Formation of magnetic discontinuities through contortion of magnetic flux surfaces

Sanjay Kumar[1], Ramit Bhattacharyya[1], P. K. Smolarkiewicz[2]

[1]Udaipur Solar Observatory, Physical Research Laboratory, India

[2]European Centre for Medium-Range Weather Forecasts, UK

Introduction

The dissipative or the non-dissipative limits of a single fluid MHD, are determined by the magnetic Reynolds number

$$\boxed{R_M = \frac{v\,L}{\eta}}$$

$R_M << 1$

Magnetic field lines diffuse out from fluid parcels.

$R_M >> 1$

Field lines are tied to fluid parcels – “flux-freezing”.

Magnetic flux across a fluid surface, identified by the material elements lying on it, is invariant.

Because of the involved large L, the astrophysical magnetofluids fall in the limit of R_M >>1 and satisfy the “flux freezing”.

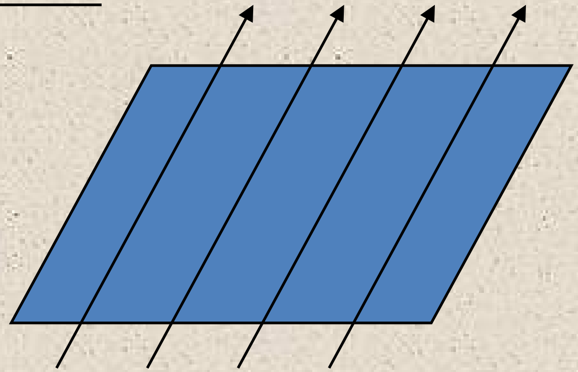
Introduction (contd.)

Select a subset Σ of the set Γ describing all fluid surfaces such that magnetic field lines are tangential to surfaces belonging to Σ.

Magnetic flux surfaces

(MFSs)

(schematic)



• In an evolving magnetofluid, these Σ surfaces retain their identity of MFSs; under the condition of flux-freezing.

• Favorable contortions can bring two oppositely directed field lines toward each other.

$\mathbf{J} = \nabla \times \mathbf{B}$ ⟹ A sharpening of field gradients then localizes the volume current density on a plane across which the field lines flip sign and develops a current sheet (CS).

The importance

- An inherent mechanism, fundamental to MHD, that generates small scales from the large scales which are natural to astrophysical plasmas.

- The corresponding diffusion of field lines leading to magnetic reconnections is then responsible for a multitude of observed solar/stellar eruptive phenomena in the form of flares or CMEs and provides a possible explanation for the million degree temperature of the solar corona.

- **The inevitability of CS formation in a static magnetofluid with infinite electrical conductivity and complex magnetic topology, is assured by the Parker's magnetostatic theorem.**

- **The theorem is based on a general inability of the magnetofluid to achieve local force balance while preserving its global topology, with a magnetic field continuous everywhere.**

(Parker, Astrophys. J. 1972, 1988)

Viscous relaxation

$$\rho_0[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}] = -\nabla p + \mathbf{J}\times\mathbf{B} + \mu_0\nabla^2\mathbf{v}$$

$$\nabla\cdot\mathbf{v} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times\mathbf{v}\times\mathbf{B}$$

$$\nabla\cdot\mathbf{B} = 0$$

MHD equations for an infinite electrical conducting, incompressible and viscous magnetofluid.

$$\frac{dW_k}{dt} = \int_V [\mathbf{J}\times\mathbf{B}]\cdot\mathbf{v}d^3x - \mu_0\int_V |\nabla\times\mathbf{v}|^2 d^3x$$

$$\frac{dW_M}{dt} = -\int_V [\mathbf{J}\times\mathbf{B}]\cdot\mathbf{v}d^3x$$

$$\frac{dW_T}{dt} = -\mu_0\int_V |\nabla\times\mathbf{v}|^2 d^3x$$

Energy budget for kinetic, magnetic and total energies for a periodic domain.

- Under the condition of flux freezing, terminal relaxed state is expected to be in magnetostatic equilibrium and identical in magnetic topology to the initial state.

Objective

Realizing the fact that deformations/contortions of MFSs can be responsible for CS formation in an evolving magnetofluid, motivation of the present study is to numerically demonstrate the formation of CSs by solving MHD equations as suitable initial value problems and describing the evolution in terms of MFSs.

Flux surface representation

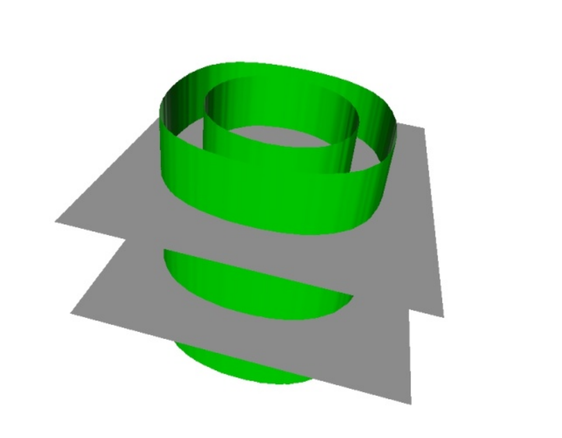
Untwisted magnetic field

$$\boxed{\mathbf{B} = \nabla\alpha \times \nabla\beta}$$

α(x,y,z) and β(x,y,z) are Euler potentials (Stern, Am. J. Phys. 1971).

$$\boxed{\begin{aligned}\mathbf{B}\cdot\nabla\alpha &= 0\\ \mathbf{B}\cdot\nabla\beta &= 0\end{aligned}}$$

α(x,y,z) = const., and β(x,y,z) = const., are two families of global magnetic flux surfaces.



Schematic of two families of flux surfaces. Intersection of these surfaces are field lines.

Twisted magnetic field

$$\mathbf{B} = \sum_i \mathbf{B}_i = \sum_i (\nabla\alpha_i \times \nabla\beta_i)$$

(Low, Astrophys. J. 2006)

Under the condition of flux-freezing, flux surfaces evolve as fluid surfaces defined by material elements lying on it.

$$\frac{d\alpha_i}{dt} = \frac{\partial\alpha_i}{\partial t} + (\mathbf{v}\cdot\nabla)\alpha_i = 0$$

$$\frac{d\beta_i}{dt} = \frac{\partial\beta_i}{\partial t} + (\mathbf{v}\cdot\nabla)\beta_i = 0$$

Initial value problem

- Initial magnetic field in a triply periodic Cartesian domain with volume $s_0\,(2\pi\,)^3$

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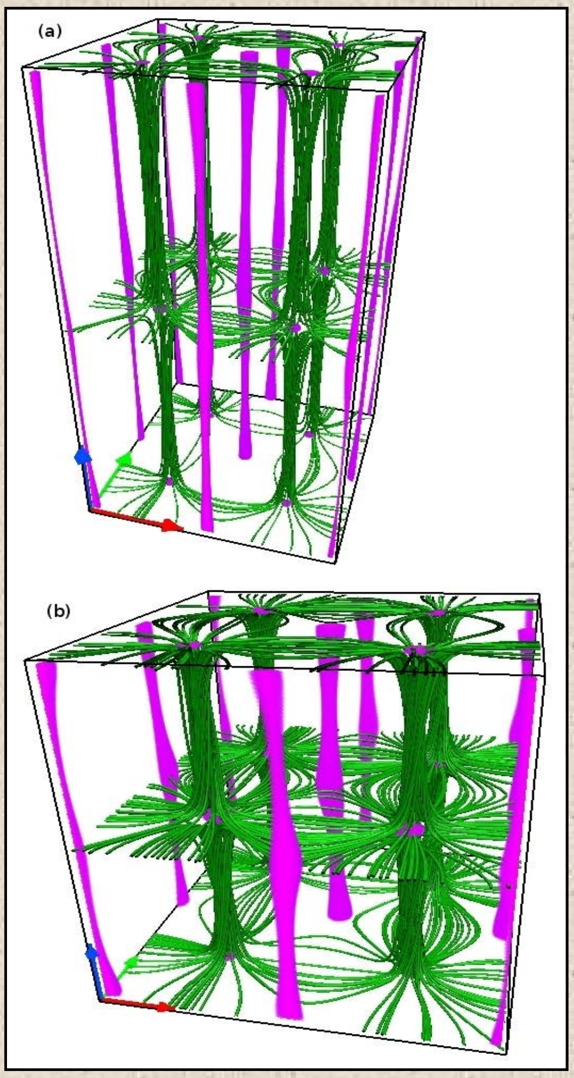
$$\begin{aligned}
B_x &= \sqrt{3}\sin(x)\cos(y)\sin(z/s_0) + \cos(x)\sin(y)\cos(z/s_0)\\
B_y &= -\sqrt{3}\cos(x)\sin(y)\sin(z/s_0) + \sin(x)\cos(y)\cos(z/s_0)\\
B_z &= 2s_0\sin(x)\sin(y)\sin(z/s_0)
\end{aligned}$$

$$\nabla \times \mathbf{B} = \alpha_0 \mathbf{B} \text{ for } s_0 = 1$$

- Field is linear force free with $\alpha_0 = 3^{1/2}$ where α_0 represents magnetic circulation per unit flux.
- Magnetic topology is complex because of the presence of 2D and 3D nulls.

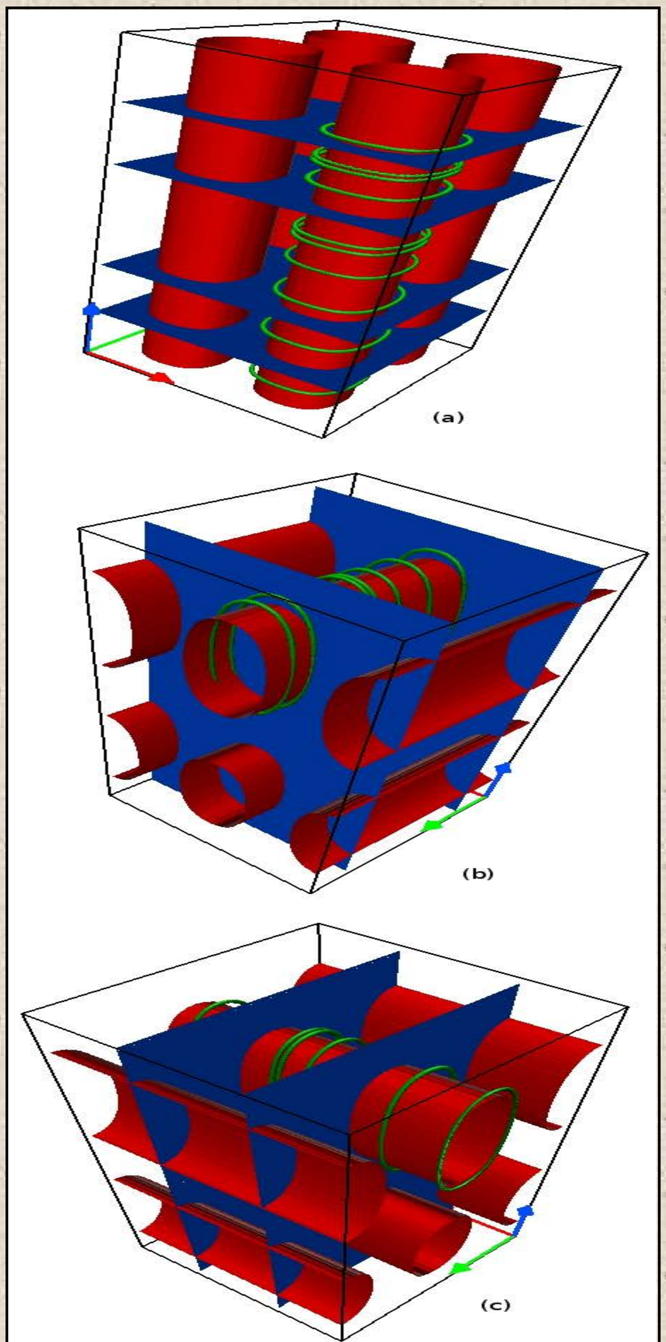
The panel (a) illustrates magnetic nulls of the initial field B for $s_0 = 3$. Panel (b) shows the same for the corresponding linear force-free field characterized by $s_0 = 1$.



$$\boxed{\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3}$$

$$\mathbf{B}_1 = \nabla\alpha_1 \times \nabla\beta_1 = \nabla(s_0\sqrt{3}\sin(x)\sin(y)) \times \nabla(-\cos(z/s_0))$$
$$\mathbf{B}_2 = \nabla\alpha_2 \times \nabla\beta_2 = \nabla(s_0\cos(x)\sin(z/s_0)) \times \nabla(\cos(y))$$
$$\mathbf{B}_3 = \nabla\alpha_3 \times \nabla\beta_3 = \nabla(s_0\cos(y)\sin(z/s_0)) \times \nabla(-\cos(x))$$

Panel (a) depicts the Euler surfaces α_1 =1.3, -1.3 (in red); and β_1=0.25,-0.25 (in blue). Panel (b) plots the Euler surfaces α_2=0.85, -0.85(in red); and β_2=0.25 (in blue). The Euler surfaces α_3=0.85,-0.85 (in red); and β_3=0.25 (in blue) are shown in panel (c). Green lines represent the corresponding field lines.



Numerical setup

- Governing equations

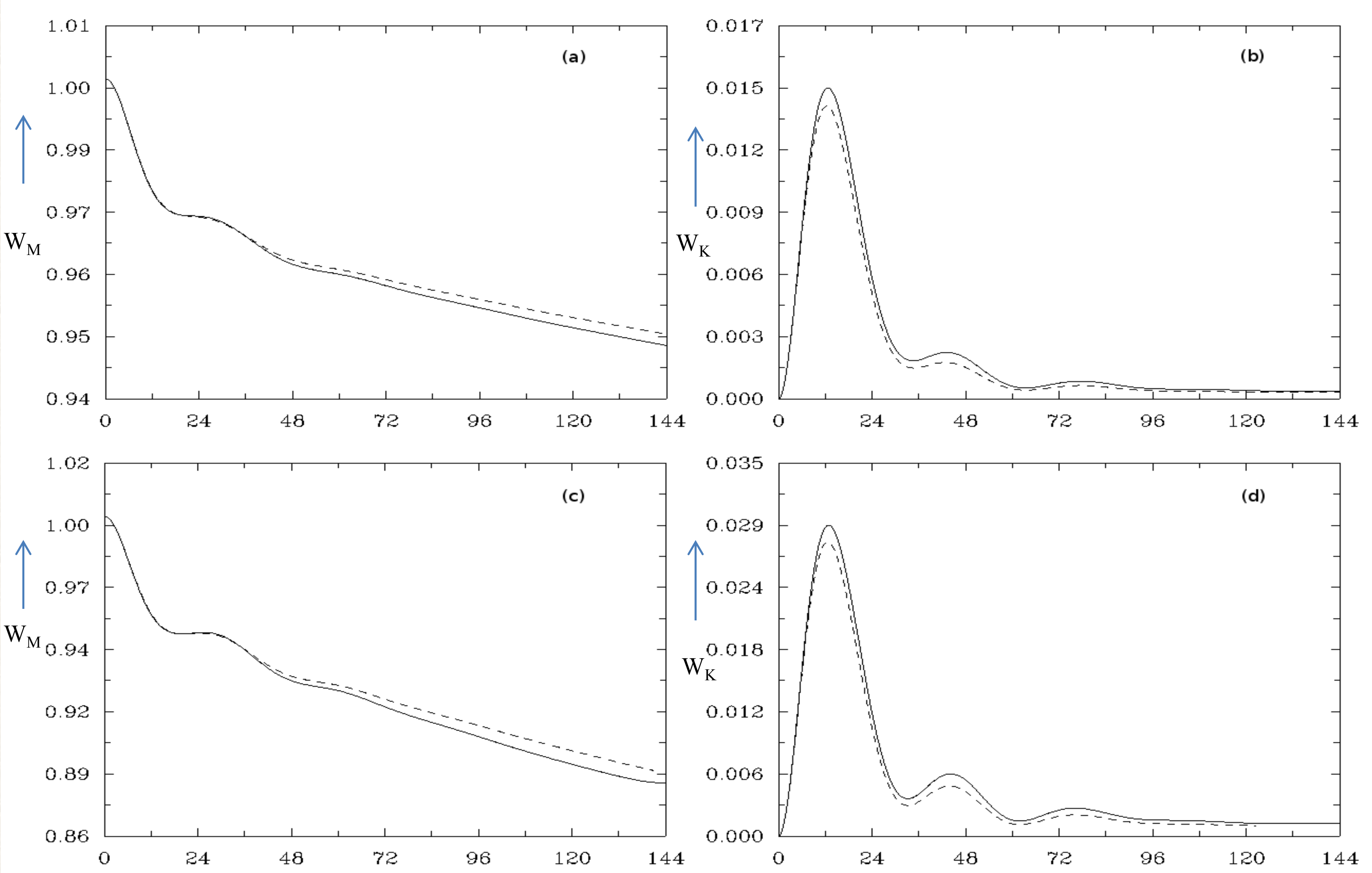
$$\rho_0\left[\frac{\partial v}{\partial t} + (v\cdot\nabla)v\right] = -\nabla p + J\times B + \mu_0\nabla^2 v$$
$$\nabla\cdot v = 0$$
$$B = \sum_{i=1}^{3}\nabla\alpha_i\times\nabla\beta_i$$

$+$

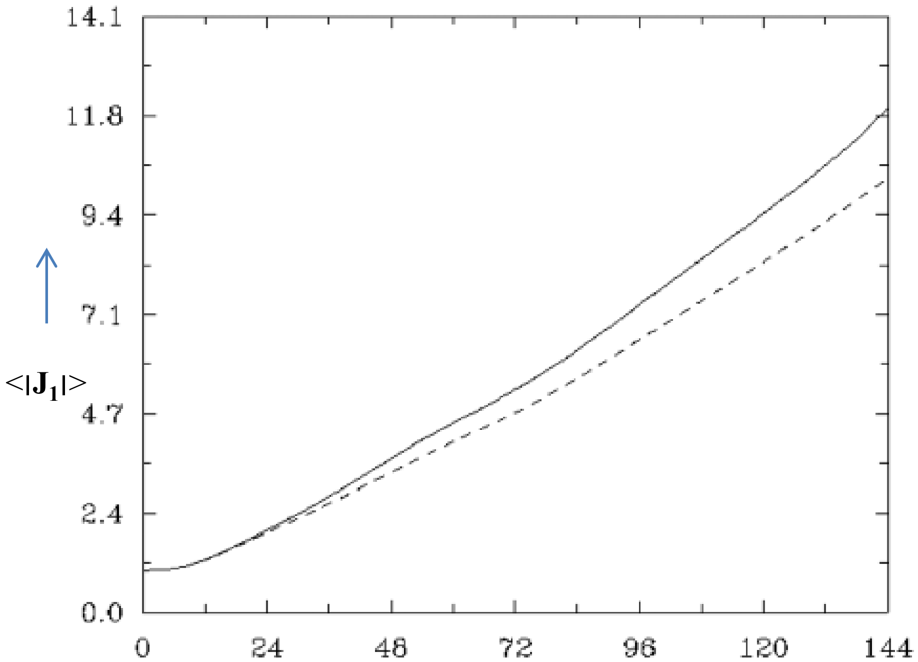
$$\frac{\partial\alpha_i}{\partial t} + (\mathbf{v}\cdot\nabla)\alpha_i = 0$$
$$\frac{\partial\beta_i}{\partial t} + (\mathbf{v}\cdot\nabla)\beta_i = 0$$

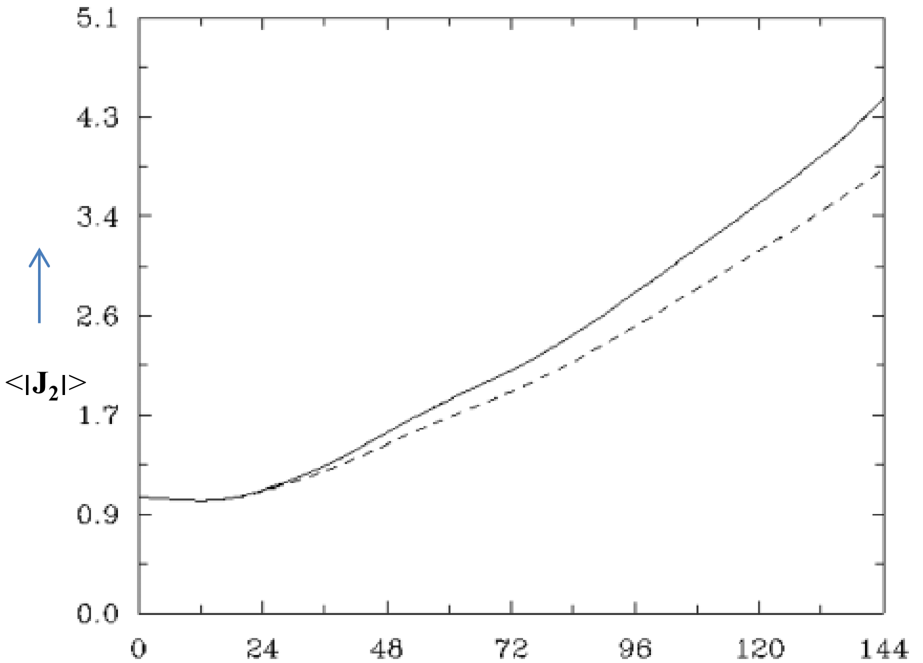
- To solve the above partial differential equations, MHD version of the general-purpose hydrodynamic numerical model, EULAG has been adopted (Prusa et al., Comput. Fluids 2008).

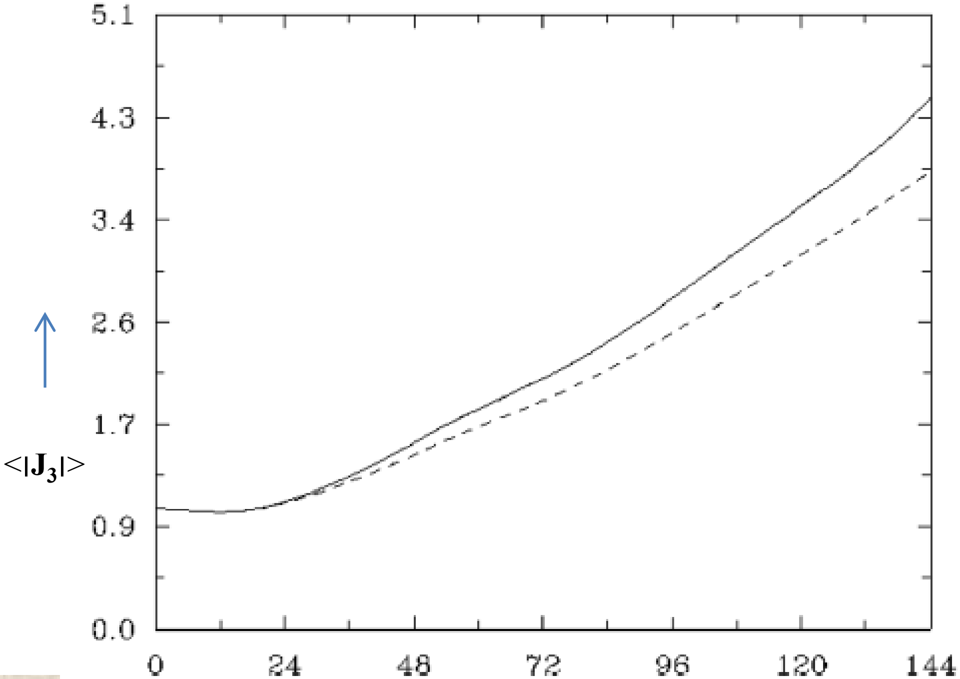
Results



Evolution of normalized magnetic and kinetic energies for $s_0=2$ (panels a and b) and $s_0=3$ (panels c and d). Each plot is for two different viscosities 0.0075 (solid line) and 0.0085 (dashed line).





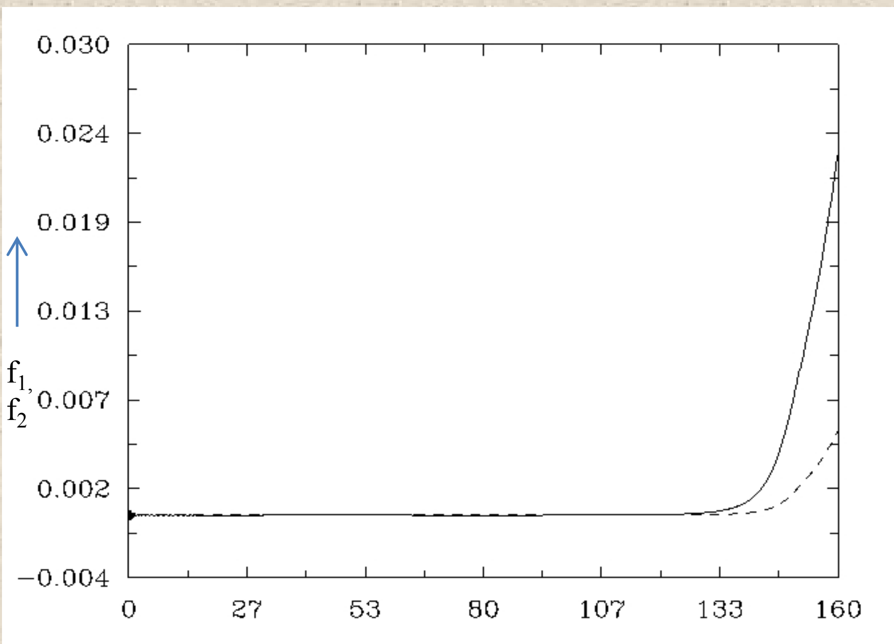


$$\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 + \mathbf{J}_3$$

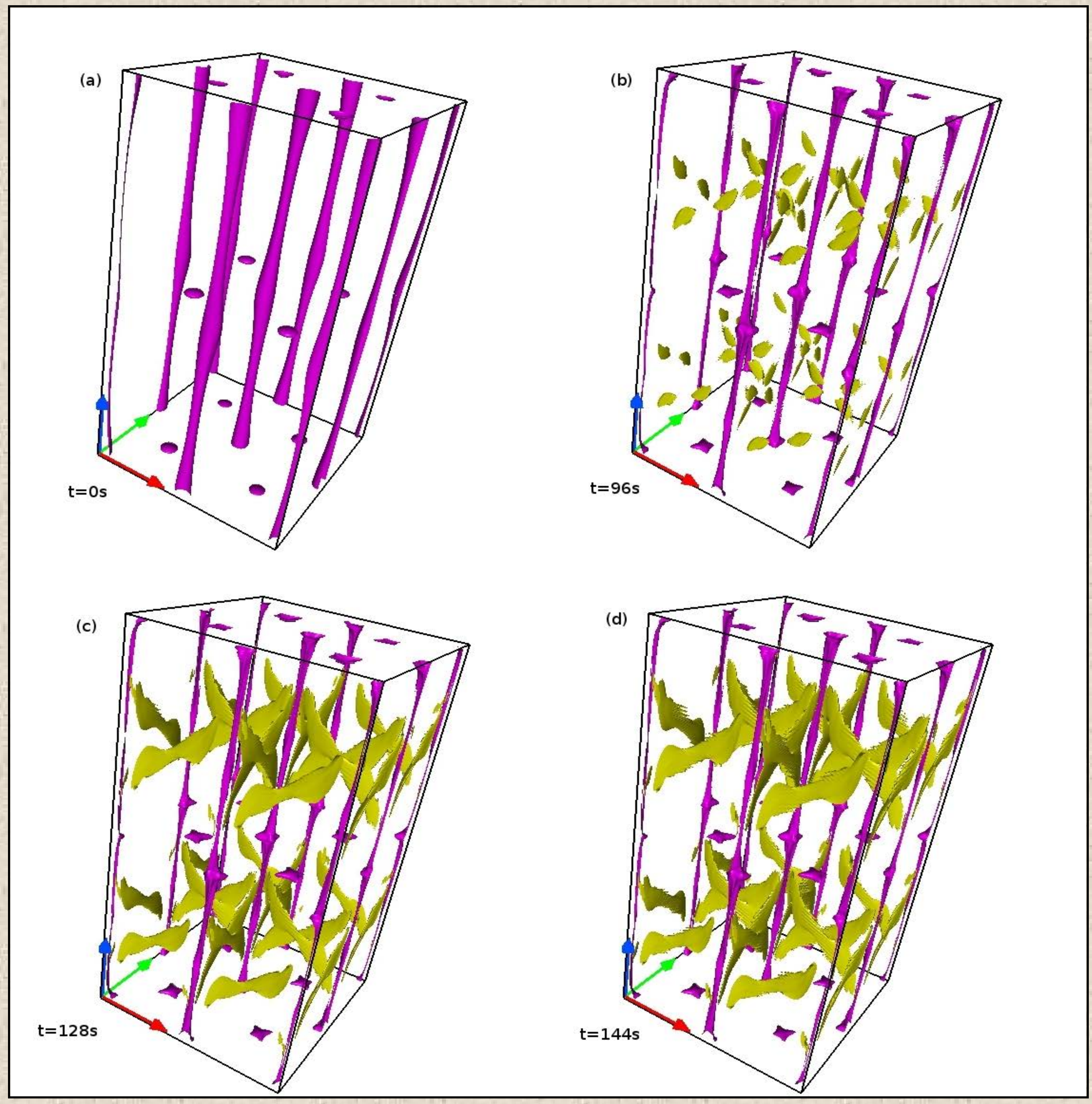
History of $<|\mathbf{J}_1|>$ (top left), $<|\mathbf{J}_2|>$ (top right), $<|\mathbf{J}_3|>$ (bottom left), for $s_0=3$; normalized with respect to their initial values.

$$\frac{dW_k}{dt} - \int_V [\mathbf{J}\times\mathbf{B}]\cdot \mathbf{v} d^3x + \mu_0 \int_V |\nabla\times\mathbf{v}|^2 d^3x = f_1$$

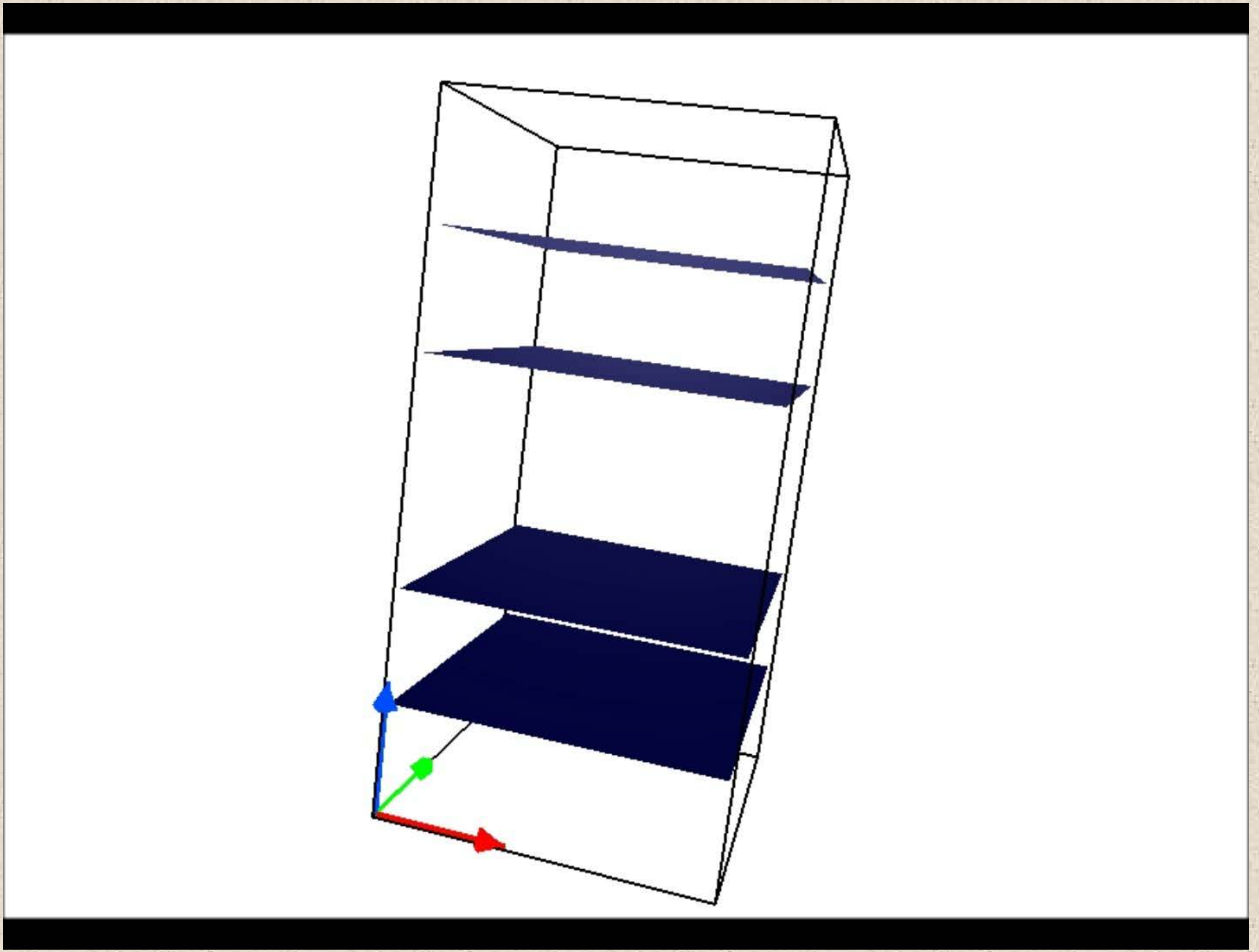
$$\frac{dW_M}{dt} + \int_V [\mathbf{J}\times\mathbf{B}]\cdot \mathbf{v} d^3x = f_2$$



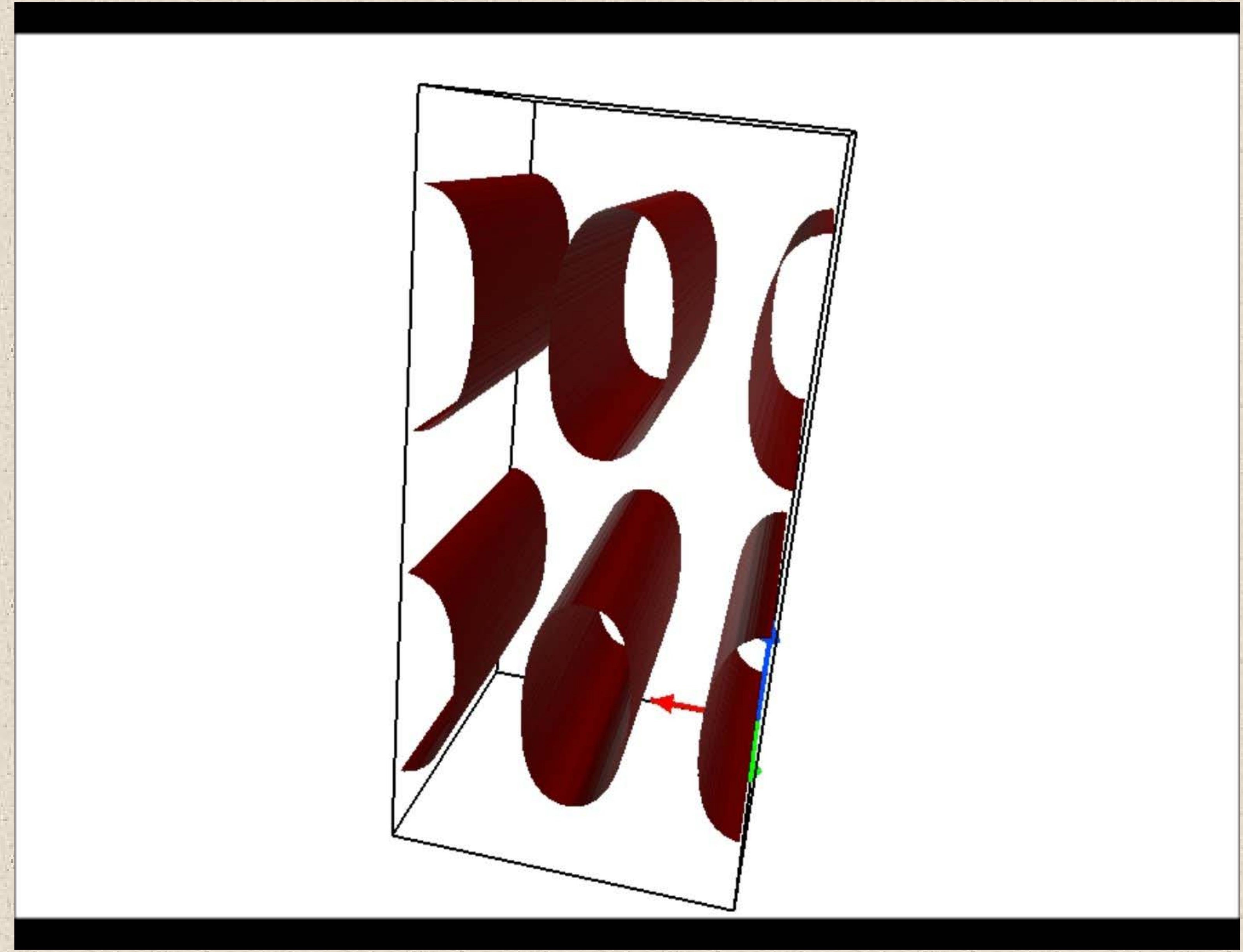
The history of energy budget for kinetic (solid) and magnetic (dashed) energies for s_0 =3.



Evolution of magnetic nulls (in pink), overlaid with isosurface (in yellow) of total current density having a magnitude of 30% of its maximum value.



Evolution of Euler surfaces β_1=0.25,-0.25 (in blue), overlaid with isosurface of J_1 (in yellow), having magnitude of 60% of $|J_{Max}|$.



Evolution of Euler surfaces α_2=0.85,-0.85 (in red), overlaid with isosurface of J_2 (in yellow), having magnitude of 60% of $|J_{Max}|$.

Summary

- Formation of CSs is demonstrated via MFSs description utilizing the viscous relaxation to obtain a terminal quasi-steady state which is identical in magnetic topology to initial state as demanded by Parker's magnetostatic theorem.

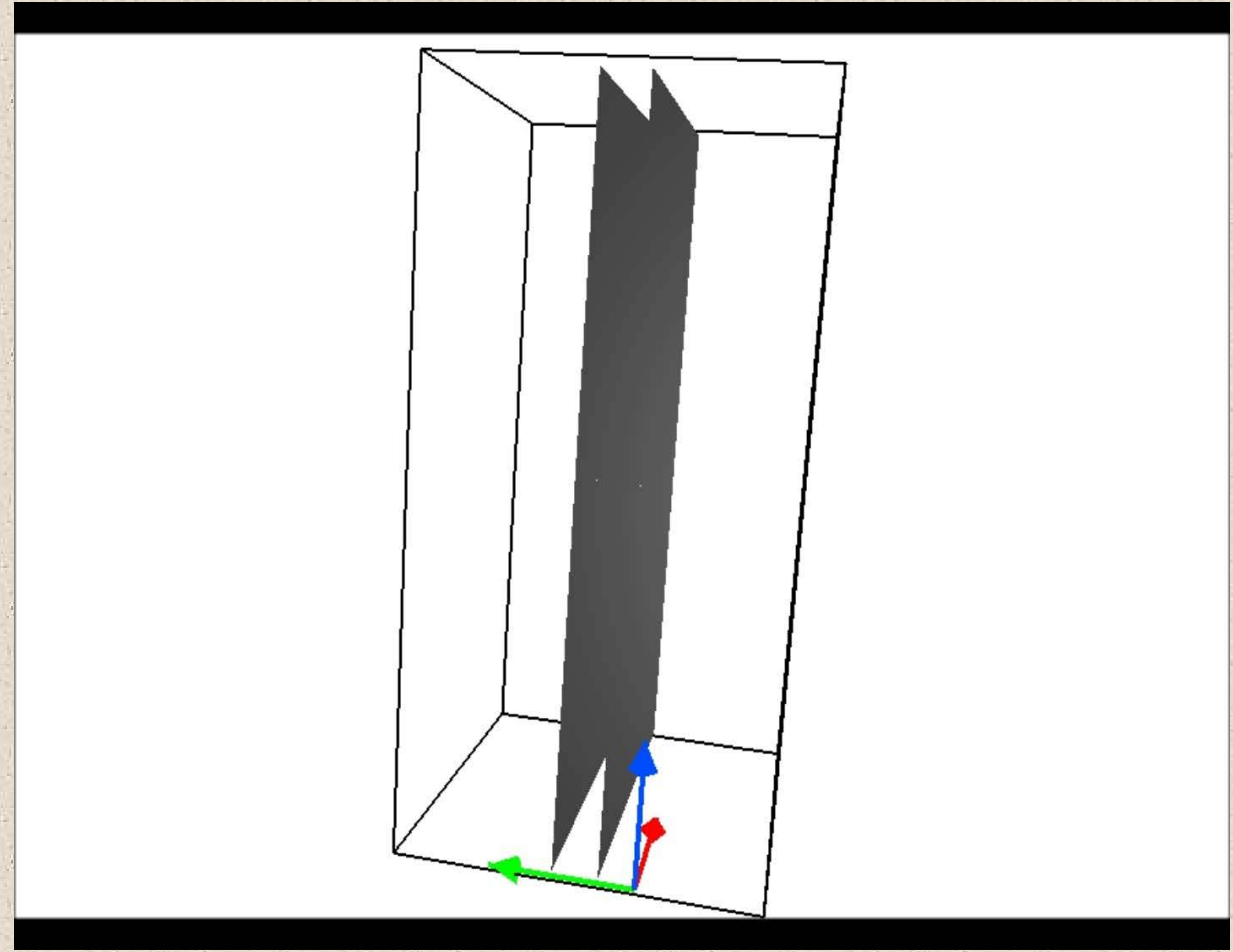
- Monotonic increase of every components current densities provides evidence of CS formation.

- Spatial locations of CSs are away from the magnetic nulls.

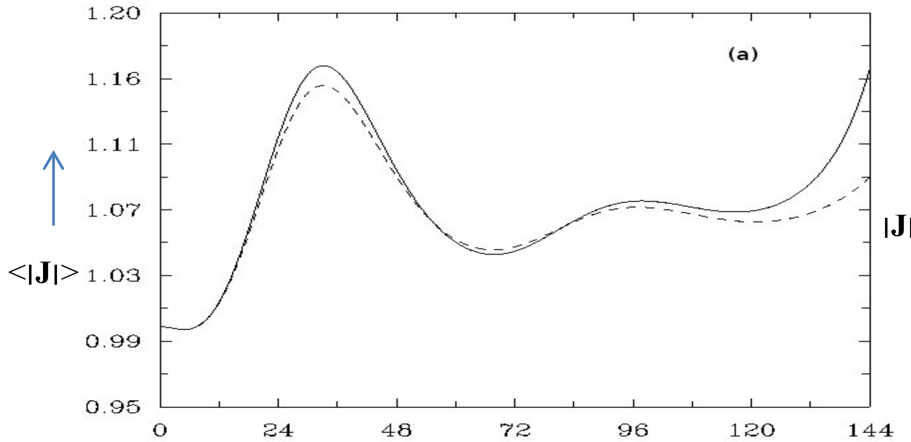
- The favourable contortions of MFSs is found to be the responsible process for development of CSs away from nulls.

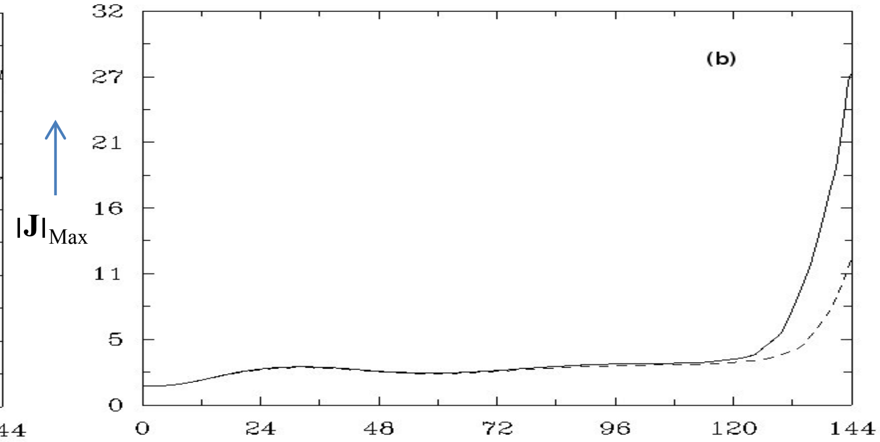
- The advection of MFSs provides the better insight of governing dynamics of CS formation.

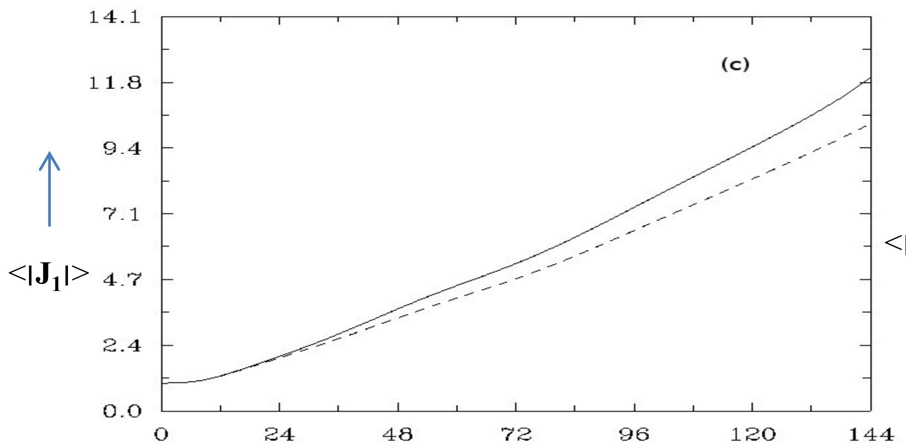
THANKS !

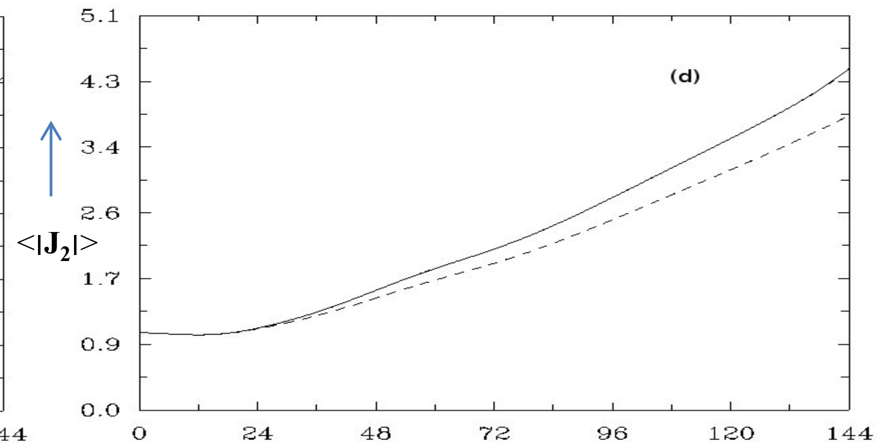


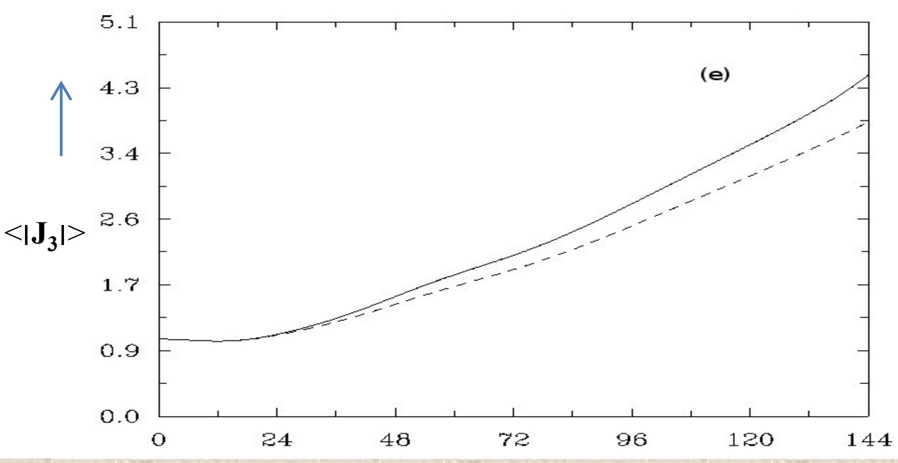
Evolution of Euler surfaces β_2=-0.40 (in grey), overlaid with isosurface of J_2 (in blue), having magnitude of 60% of $|J_{Max}|$ (J_3 -60).





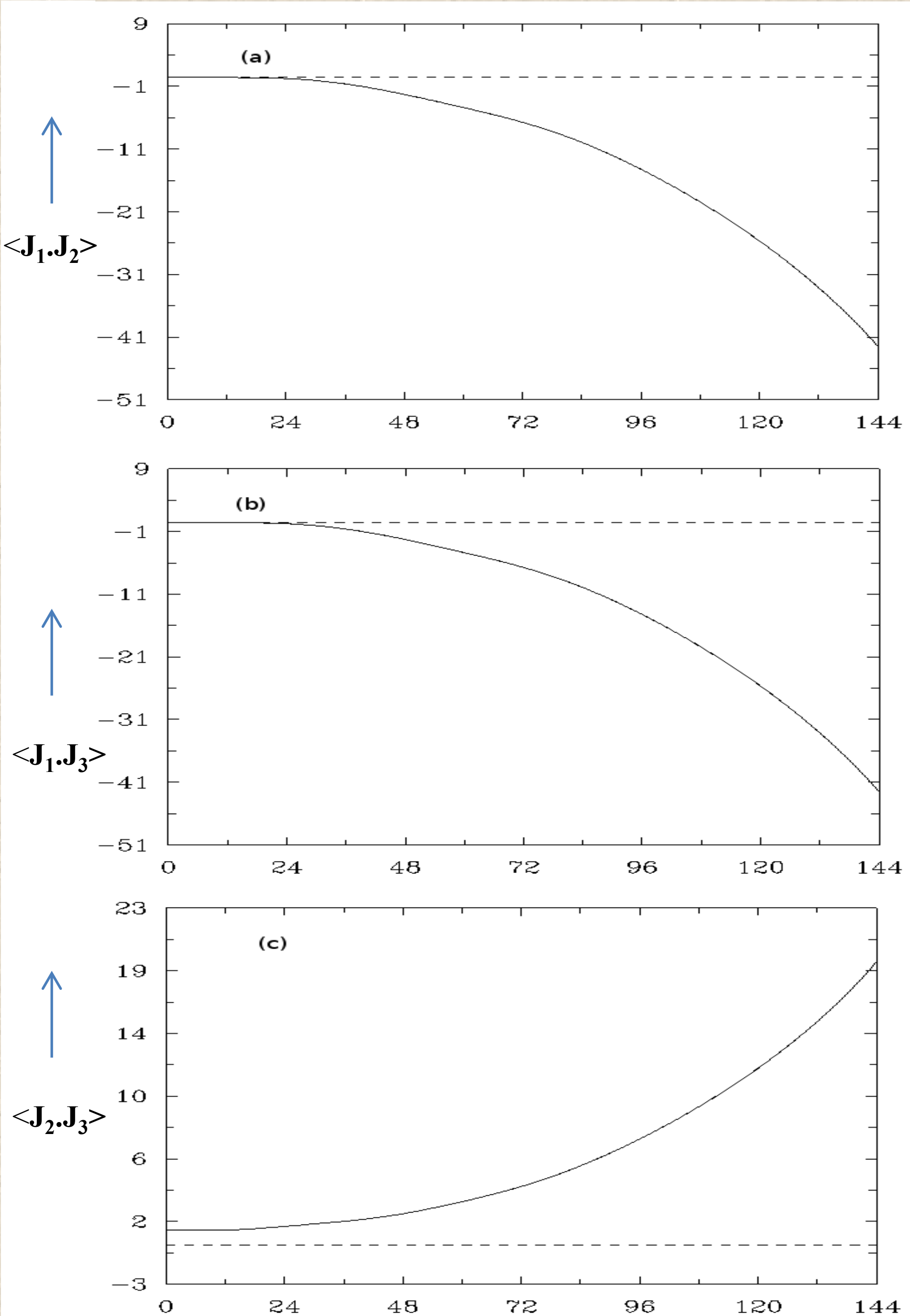






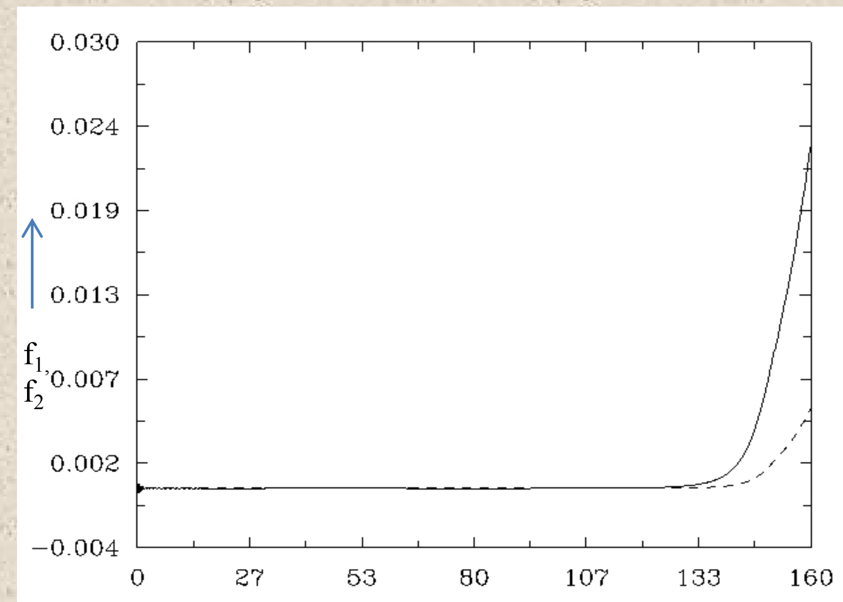
$$\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 + \mathbf{J}_3$$

History of (a): $\langle|\mathbf{J}|\rangle$, (b): $|\mathbf{J}|_{Max}$, (c): $\langle|\mathbf{J}_1|\rangle$, (d): $\langle|\mathbf{J}_2|\rangle$, (e): $\langle|\mathbf{J}_3|\rangle$, for $s_0=3$; normalized with respect to their initial values.



$$\frac{dW_k}{dt} - \int_V [\mathbf{J}\times\mathbf{B}]\cdot \mathbf{v} d^3x + \mu_0 \int_V |\nabla\times\mathbf{v}|^2 d^3x = f_1$$

$$\frac{dW_M}{dt} + \int_V [\mathbf{J}\times\mathbf{B}]\cdot \mathbf{v} d^3x = f_2$$



The history of energy budget for kinetic (solid) and magnetic (dashed) energies for $s_0 = 3$.

Time profiles of the normalized (a): $<\mathbf{J}_1.\mathbf{J}_2>$, (b): $<\mathbf{J}_1.\mathbf{J}_3>$, and (c): $<\mathbf{J}_2.\mathbf{J}_3>$, for $s_0 = 3$.

Numerical Model

- Governing equations

$$\boxed{\begin{aligned}&\rho_0\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla p + \mathbf{J}\times\mathbf{B} + \mu_0\nabla^2\mathbf{v}\\ &\nabla\cdot\mathbf{v} = 0\end{aligned}}$$

$$\boxed{\begin{aligned}&\frac{\partial \alpha_i}{\partial t} + (\mathbf{v}\cdot\nabla)\alpha_i = 0\\ &\frac{\partial \beta_i}{\partial t} + (\mathbf{v}\cdot\nabla)\beta_i = 0\end{aligned}}$$

- To solve the above partial differential equations, MHD version of the general-purpose hydrodynamic numerical model, EULAG has been adopted (Prusa et al., Comput. Fluids 2008).

$$\boxed{\frac{\partial(\rho_0\psi)}{\partial t} + \nabla\cdot(\rho_0\mathbf{v}\psi) = \rho_0 R}$$

where ψ is the dependent fluid variable and R symbolizes the forcing terms.

$$\boxed{\psi_i^{n+1} = LE_i(\psi^n + 0.5\delta t R^n) + 0.5\delta t R_i^{n+1} \equiv \hat{\psi}_i + 0.5\delta t R_i^{n+1}}$$

EULAG template