

APSYS — an Artificial Planetary System in Space to probe extra-dimensional gravity and MOND

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A proposal is made to test Newton's inverse-square law using the perihelion shift of test masses (planets) in free fall within a spacecraft located at the Earth–Sun L2 point. Such an *Artificial Planetary System In Space* (APSYS) will operate in a drag-free environment with controlled experimental conditions and minimal interference from terrestrial sources of contamination. We demonstrate that such a space experiment can probe the presence of a ‘hidden’ fifth dimension on the scale of a micron, if the perihelion shift of a ‘planet’ can be measured to sub-arc-second accuracy. Some suggestions for spacecraft design are made.

I. INTRODUCTION

Cosmology at the turn of this century appears to stand at new crossroads.

Remarkably precise recent observations have, on the one hand, confirmed long standing theoretical ideas (inflation, baryon oscillations) and, on the other hand, provided glimpses into unexpected territories and landscapes (dark matter and dark energy). One of the central issues facing the cosmologist in the new century is an explanation for the multifarious properties of an accelerating universe and the plausible existence of extra dimensions. Indeed, it now appears [1, 2, 3] that only about 4% of the matter content of the universe is baryonic in form. To account for the remaining 96%, one usually invokes the presence of

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‘dark matter’ ($\sim 26\%$) and ‘dark energy’ ($\sim 70\%$). While dark matter successfully explains several different sets of observations, its present avatar — cold dark matter — is currently facing an increasing number of observational challenges including galaxy cores which appear to be shallower than the cuspy cores predicted by CDM, and an over-abundance of dwarf galaxies predicted by this scenario to exist both in voids and in our local group, and not seen in either. While it may be that traditional remedies to these problems (baryonic feedback, making the dark matter ‘warm’ instead of ‘cold’, using a scalar field to describe dark matter, etc.) may alleviate some of the tension between theory and observations [4, 5, 6, 7, 8], it could also be that the current situation warrants a more fundamental revision of our understanding of the basic laws governing gravity. An example of a radical approach which attempts the latter is MOdified Newtonian Dynamics (MOND), a phenomenological model originally suggested in 1983, which gives impressive results in explaining the flat rotation curves of galaxies and some other observations [9, 10].

In contrast to dark matter, which is assumed to be pressureless and prone to gravitational clustering, dark energy is virtually unclustered and endowed with a large negative pressure which allows it to explain the current acceleration of the universe. Like it is the case with dark matter, the existence of dark energy is largely hypothetical, its *raison d’être* being observations of cosmic acceleration, which are not easily explained by a more conventional matter source. However, the unevolving nature of the simplest dark energy candidate — the cosmological constant — implies that the ratio of the density in the latter ($\rho_\Lambda \simeq 10^{-47} \text{ GeV}^4$) to the radiation/matter density is an increasingly small number at early times. For instance, $\rho_\Lambda \simeq 10^{-123} \rho_P$ at the Planck time $\sim 10^{-43}$ seconds after the Big Bang (here, ρ_P is the Planck density, the only natural value at that time). This gives rise (according to one’s perspective) either to an initial ‘fine-tuning’ problem or to a ‘cosmic coincidence’ conundrum [11].

Keeping these issues in mind, the prevailing views on dark matter and dark energy have, in recent years, been supplemented by new ideas, which see the recent observations as lending support to the possibility that our traditional theories of gravity may need reformulation either in regions of small acceleration (MOND) or on very large scales (braneworld models [12], modified gravity theories [13], etc).¹

For instance, MOND assumes that Newton’s law of inertia ($F = ma$) is modified at

¹ For other alternative explanations of dark matter and dark energy see [14, 15]

sufficiently low accelerations ($a < a_0$), so that

$$\mathbf{F} = m\mathbf{a}\mu\left(\frac{a}{a_0}\right), \quad (1)$$

where $\mu(x) = x$ when $x \ll 1$, and $\mu(x) = 1$ when $x \gg 1$ [9, 10]. It is easy to see that this leads to the following limiting velocity of a body orbiting a point mass M :

$$v^4 = GMa_0. \quad (2)$$

In other words, for sufficiently low values of acceleration, this theory predicts flat rotation curves (which are formally infinite in extent). Surprisingly, the value needed to explain observations is $a_0 \sim 10^{-8}$ cm/s², which is of the same order as cH_0 ! The increasing success rate of MOND in explaining observational data has led to an appreciable growth in the number of recent research publications on this subject. Clearly of interest would be tests which might simulate MOND-like conditions in the controllable environment of a laboratory. One such space experiment will be discussed later in this paper.

We end this introduction by noting a strange coincidence: the MOND acceleration a_0 is tantalizingly close to that associated with the Pioneer anomaly [16]. This is the anomalous acceleration experienced by the spacecraft Pioneer 10 and 11, which was noticed in 1980 after Pioneer 10 had passed a distance of ~ 20 astronomical units from the Sun. (Pioneer 10 has now left the solar system.) The acceleration is directed towards the Sun and has the value [16] $(8.60 \pm 1.34) \times 10^{-8}$ cm/s². Efforts to find a conventional explanation for this effect in terms of spacecraft design, leakage, and the influence of the solar wind have so far proved elusive and proposals have been made, both by NASA and the European Space Agency, for a dedicated space mission to probe the Pioneer anomaly [17, 18].

II. EXTRA DIMENSIONS

The possibility that space could have more than three dimensions was originally suggested in the seminal works of Kaluza (1921) and Klein (1926), who demonstrated that a compact (circle-like) fifth dimension would unify gravity with the electromagnetic force [19]. The Kaluza–Klein program was pursued with great enthusiasm during the 1980’s, the main objective then being the recovery of gauge fields and symmetries of the standard model from compact (hidden) dimensions. However, since the size of the extra dimensions was

close to the Planck scale, $\mathcal{R} \sim \ell_P \simeq 10^{-33}$ cm, direct observational evidence of these dimensions was virtually impossible, and extra dimensions in such theories were kept well hidden. A paradigm shift in our perception of a multi-dimensional cosmos occurred when it was suggested that extra dimensions, though compact, may be much larger than the Planck size [20], and even macroscopic [21], $\mathcal{R} \lesssim 1$ mm. The rationale for such an approach was the long-standing hierarchy problem in physics which arises because the Planck scale is so much higher than other mass scales in particle physics (for instance $M_P/M_W \sim 10^{17}$, where M_W is the mass of the vector bosons which mediate the weak force). Within this new higher-dimensional framework, the fundamental scale of gravity can be much lower than M_P , and a simple example shows how this can be achieved.

Consider two test masses m_1 and m_2 separated by a distance $r \ll \mathcal{R}$ in a $(4+n)$ -dimensional universe and interacting via the gravitational potential²

$$V(r) \sim \frac{m_1 m_2}{M^{n+2}} \frac{1}{r^{n+1}}, \quad r \ll \mathcal{R}, \quad (3)$$

where M is the $(4+n)$ -dimensional Planck mass. If the same two particles are placed much further apart, then, because the gravitational field lines associated with m_1 and m_2 do not have room to propagate in the extra dimensions at such large distances, the potential at large separations becomes

$$V(r) \sim \frac{m_1 m_2}{M^{n+2} \mathcal{R}^n} \frac{1}{r} \sim \frac{m_1 m_2}{M_P^2} \frac{1}{r}, \quad r \gg \mathcal{R}. \quad (4)$$

From (3) & (4) one finds that the effective four-dimensional Planck mass is simply given by $M_P^2 \sim M^{n+2} \mathcal{R}^n$. From (3) we find that gravity becomes higher-dimensional on length scales smaller than \mathcal{R} ; substituting $\mathcal{R} \sim 1$ mm and $n = 2$, one gets $M \sim 1$ TeV. The theory of macroscopic compact extra dimensions can be tested by traditional ‘table top’ experiments which probe the inverse-square law on scales down to ~ 0.1 mm [23]. Other tests include sufficiently energetic collisions on the LHC, NLC etc. [21].

Of course, the scheme described above could not be realized in the original Kaluza–Klein approach since the presence of extra dimension of millimeter size would be well observable in ordinary, non-gravitational, physics. Therefore, an interesting alternative approach adopted in [21] was connected with the braneworld concept. In this scenario, our $(3+1)$ -dimensional universe is thought to be a brane (from the word ‘membrane’) embedded in

² The discussion here closely follows that in [21, 22].

a higher-dimensional ‘bulk’ space time with large extra spatial dimensions. The fields of Standard Model are confined to move along the brane and, therefore, do not ‘feel’ the presence of extra dimensions, whereas gravity can propagate in the bulk and have the properties described above.³

The next fruitful idea was to consider bulk spaces with *infinite* noncompact extra dimensions [24, 25, 26, 27]. In this paradigm, gravity on the brane remains four-dimensional on sufficiently large scales due to the effect of curvature of the bulk space. A seminal model of this kind was put forward by Randall and Sundrum [24]. It has one infinite extra dimension, and the space-time metric in this model has the form⁴

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 . \quad (5)$$

Writing the full gravitational potential between two point masses on the brane in this model as $V(r) = V_0(r) [1 + \Delta(r)]$, one can calculate the series expansion for the correction term $\Delta(r)$ in the form [28]

$$kr \ll 1 : \quad \Delta = \frac{4}{3\pi kr} - \frac{1}{3} - \frac{1}{2\pi} kr \ln kr + 0.089237810 kr + \mathcal{O} [(kr)^2] , \quad (6)$$

$$kr \gg 1 : \quad \Delta = \frac{2}{3(kr)^2} - \frac{4 \ln kr}{(kr)^4} + \frac{16 - 12 \ln 2}{3(kr)^4} + \mathcal{O} \left[\frac{(\ln kr)^2}{(kr)^6} \right]. \quad (7)$$

Thus, on length scales r smaller than the curvature radius k^{-1} of the fifth dimension, gravity becomes five-dimensional, and the gravitational potential changes from its familiar four-dimensional form $V(r) \propto 1/r$ to the five-dimensional $V(r) \propto 1/r^2$. On scales larger than the curvature radius, the leading correction to the potential is $V(r) \propto 1/r^3$.

Considerable support for the multi-dimensional viewpoint comes from string and M-theory, in which extra dimensions play a crucial role in the unification of all forces at a fundamental level. In fact, it was the famous supergravity model of Hořava and Witten [29], representing two ten-dimensional branes connected by large eleventh dimension, that inspired the model by Randall and Sundrum [24].

It is interesting to note that, while higher-dimensional theories generically predict departures from the inverse-square law on small scales [30, 31], such departures can also arise in

³ In an earlier proposal of this kind [20], the matter fields of the Standard Model were localized at the orbifold fixed points while the gauge fields propagated also in extra dimensions of TeV size.

⁴ In the Randall–Sundrum model the small value of the true five-dimensional Planck mass is related to its large effective four-dimensional value by the extremely strong warp of the five-dimensional space.

other theories. For instance, models which modify the general-relativistic Einstein–Hilbert action can lead to MOND-type effects at low accelerations and/or departures from Newton’s law on small scales (see, for instance, [32]).

It is important to add that extra-dimensional models as well as four-dimensional models which modify the general-relativistic Einstein–Hilbert action, have several important cosmological properties. Amongst these is the attractive possibility of explaining cosmic acceleration without dark energy and of describing both early and late-time acceleration within a single unified setting (see [33, 34] and references therein).

Having briefly discussed the theoretical motivation for expecting departures from Newton’s laws on small scales and/or at low accelerations, we now outline a space experiment which attempts to detect these new phenomena.

III. AN ARTIFICIAL PLANETARY SYSTEM IN SPACE (APSYS)

A. The perihelion shift

As briefly discussed above, several alternative theories of gravity predict a force law which differs from Newton’s inverse-square law on small scales. This departure usually occurs in one of two ways:

1. Due to the presence of a Yukawa-like force on small scales

$$V(r) = -\frac{G_\infty m_1 m_2}{r} [1 \pm \alpha \exp(-r/r_0)] \quad (8)$$

which falls off exponentially with increasing distance from the source. Here, G_∞ is the value of Newton’s gravitational constant (formally) measured at infinity.

2. A power-law modification with a more gradual fall-off with distance is usually associated with higher-dimensional cosmological models:

$$U(r) = -\frac{Gm_1m_2}{r} \left[1 \pm \left(\frac{r_0}{r} \right)^n \right]. \quad (9)$$

Both (8) and (9) predict a perihelion shift in the orbit of two bodies which shall be the focus of our discussion in the present section. For simplicity, we shall restrict our attention to (9), although the entire discussion carries over to (8) quite simply.

Our experiment proposes to test Newtonian laws of motion by means of two (or more) test bodies which freely fall within the drag-free environment of a spin-axis stabilized spacecraft. This spacecraft could be placed either in a geosynchronous orbit or at the L2 Lagrange point of the Earth–Sun system (where WMAP is currently deployed). As we proceed to show, such a deployment of an artificial small-scale planetary system in free fall allows us to probe very small relative accelerations and test Newton’s law on sub-millimeter scales. A significant advantage of our set-up vis a vis terrestrial experiments is its conceptual simplicity and its relative freedom from sources of contamination. (For instance, Casimir forces [35], which can contribute significantly to the signal on sub-millimeter scales in Cavendish-type experiments and are difficult to model and subtract, are wholly absent in our case.)

To illustrate the basic physical principle behind our experiment, we consider two masses m_1 and m_2 freely moving in a drag-free environment and interacting with the potential

$$U(r) = -\frac{\alpha}{r} + U_{\text{mod}}(r) \quad (10)$$

where $\alpha = Gm_1m_2$, G being the Newton’s constant, and $U_{\text{mod}}(r)$ is the new term which modifies the inverse-square law. We will consider potentials of the type

$$U(r) = -\frac{\alpha}{r} \left[1 \pm \left(\frac{r_0}{r} \right)^n \right], \quad r \gg r_0, \quad n \geq 1 \quad (11)$$

where r_0 is the relevant scale below which gravity becomes essentially non-Newtonian, and ‘ \pm ’ determines the attractive or repulsive character of the additional potential. The power n will depend on a particular model; for instance, $n = 2$ in the five-dimensional Randall–Sundrum model [24].

Let us now calculate the perihelion shift in the orbit of these two masses. Assuming the correction to the potential to be small, we have [36]

$$\delta\phi = \frac{\partial}{\partial \mathcal{M}} \left(\frac{2m}{\mathcal{M}} \int_0^\pi r^2 U_{\text{mod}}(r) d\phi \right), \quad (12)$$

where

$$m = \frac{m_1 m_2}{m_1 + m_2}, \quad (13)$$

is the reduced mass, \mathcal{M} is the angular momentum, and the integral is taken over the unperturbed orbit

$$r = \frac{p}{1 + e \cos \phi}, \quad p = \frac{\mathcal{M}^2}{m\alpha}, \quad e = \sqrt{1 + \frac{2E\mathcal{M}^2}{m\alpha^2}}, \quad (14)$$

where e is the eccentricity, and $E < 0$ is the total energy. The partial derivative with respect to \mathcal{M} in (12) is calculated under the assumption of constant E .

In general, the integral in (12) can be evaluated numerically. But in the simplifying case $e \ll 1$, the result can also be obtained analytically as follows:

$$\int_0^\pi r^2 U_{\text{mod}}(r) d\phi = \mp \alpha r_0^n \int_0^\pi \frac{d\phi}{r^{n-1}} = \mp \alpha r_0^n p^{1-n} \int_0^\pi (1 + e \cos \phi)^{n-1} d\phi \approx \mp \pi \alpha r_0^n p^{1-n}, \quad (15)$$

where the approximation assumes $e = 0$, and is exact in the case $n = 2$. Substituting (15) into (12), using (14), and taking into account the relation

$$\mathcal{M} = \sqrt{\alpha m r}, \quad (16)$$

valid for an (almost) circular orbit, we obtain the contribution to perihelion shift from $U_{\text{mod}}(r)$:

$$\delta\phi = \pm 2\pi(2n - 1) \left(\frac{r_0}{r}\right)^n. \quad (17)$$

Notice that the perihelion shift depends only upon the new parameter n (which is related to the particular theory of modified gravity) and upon the ratio r_0/r , where r is the radius of the orbit and r_0 the length scale below which departures from the $1/r^2$ force-law occur. From (17) we notice that in order to make $\delta\phi$ large we should (for a fixed n) try and make the radius of the orbit as small as technologically feasible.

A small radius of the orbit is also advantageous from another perspective. Recall that the orbital period of a test body around a more massive central mass M is

$$\tau = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}}, \quad (18)$$

r being the radius of the (almost) circular orbit. Relative to Earth's motion around the Sun, we have

$$\frac{\tau_{\text{orbit}}}{\tau_{\text{Earth orbit}}} = \left(\frac{r}{1 \text{ au}}\right)^{3/2} \left(\frac{M_\odot}{M}\right)^{1/2} = 0.02428 \times \left(\frac{r}{1 \text{ m}}\right)^{3/2} \left(\frac{1 \text{ kg}}{M}\right)^{1/2}, \quad (19)$$

where $1 \text{ au} = 1.5 \times 10^{11} \text{ m}$ is the astronomical unit, and $M_\odot = 1.99 \times 10^{30} \text{ kg}$ is the Solar mass. The mass of a tungsten sphere ($\rho = 19.6 \text{ g/cm}^3$) of four-centimeter radius is $M = 5254 \text{ g}$; hence, if our test body orbits it at a distance of $r = 10 \text{ cm}$, we get

$$\frac{\tau_{\text{Earth orbit}}}{\tau_{\text{orbit}}} = 2985, \quad (20)$$

i.e., our test mass will make almost 3000 revolutions per year — a significant number!

Since the perihelion shift is an additive quantity, the *total* perihelion shift in our space-based two-body system in a single year becomes

$$\delta\phi_{\text{one year}} = \mathcal{N}\delta\phi, \quad (21)$$

where $\mathcal{N} = \tau_{\text{Earth orbit}}/\tau_{\text{orbit}}$. Substituting from (17) and (19), we find

$$\delta\phi_{\text{one year}} = \pm 2\pi(2n - 1) \left(\frac{1 \text{ au}}{r}\right)^{3/2} \left(\frac{M}{M_{\odot}}\right)^{1/2} \left(\frac{r_0}{r}\right)^n, \quad (22)$$

Since

$$\delta\phi_{\text{one year}} \propto \left(\frac{M}{r^{3+2n}}\right)^{1/2}, \quad (23)$$

a larger perihelion shift is obtained by: (i) making the central mass as large as possible and (ii) by simultaneously shrinking the radius of the orbit. (Of course, in practice, (i) and (ii) work in opposite directions so a judicial consideration needs to be applied to make an optimum choice.)

Consider next the extra-dimensional scenario due to Randall and Sundrum [24], in which Newton's law becomes five-dimensional on sub-micron scales and is modified by the correction $U_{\text{mod}} \propto 1/r^3$ on scales larger than a micron. Substituting $r_0 = 10^{-4}$ cm and $n = 2$ in (22), we obtain

$$\delta\phi_{\text{one year}} \simeq 1 \text{ arc sec}. \quad (24)$$

We therefore find that if Newton's law becomes five dimensional on sub-micron scales, then the cumulative perihelion shift in our space-based two-body system is about an arc second per year, which is a reasonable quantity to try and measure.

It is interesting to note that the accelerations experienced by our miniature planetary system are typically very small. For instance, a test body at a distance of one meter from the central mass M will experience an acceleration of only 3×10^{-8} cm/s², which is close to the MOND value! So, departures from the Newtonian inertia law should be verifiable for a MOND-type theory using our space-borne planetary system. In this case, one places two (or more) bodies in orbit at different radii around M . Clearly, the closer body will probe departures from the inverse-square law whereas the more distant ones will probe MOND.⁵

⁵ Of central importance here is the question of whether MOND works in regions of small relative or absolute accelerations. If the latter is the case, then the much larger acceleration (relative to the MOND value)

B. Spacecraft Design — some preliminary ideas

The following points need to be noted in connection with the design of our space experiment:

1. The payload will include, in addition to the test masses, a tracking camera which will monitor the motion of the miniature planetary system floating within the spacecraft.⁶
2. Since the masses involved are to be in free fall, it is essential that all non-gravitational forces are minimized. For this purpose, the spacecraft will play the role of a Faraday cage and screen the experiment from any external electric field as well as cosmic rays. In addition, any residual gas present within the spacecraft can easily be released by means of a small opening.
3. Since it is essential that the test bodies be allowed to execute their motion in a drag-free environment, one must account for all non-gravitational forces which could act on the spacecraft. One of the main perturbations in outer space is the solar radiation pressure. If the spacecraft is placed in a geosynchronous orbit then, for an area-to-mass ratio of $0.26 \text{ cm}^2/\text{g}$, the perturbing acceleration generated by the solar wind is $\sim 10^{-5} \text{ cm/s}^2$, which is somewhat larger than the relative acceleration between the test masses in our experiment. Therefore our drag-free spacecraft should be designed to compensate for this non-gravitational acceleration perhaps by activating jets which ensure that the spacecraft follows the free-fall motion of the miniature planetary system. The influence of solar wind can also be minimized by placing the spacecraft near the L2 Lagrange point which is shielded from the Sun by the Earth's shadow.

Also note that it is possible for charged particles (mostly protons) to penetrate the spacecraft thereby creating strong electrostatic fields which could disturb the experiment. Of the three sources of such contamination: cosmic rays, solar flares and the

experienced by the spacecraft due to its vicinity to the Sun will make extra-terrestrial tests of MOND quite useless. On the other hand, if the local acceleration holds the key to the MOND phenomenon, then experiments like the one proposed in this paper should be able to test this hypothesis. (See [37] for recent attempts to provide a theoretical foundation to MOND and [10] for an excellent review.)

⁶ Our experimental setup has been adapted from an earlier suggestion for a space experiment [38, 39], in which the idea of a space-based mini-planetary system was advocated with the purpose of obtaining accurate measurements of Newton's gravitational constant G .

Van Allen belts, the latter are the most dangerous [39]. However because the belts are associated with the Earth, one can avoid this effect by having the spacecraft at the L2 point instead of in a geosynchronous orbit. This could also decrease the solar flare component, although for complete safety, the experiment should be ‘switched off’ during (rare) periods of intense solar activity.

4. An important non-rigid perturbation source is the fuel whose gravity gradient effects on the experiment need to be modelled. Clearly, in order to minimize ‘force-contamination’, the fuel tank must be placed as far away from the experiment as possible. Keeping in mind that the radius of the experiment is < 1 m, we suggest (following Nobili *et al.* [38]) that the spacecraft be cylindrical: 3.5 meters in height with a base diameter of 3 meters and a mass of 400 kg. Furthermore, the spin axis of the spacecraft is stabilised with a spin period of 60 seconds.⁷ It may be noted that, in order for an experiment to last an interval of time ΔT , the mass of liquid propellant fuel required is [38]

$$\left(\frac{\Delta M}{1 \text{ kg}}\right) = 5.2 \times 10^{-3} \left(\frac{\Delta T}{1 \text{ day}}\right). \quad (25)$$

Since our experiment is likely to be operational for a two-year duration (the longer the better!), its fuel requirement is $\Delta M \simeq 4$ kg.

Also note that tidal effects on the spacecraft due to the gravitational field of the environment could be a potential source of perihelion shift of our planetary system and need to be properly understood and incorporated into the analysis.

5. Some departure from spherical symmetry is likely to occur for the ‘planets’ due to purely technological reasons. This effect needs to be included in the analysis perhaps by expanding the gravitational field in spherical harmonics — a standard exercise in satellite geodesy.
6. A mass-release mechanism which will release the ‘planets’ and place them in an elliptical orbit is essential. Since the spacecraft is spinning much more rapidly than the orbital period of the planets, the mechanism which effects mass-release must do so

⁷ Spinning the spacecraft is important because it averages forces which are body-based in the spin plane which is also the orbital plane of the planets. Consequently only the zonal harmonics of the gravitational field of the spacecraft are relevant in this case [38, 40].

gently, so that planets are injected with a relative velocity which is smaller than their escape velocity (with respect to each other). Some ideas for this have been put forth in [38] and the reader is referred to these papers for more details. Proper account must also be taken of the planets' spin in order to preclude the occurrence of a spin-orbital resonance. A central role in this experiment will be an accurate measurement of the semi-major axis required for determining the perihelion shift.

7. Even with an excellent mass-release mechanism, it is still possible that the two planets will not be placed into the proper orbital plane. (To ensure proper tracking it is essential that the inclination of the orbital plane be close to the equatorial plane of the spacecraft which will also be the camera's focal plane.) In this case, orbit correction devices, such as light sources capable of exerting a gentle pressure on the 'planets', need to be incorporated into the design of the spacecraft. (An acceleration of $\sim 10^{-8}$ cm/s² can be induced on our 'planet' by means of a 10 W source of light concentrated in a 10 degree cone. So one might expect that intermittent bursts of light could be used to gently change the inclination of the orbit.)
8. Note that our experimental set-up requires a prior determination of M which can easily be carried out in a laboratory on Earth. Thus all essential experimental inputs can be determined to high accuracy terrestrially while the motion of our miniature planetary system is easy to predict theoretically and can therefore be compared with observational measurements made within the spacecraft.
9. Finally, note that at least some of the technology required for APSIS is already being developed in connection with the LISA Pathfinder (LP) mission, which envisages two test objects in gravitational free fall in a drag-free environment, and in the Gravity Probe B (GP-B) gyroscope experiment. The LP test objects are shielded from non-gravitational forces and will be discharged at regular intervals using fibre-coupled UV lamps [41]. A gentle release mechanism for the test masses is crucial for the LISA Pathfinder, in which one hopes to achieve a release speed of less than 5 μ m/s (18 mm per hour) and thereafter measure the position of the test masses with respect to the spacecraft (or each other) to an accuracy of 10^{-12} m. The designers of the GP-B experiment have achieved sphericity of the gyroscope spheres (made of homogeneous

fused quartz of 3.81 centimeter in diameter) less than 40 atomic layers from perfect.⁸

IV. CONCLUSIONS

It is well known that, within Newtonian mechanics, closed particle orbits occur only for two types of central force fields [36], namely, the inverse-square law $F \propto r^{-2}$ and the harmonic oscillator $F \propto r$. A force law which deviates on small scales from $1/r^2$ is therefore expected to give rise to particle orbits which show small perihelion shifts. This effect is additive in nature since the perihelion shift for successive orbits is added to give the *total* shift in the major (minor) axis of a planet within a stipulated time scale (say, a year). Since smaller orbits also have shorter periods of rotation, it follows that the net magnitude of this effect is larger for planets with smaller orbits. Keeping this in mind, we are of the view that a miniature planetary system placed in free fall within a spacecraft located at the L2 Lagrange point of the Earth–Sun system, provides an ideal testing ground for possible departures of Newtonian gravity from the familiar inverse-square law. Since many higher-dimensional cosmological models (and several models of modified gravity) predict such departures on small scales, our Artificial Planetary System In Space (APSIS) could provide a space laboratory with which to test such theories. The small accelerations prevalent in such systems ($\sim 10^{-8}$ cm/s²) may also be useful for probing the MOND hypothesis. In our experimental setup, one (or more) test bodies (planets) orbit a more massive central ~ 5 kg mass at a distance of ~ 10 cm. The perihelion shift for this configuration is ~ 1 arc second/year if the spatial scale for the modification of gravity law is of the order of a micron ($1 \mu\text{m}$).

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⁸ See <http://einstein.stanford.edu>

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