 0$ . Then one can show that, near the horizon,  $r \simeq l$ ,  $r' \simeq l$  and the two-point function will have the limiting form

$$G(t - t'; r, r'; \mathbf{x}_\perp, \mathbf{x}'_\perp) \simeq \left\{ \sum_\lambda f_\lambda(\mathbf{x}_\perp) f_\lambda(\mathbf{x}'_\perp) \right\} \left\{ \sum_\omega e^{-i\omega[(t-t') \pm (\xi - \xi')] } \right\} \quad (18)$$

where the function  $f$  is the eigenfunction of the transverse Laplacian with (a set of) eigenvalue(s)  $\lambda$ ; that is  $\nabla_{D-1}^2 f = -\lambda^2 f$ . In other words, the two-point function factorizes into a transverse and radial part with the radial part being that of a two-dimensional massless scalar field. The latter is the same as the Green function of the standard conformal field theory.

The role of a horizon in producing a CFT can be summarized as follows: in a general  $(D + 1)$ -dimensional theory with  $D \geq 2$ , we do not have conformal invariance. If we can kill the transverse dimensions and reduce the theory to a two-dimensional theory, then we would have automatically enhanced the symmetries to that of a CFT. The metrics in (11) with  $g_{00}$  having a simple zero at  $r = l$  achieves exactly this. Since such metrics have a horizon, we obtain a connection between CFT and horizons quite generically. All the results of CFT (especially the behaviour of two-point functions) can now be used to study the field theory near the horizon.

Trapped surfaces also highlight the role of boundary conditions (called the ‘holographic principle’ in some contexts) in QFT. The structure of a free-field propagating in an arbitrary spacetime can be completely specified in terms of, say, the Feynmann Greens function  $G_F(x, y)$  which satisfies a local, hyperbolic, inhomogeneous partial differential equation. Each solution to this equation provides a particular realization of the theory so that there exists a mapping between the realizations of the quantum field theory and the relevant boundary conditions for this equation which specify a useful solution. When trapped surfaces exist, the differential operator governing the Greens function will be singular on these surfaces (in some coordinate chart) and the issue of boundary conditions becomes far more complex. It is nevertheless possible—at least in simple cases with compact trapped surfaces—to provide a one-to-one correspondence between the ground states of the theory and the boundary conditions for  $G_F$  on the compact trapped surface. In fact, the Greens function connecting events outside the trapped surface can be completely determined in terms of a suitable boundary condition on the trapped surface, indicating that the trapped surfaces acquire a life of their own even in the context of QFT in CST. In a way, the procedure is reminiscent of the renormalization group approach, but now used in real space to integrate out information inside the trapped surface and replace it by some suitable boundary condition.

Next, one would like to know whether one can associate an entropy with these spacetimes in a sensible manner, given that the notion of temperature arises very naturally. Conventionally there are two *very different* ways of defining the entropy. In statistical mechanics, the entropy  $S(E)$  is related to the degrees of freedom (or phase volume)  $g(E)$  by  $S(E) = \ln g(E)$ . Maximization of the phase volume for systems which can exchange energy will then lead to equality of the quantity  $T(E) \equiv (\partial S / \partial E)^{-1}$  for the systems. It is conventional to identify this variable as the thermodynamic temperature. In classical thermodynamics, on the other hand, it is the *change in* the entropy which can be operationally defined via  $dS = dE / T(E)$ . Integrating this equation will lead to the function  $S(E)$  except for an additive constant which needs to be determined from additional considerations.

In the case of time-symmetric spacetimes, if one chooses a vacuum state of QFT which is also time-symmetric, there will be no change of entropy  $dS$  and the thermodynamic route is blocked. But the alternative definition of  $S$ —in terms of certain degrees of freedom—is possible even in the time-symmetric context. Unfortunately, identifying these degrees of freedom is a nontrivial task. More importantly, the QFT described in the last few paragraphs makes absolutely no mathematical distinction between the horizons which arise in the Schwarzschild, de Sitter and Rindler spacetimes. Any honest identification of degrees of freedom in conventional QFT will lead to a definition of entropy *for all the three cases*. While the black-hole result is acceptable (the horizon being compact and observer independent), the de Sitter spacetime will have an observer-dependent entropy (the horizon is compact but

coordinate dependent) and the Rindler frame will have an infinite, observer-dependent entropy (since the horizon is non-compact and observer-dependent). While there is a voluminous literature on the temperature associated with these spacetimes, there is virtually no clear, published, discussion on the question: *Do the de Sitter and Rindler spacetimes possess observer-dependent entropies, which can be interpreted sensibly?*

There is an alternative point of view which one can take regarding this issue. The Schwarzschild metric, for example, can be thought of as an asymptotic metric arising from the collapse of a body forming a black hole. While developing the QFT in such a spacetime we need not maintain time reversal invariance for the vacuum state and, in fact, it is more natural to choose a state with purely ingoing modes at early times like the Unruh vacuum state. The study of QFT in such a spacetime shows that, at late times, there will exist a thermal outgoing radiation of particles which is totally independent of the details of the collapse. The temperature in this case will be  $T(M) = 1/8\pi M$ , which is the same as the one found in the case of the state of thermal equilibrium around an ‘eternal’ black hole. In the Schwarzschild spacetime, which is asymptotically flat, it is also possible to associate an energy  $E = M$  with the black hole. Though the QFT in CST calculation was done in a metric with a fixed value of energy  $E = M$ , it seems reasonable to assume that as the energy flows to infinity at late times, the mass of the black hole will decrease. *If we make this assumption—that the evaporation of black hole will lead to a decrease of  $M$ —then one can integrate the equation  $dS = dM/T(M)$  to obtain the entropy of the black hole to be  $S = 4\pi M^2 = (1/4)(A/L_P^2)$  where  $A = 4\pi(2M)^2$  is the area of the event horizon and  $L_P = (G\hbar/c^3)^{1/2}$  is the Planck length. (This integration can determine the entropy only up to an additive constant. To fix this constant, one can make the additional assumption that  $S$  should vanish when  $M = 0$ . One may think that this assumption is eminently reasonable since the Schwarzschild metric reduces to the Lorentzian metric when  $M \rightarrow 0$ . But, note that in the same limit of  $M \rightarrow 0$ , the temperature of the black hole diverges! Treated as a limit of Schwarzschild spacetime, normal flat spacetime has infinite—rather than zero—temperature.) The procedure outlined above is similar in spirit to the approach of classical thermodynamics rather than statistical mechanics.*

It is rather intriguing that there exist analogues for the collapsing black hole in the case of de Sitter and even Rindler spacetimes [31]. The analogue in the case of de Sitter spacetime will be an FRW universe which behaves like a de Sitter universe only at late times. (This is probably the actual state of our universe, which has become dominated by a cosmological constant in the recent past.) Mathematically, we only need to take  $a(t)$  to be a function that has the asymptotic form  $\exp(Ht)$  at late times. Such a spacetime is, in general, time asymmetric and one can choose a vacuum state at early times in such a way that a thermal spectrum of particles exists at late times. Emboldened by the analogy with black-hole spacetimes, one can also directly construct quantum states (similar to Unruh vacuum of black-hole spacetimes) which are time asymmetric, even in the exact de Sitter spacetime, with the understanding that the de Sitter universe came about at late times through a time asymmetric evolution.

The analogy also works for Rindler spacetime, which is also time symmetric. The standard vacuum state respects this symmetry and we arrive at a situation in thermal equilibrium. The coordinate system for an observer with *time-dependent* acceleration will, however, generalize the standard Rindler spacetime in a time-dependent manner. In particular, one can have an observer who was inertial (or at rest) at early times and is uniformly accelerating at late times. In this case an event horizon forms at late times exactly in analogy with a collapsing black hole. It is now possible to choose quantum states which are analogous to the Unruh vacuum—which will correspond to an inertial vacuum state at early times and will appear as a thermal state at late times. The correspondence with CFT can now be used to compute  $\langle T_{ab} \rangle$  in different

‘vacuum’ states to show [31] that radiative flux exists in the quantum states which are time asymmetric analogues of the Unruh vacuum state.

But in de Sitter or Rindler spacetimes there is *no* natural notion of energy (unlike in black-hole spacetimes which are asymptotically flat). In fact, it is not clear whether these spacetimes have an ‘energy source’ analogous to the mass of the black hole. While the deSitter spacetime is curved and one might consider the cosmological constant to change with evaporation, the Rindler spacetime is flat with (presumably) zero energy. Hence one is *forced* to interpret the quantum field theory in these spacetimes in terms of a state of thermal equilibrium with constant temperature but no radiation (‘evaporation’). It seems correct to conclude that the horizons *always* have temperature, but it may not be conceptually straightforward to associate an entropy (or evaporation) with the horizon in all cases.

This might tempt one to take the following point of view: in the case of black holes, one considers the collapse scenario as ‘physical’ and the natural quantum state is the Unruh vacuum. The notions of evaporation, entropy, etc then follow in a concrete manner. The eternal black hole (and the Hartle–Hawking vacuum state) is taken to be just a mathematical construct not realized in nature. In the case of Rindler, one may like to think of a time-symmetric vacuum state as natural and treat the situation as one of thermal equilibrium. This forbids using quantum states with outgoing radiation which could make the Minkowski spacetime radiate energy—which seems unlikely.

The real trouble arises for de Sitter spacetime, which is gaining in popularity. If the spacetime is asymptotically de Sitter, should one interpret it as ‘evaporating’ at late times with the cosmological constant changing with time? This will make the cosmological constant behave like quintessence models [31]. The energy source for expansion at early times (say, matter or radiation) is irrelevant just as the collapse details are irrelevant in the case of a black hole. If this is the case, can one sensibly integrate the  $dS = dE/T$  equation and obtain an entropy for deSitter spacetime, even though the spacetime is not asymptotically flat? And finally, how does one reconcile the fact that the horizon in this case is observer dependent? These issues are not analysed in adequate detail in the literature and are under study [31].

The discussion so far was based on the thermodynamic approach to entropy. One could ask whether it is possible to provide an alternative statistical mechanics interpretation of the entropy (via the equation  $S = \ln g$ ) to complement the thermodynamical derivation and, if so, does it make a distinction between Schwarzschild, de Sitter and Rindler. The simplest case, of course, will be that of a black hole. The study of the extensive literature currently available on this topic, with varying points of view, shows that we do *not* have a rigorous and unambiguous interpretation of the entropy of black hole in statistical mechanics terms *within the context of QFT in CST*. The situation is more unclear in the case of de Sitter and Rindler. It is not even clear what are the degrees of freedom one is talking about, though the best bet seems to be that they reside near the surface of the event horizon rather than inside or outside.

In the context of quantum gravity there have been attempts to relate  $S$  to underlying degrees of freedom of spacetime [15–17]. String theory provides an interpretation of  $S$  in the very special case of an extremal black hole, while the approach based on loop gravity leads to the proportionality between entropy and the horizon area in a general context though it cannot provide the proportionality constant unambiguously. In both these quantum gravitational approaches, certain degrees of freedom are identified and the logarithm of these degrees of freedom leads to an entropy. These quantum gravitational approaches, unfortunately, are not of much help in comparing de Sitter and Schwarzschild spacetimes. String theory has difficulty in accommodating a positive definite cosmological constant [20] and—in any case—the formalism cannot even handle a normal Schwarzschild black hole rigorously at present. The loop gravity approach suffers from the difficulty that—while it attributes an entropy to

any horizon—the derivation is kinematical in the sense that there could be selection rules in the theory which have a bearing on the emission of radiation. Until these are incorporated, it is not possible to proceed from the entropy to temperature. In short, QG models obtain an entropy but not temperature while QFT in CST can lead to temperature but not to an entropy in a straightforward manner.

There is another intriguing connection between trapped surfaces and quantum gravity. I have given detailed arguments elsewhere [18] to show that the event horizon of a Schwarzschild black hole acts as a magnifying glass, allowing us to probe Planck scale physics. Consider, for example, a physical system described by a low-energy Hamiltonian,  $H_{\text{low}}$ . By constructing a black hole made from the system with this Hamiltonian and requiring that the black hole should have a density of states that is immune to the details of the matter of which it is made, one can show that the Hamiltonian,  $H_{\text{true}}$  describing the interactions of the system at transplanckian energies must be related to  $H_{\text{low}}$  by  $H_{\text{true}}^2 = \alpha E_p^2 \ln [1 + (H_{\text{low}}^2 / \alpha E_p^2)]$  where  $\alpha$  is a numerical factor. Of course, the description at transplanckian energies cannot be in terms of the original variables in the rigorous theory. The above formula should be interpreted as giving the mapping between an effective field theory (described by  $H_{\text{true}}$ ) and a conventional low-energy theory (described by  $H_{\text{low}}$ ) such that the black-hole entropy will be reproduced correctly.

In fact, one can do better and construct a whole class of effective field theories [18] such that the one-particle excitations of these theories possess the same density of states as a Schwarzschild black hole. All such effective field theories are nonlocal in character and possess a universal two-point function at small scales. The nonlocality appears as a smearing of the fields over regions of the order of Planck length, thereby confirming one's intuition about microscopic structures, trapped surfaces and black-hole entropy.

## 7. Cosmological constant—to be or not to be

The last theme I want to mention is the issue of the cosmological constant, which is the deepest question that confronts any attempt to combine the principles of general relativity and quantum theory. If the current observational evidence—suggesting the existing of a small but nonzero cosmological constant—does not go away, theoreticians have a serious problem in their hands. Let me briefly review the difficulties and possibilities.

To begin with, the cosmological constant is *not* a problem in classical general relativity. Classical physics has constants which stay as constants and one is allowed to give any value to them. If we write the Einstein equations as  $G_{ik} + \Lambda g_{ik} = 8\pi G_N T_{ik}$  we are free to choose any value we like for the two constants ( $G_N$ ,  $\Lambda$ ). Further, one cannot construct any quantity with dimension of  $\Lambda$  from  $G_N$  and  $c$  alone, so there is no question of fine tuning.

The situation changes in three respects when one brings in quantum theory. (i) With ( $G_N$ ,  $c$ ,  $\hbar$ ) one can construct a dimensionless combination  $\Lambda L_p^2$  and one may be justifiably curious as to why this quantity is somewhat small—being less than  $10^{-120}$ . (ii) The coupling constants in quantum theory ‘run’. In any sensible model, with a UV cut-off around the Planck scale, the value of  $\Lambda L_p^2$  will run to a number of order unity; that is, a tiny value is unnatural in the technical sense of the term. (iii) Any finite vacuum energy density  $V_0$ , including the constants added to potential energy terms of scalar fields, say, will contribute a term ( $-V_0 g_{ik}$ ) to the right-hand side of Einstein's equations. This is mathematically indistinguishable from the cosmological constant. It is not clear why  $V_{0,\text{net}} L_p^2$  is less than  $10^{-120}$ . The situation is aggravated by the fact that we do not know of any symmetry which requires  $V_{0,\text{net}} L_p^2$  to be zero.

Ever since recent cosmological observations suggested the existence of a nonzero cosmological constant, there has been a flurry of theoretical activity to ‘explain’ it, none

of which even gets to the first base. One class of models invokes some version of the anthropic principle; but since the anthropic principle never predicted anything, I do not consider it part of scientific methodology. The second class of models uses a scalar field with an ‘appropriate’ potential  $V(\phi)$  to ‘explain’ the observations. These models are all trivial and have no predictive power because it is always possible to choose a  $V(\phi)$  to account for any sensible dynamical evolution of the universe. Since the triviality of these models (which are variously called ‘quintessence’, ‘dark energy’, etc) does not seem to have been adequately emphasized in literature, let me briefly comment on this issue [32].

Consider any model for the universe with a *given*  $a(t)$  and some known forms of energy density  $\rho_{\text{known}}(t)$  (made of radiation, matter, etc) both of which are observationally determined. It can happen that this pair does not satisfy the Friedmann equation for an  $\Omega = 1$  model. To be specific, let us assume  $\rho_{\text{known}} < \rho_c$  which is substantially the situation in cosmology today. If we now want to make a consistent model of cosmology with  $\Omega = 1$ , say, we can invoke a scalar field with the potential  $V(\phi)$ . It is trivial to choose  $V(\phi)$  such that we can account for *any* sensible pair  $[a(t), \rho_{\text{known}}(t)]$  along the following lines: using the given  $a(t)$ , we define two quantities  $H(t) = (\dot{a}/a)$  and  $Q(t) \equiv 8\pi G\rho_{\text{known}}(t)/3H^2(t)$ . The required  $V(\phi)$  is given parametrically by the equations

$$V(t) = (1/16\pi G)H(1 - Q)[6H + (2\dot{H}/H) - (\dot{Q}/(1 - Q))] \quad (19)$$

$$\phi(t) = \int dt [H(1 - Q)/8\pi G]^{1/2} [\dot{Q}/(1 - Q) - (2\dot{H}/H)]^{1/2}. \quad (20)$$

All the potentials invoked in the literature are special cases of this formula [32]. This result shows that *irrespective of what the future observations reveal about  $a(t)$  and  $\rho_{\text{known}}(t)$*  one can always find a scalar field which will ‘explain’ the observations. Hence this approach has no predictive power. What is worse, most of the  $V(\phi)$  suggested in the literature have no sound particle physics basis and, in fact, the quantum field theory for these potentials are very badly behaved or non-existent.

It is worth realizing that the existence of a nonzero cosmological constant will be a statement of fundamental significance and constitutes a conceptual contribution of cosmology to quantum gravity. The tendency of some cosmologists to treat  $\Omega_\Lambda$  as one among a set of, say, 17 parameters (such as  $\Omega_{\text{rad}}, \Omega_B, n, \dots$ ) which need to be fixed by observations, completely misses the point. The cosmological constant is special and its importance transcends cosmology.

At present, we do not have a fundamental understanding of the cosmological constant from any approaches to quantum gravity. There are no nontrivial string theoretical models incorporating  $\rho_V > 0$ ; loop gravity can incorporate it but does not throw any light on its value. It should be stressed that the nonzero value for  $\rho_V \neq 0$  does *not* imply de Sitter (or even asymptotically de Sitter) spacetime. Hence the formalism should be capable of handling  $\rho_V$  without de Sitter geometry.

To give an example of a more fundamental way of thinking about the cosmological constant, let me describe an idea in which the cosmological constant is connected with the microstructure of spacetime. In this model, we start with  $\Lambda = 0$  but generate a small value for this parameter from two key ingredients: (i) a discrete spacetime structure at Planck length and (ii) a quantum gravitational uncertainty principle. To do this, we first note that the cosmological constant can be thought of as a Lagrange multiplier for the proper volume of spacetime in the action functional for gravity:

$$A_{\text{grav}} = \frac{1}{2L_P^2} \int d^4x R\sqrt{-g} - \frac{\Lambda}{L_P^2} \int d^4x \sqrt{-g}. \quad (21)$$

In any quantum cosmological model which leads to large volumes for the universe, the phase of the wavefunction will pick up a factor of the form  $\Psi \propto \exp(-i(\Lambda/L_P^2)\mathcal{V})$ , where  $\mathcal{V}$  is the four-volume, from the second term in (21). Treating  $(\Lambda/L_P^2, \mathcal{V})$  as conjugate variables  $(q, p)$ , we can invoke the standard uncertainty principle to predict  $\Delta\Lambda \approx L_P^2/\Delta\mathcal{V}$ . Now we make the crucial assumption regarding the microscopic structure of the spacetime: assume that there is a zero-point length of the order of  $L_P$  so that the volume of the universe is made of numerous cells, each of volume  $L_P^4$ . Then  $\mathcal{V} = NL_P^4$ , implying a Poisson fluctuation  $\Delta\mathcal{V} \approx \sqrt{\mathcal{V}}L_P^2$ , and leading to

$$\Delta\Lambda = \frac{L_P^2}{\Delta\mathcal{V}} = \frac{1}{\sqrt{\mathcal{V}}} \approx H_0^2 \quad (22)$$

which is exactly what cosmological observations imply! The Planck length cut-off (UV limit) and volume of the universe (IR limit) combine to give the correct  $\Delta\Lambda$ . Of course, this makes  $\Lambda$  a stochastic variable and one needs to solve the Friedmann equation using a stochastic source [31]<sup>2</sup>.

While I am not optimistic about the *details* of my model—especially since models with  $\Lambda \approx H^{-2}$  at all epochs including the era of nucleosynthesis are severely constrained—I find it attractive to think of the observed cosmological constant as arising from quantum *fluctuations* of some energy density rather than from the *bulk* energy density. This is relevant in the context of standard discussions of the contribution of zero-point energies to the cosmological constant. I would expect the correct theory to regularize the divergences and make the zero-point energy finite and about  $L_P^{-4}$ . This contribution is most likely to modify the microscopic structure of spacetime (e.g. if the spacetime is naively thought of as due to the stacking of Planck scale volumes, this will modify the stacking or shapes of the volume elements) and will not affect the bulk gravitational field when measured at scales coarse grained over sizes much bigger than the Planck scales. In other words, large amounts of energy are soaked up by the quantum microstructure of the spacetime itself like a sponge soaking up water. This process, however, will leave a small residual fluctuation which will depend on the volume of the spacetime region which is probed. I would conjecture that the cosmological constant we measure corresponds to this residue. It is small, in the sense that it has been reduced from  $L_P^{-4}$  to  $L_P^{-4}(L_P H_0)^2$ , which indicates the fact that fluctuations, when measured over a large volume, are small compared to the bulk value. It is the wetness of the sponge we notice, not the wetness inside.

## 8. Conclusions

So, where do we stand after decades of attempts to combine general relativity and quantum theory? The good news is that we have made some progress on the following fronts. (i) It is clear that spacetime at  $L \approx L_P$  has a non-trivial structure and the metric  $g_{ik}$  is only a low-energy concept; the description of the microstructure is very likely to demand our going beyond the language of QFT with a finite number of fields. (ii) The Planck length acts as a zero-point length of the spacetime in one manner or another. (iii) Black-hole entropy is very likely to be related to the degrees of freedom within one Planck length of the event horizon, and event horizons could provide a glimpse to spacetime microstructure. (iv) The consistent interpretation of semiclassical cosmology requires a concept like decoherence.

The bad news is that there are still several serious issues about which we have no clue. (i) We do not know how to describe the sub-Planckian structure of spacetime or how to obtain a smooth metric structure from the fundamental degrees of freedom. (ii) The connection between

<sup>2</sup> After I gave this talk, I discovered that a similar result was obtained earlier by Sorkin [34] based on a completely different model.

horizons and thermodynamics is suggestive of some deep principle which has remained elusive. As a result, years of work in QFT in curved spacetime has not led to any real insight into quantizing gravity. (iii) All fundamental questions related to the origin of the universe remain unanswered, and conventional quantum cosmology has not led to useful insights in quantum gravity. (iv) Quantum gravity has remained disjoint from high-energy particle physics and we do not know how to test it experimentally. (v) Finally, we have no idea at all about the cosmological constant, which is probably *the* most intriguing problem in quantum gravity.

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