

Study of the Magnitude-Redshift Relation for type Ia Supernovae in a Model resulting from a Ricci-Symmetry

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Abstract:

Models with a dynamic cosmological term $\Lambda(t)$ are becoming popular as they solve the cosmological constant problem in a natural way. Instead of considering any ad-hoc assumption for the variation of Λ , we consider a particular symmetry, the contracted Ricci-collineation along the fluid flow, in Einstein's theory. We show that apart from having interesting properties, this symmetry does demand Λ to be a function of the scale factor of the Robertson-Walker metric. In order to test the consistency of the resulting model with observations, we study the magnitude-redshift relation for the type Ia supernovae data from Perlmutter et al. The data fit the model very well and require a positive non-zero Λ and a negative deceleration parameter. The best-fitting flat model is obtained as $\Omega_0 \approx 0.5$ with $q_0 \approx -0.2$.

KEY WORDS: Ricci symmetries, variable cosmological term, R-W models.

1. INTRODUCTION

The recent measurements of the CMB anisotropy [1] and the observations of type Ia supernovae made, independently, by Perlmutter et al [2] and Riess et al [3] consistently demand a significant and positive cosmological constant Λ . These observations suggest Friedmann models with negative pressure-matter in which the expansion of the universe is accelerating. Observations of gravitational lensing [4] also indicate the presence of a non-zero Λ .

On the other hand, a dynamical $\Lambda(t)$ has been considered in numerous papers in order to explain its observed small value which is about 120 orders

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of magnitude below the value for the vacuum energy density predicted by quantum field theory- the so-called cosmological constant problem [5]. This phenomenological solution is based on the argument that Λ relaxed to its present estimate due to the expansion of the universe. (It is customary to associate a positive cosmological constant Λ with a vacuum energy density $\rho_v \equiv \Lambda/8\pi G$.)

As the dynamics of the variable Λ -models depends sensitively on the chosen dynamic law for the variation of Λ and, in general, becomes altogether different from the dynamics of the corresponding constant Λ -models, there is no reason to believe that the observations of distant objects would also agree with the variable Λ -models, given that they agree with the corresponding constant Λ -ones, especially for the same estimates of the parameters. In this view, it would be worth while to test the consistency of the above-mentioned observations with the variable Λ -models which solve the cosmological constant problem in a natural way. It may be mentioned that the quintessence or the exotic 'x-fluid' models ($-1 < p_x/\rho_x < 0$) are also capable to explain, in a natural way, the smallness of the present vacuum energy density, since the associated vacuum energy density dynamically evolves towards zero with the expansion of the universe. However, as Garnavich et al [6] have shown, if one attempts to constrain the equation of state of the dark energy component (that may have contributed to accelerating the cosmic expansion), with the Ia supernova data, one finds that the data is consistent with either a cosmological constant, or a scalar field that has, on average, an equation-of-state parameter similar to the one for the cosmological constant, i.e., -1. However, we note that $p_x/\rho_x = -1$ corresponds to the cosmological Λ which, in view of the conservation of the energy momentum tensor of the exotic component, becomes a true constant and, hence, cannot solve the cosmological constant problem. However, as we shall see later, the usual kinematical Λ can always be, in general, a function of time as the conserved quantity in this case is matter plus vacuum, and not the vacuum only [7].

We note that a number of models with Λ as a function of time have been presented in recent years [8, 10-13]. Different phenomenological laws, for the decay of Λ , have been proposed in these models which are either from dimensional arguments or from ad-hoc assumptions. Though the precise mechanism of vacuum-decay, which should come from the fundamental interactions, is not yet known, it would, however, be worth while to look for some symmetry principles which actually demand the variation of Λ . More-

over, it is always reasonable to consider symmetry properties of spacetime rather than considering ad-hoc assumptions for the variation of Λ .

In this paper, we consider, on the level of classical general relativity, a particular Ricci-symmetry which is the contracted Ricci-collineation along the fluid flow vector and show that, apart from having interesting properties, this symmetry does demand Λ to be a function of time (and space, in general). This is done in section 2. In section 3, we derive the magnitude (m) - redshift (z) relation in the resulting model to test its consistency with the Ia supernovae data from Perlmutter et al [2]. The model fits the data very well. The numerical results are discussed in section 4 followed by a concluding section.

Earlier some cosmological models with this symmetry were discussed by considering particular sets of initial and boundary conditions and by assuming that $\Lambda = \Lambda(t)$ in some of them [8-10]. However, as we shall see in section 2, the incorporation of this particular symmetry in the Einstein theory does demand Λ to be a variable and there is no need to assume $\Lambda = \Lambda(t)$ a priori. For the ready reference and completeness, we reproduce the model in the following.

2. SYMMETRY CONSIDERATIONS AND THE RESULTING MODEL

We begin our discussion by considering the contracted Ricci-collineation. The motivation for considering this particular symmetry will be discussed later. We note that a spacetime is said to admit a Ricci-collineation along a field vector η^i if [14]

$$\mathcal{L}_\eta R_{ij} = 0, \tag{1}$$

where \mathcal{L}_η denotes the Lie-derivative along η_i . Further, a spacetime is said to admit a family of contracted Ricci-collineation if

$$g^{ij} \mathcal{L}_\eta R_{ij} = 0, \tag{2}$$

which leads to the conservation law generator²

$$\left[T_m^j \eta^m + \left(\frac{\Lambda}{8\pi G} - \frac{1}{2} T \right) \eta^j \right]_{;j} = 0, \quad (3)$$

if the Einstein field equations

$$R^{ij} - \frac{1}{2} R g^{ij} = -8\pi G \left(T^{ij} - \frac{\Lambda}{8\pi G} g^{ij} \right) \quad (4)$$

are satisfied. We consider units with $c = 1$. Recalling that the energy density associated with vacuum is $\rho_v = \Lambda/8\pi G$ with its pressure $p_v = -\rho_v$ implying $T_v^{ij} \equiv -(\Lambda/8\pi G)g^{ij}$, the quantity appearing in the parentheses on the right hand side of equation (4) can be written as

$$T_{\text{tot}}^{ij} \equiv T^{ij} + T_v^{ij} = (\rho_{\text{tot}} + p_{\text{tot}})u^i u^j + p_{\text{tot}} g^{ij}, \quad (5)$$

which represents the energy momentum tensor of the total matter, i.e., ordinary matter plus vacuum. Here $\rho_{\text{tot}} = \rho + \rho_v$, $p_{\text{tot}} = p + p_v = p - \rho_v$ and u^i is the normalized velocity 4-vector. The Bianchi identities then confirm the conservation of T_{tot}^{ij} and not the conservation of T^{ij} and T_v^{ij} separately unless $\Lambda = \text{constant}$.

If we consider $\eta^i \propto u^i$, equation (3) reduces to

$$\{(\rho_{\text{tot}} + 3p_{\text{tot}})u^j\}_{;j} = 0, \quad (6)$$

which may be interpreted as the conservation of generalized *momentum density*. This is an important result in its own right, but it implies even more. To understand the full meaning of this conservation law and compare it with the existing results, let us consider the Robertson-Walker metric

$$ds^2 = -dt^2 + S^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right\}, \quad (7)$$

characterized by the scale factor $S(t)$ and the curvature index $k \in \{-1, 0, 1\}$ of the spatial hypersurfaces $t = \text{constant}$. Now the Einstein field equations (4) yield two independent equations:

²Due to a typographical error in [8], the same definition of the conservation law generator for a contracted Ricci-collineation was also assigned to the Ricci-collineation. But this did not affect the results of the paper as the Ricci-collineation was never used in the paper.

the Raychaudhuri equation:

$$-\frac{\ddot{S}}{S} = \frac{4\pi G}{3}(\rho_{\text{tot}} + 3p_{\text{tot}}) \quad (8)$$

and the Friedmann equation:

$$\frac{\dot{S}^2}{S^2} + \frac{k}{S^2} = \frac{8\pi G}{3}\rho_{\text{tot}}. \quad (9)$$

By using (7), the conservation law (6) reduces to

$$(\rho_{\text{tot}} + 3p_{\text{tot}})S^3 = \text{constant} = A \quad (\text{say}), \quad (10)$$

which is the central equation of our analysis and may be interpreted as the conservation of *total active gravitational mass*, taken matter and vacuum together, of a comoving sphere of radius S . Note that $(\rho + 3p)$ is defined as the *active gravitational mass density* of the universe [15]. Obviously the conservation law (10) reduces to the pressure-less phase of the standard big bang (FLRW) model for $p = \Lambda = 0$.

To understand the presence of pressure and vacuum terms in equation (10), we consider the Raychaudhuri equation (8) which, being the analogue of Newtonian gravitation, suggests that the gravitational attraction, experienced by a unit test mass, is exerted in fact not only by ρ as in the Newtonian theory but rather by the *active gravitational mass density* $(\rho + 3p)$, which exhibits the relativistic effects. Associated with the attractive force $G\mathcal{A}/S^2$ ($\mathcal{A} \equiv \frac{4\pi}{3}(\rho + 3p)S^3$ being the *active gravitational mass* from the gravitating matter) in equation (8), there is a repulsive force $-\Lambda S/3$ due to a positive Λ (or an additional attraction with a negative Λ). It is this repulsive force which drives inflationary expansion in the early vacuum-dominated universe. It is obvious that the constant A (the *total active gravitational mass*) might take different values in different phases of evolution depending upon the relative dominance of these two terms of opposite character in equation (8). This implies that the *total active gravitational mass* might be conserved phase-wise only and not throughout the evolution. (For details, see [8]).

One may also note that the left hand side of equation (8) is the Gaussian curvature of the two-dimensional surface specified by varying r and t , keeping θ and ϕ constant, in equation (7) and may be considered as the curvature of the homogeneous and isotropic spacetime. Thus equation (8) implies that

the curvature of spacetime is governed by the total *active gravitational mass density* of the universe. In this view, a naive assumption would be that the cause of curvature of spacetime, i.e., the *total active gravitational mass* be conserved, justifying our symmetry consideration leading to the conservation law (10). Equation (10) also implies, via equation (8), that the curvature of spacetime evolves as S^{-3} , which quickly transforms the spacetime from a state of large curvature to a state of flatness.

It would be worth while to mention that if one wishes to describe the radiation-dominated universe in the Newtonian framework (extending the work of McCrea and Milne), one needs to use $(\rho + 3p)$ instead of ρ only in Poisson's equation to get the right answer. This is consistent with our using $(\rho + 3p)$ for gravitational mass density.

Equations (8) and (10), taken together, yield

$$-\ddot{S} = \frac{4\pi GA}{3S^2}, \quad (11)$$

which integrates to

$$\dot{S}^2 = \frac{8\pi GA}{3S} + B, \quad (12)$$

where B is a constant of integration. This supplies the dynamics of the scale factor. Equations (9), (10) and (12) may be used to obtain

$$\rho_{\text{tot}} = \frac{A}{S^3} + \frac{3(B+k)}{8\pi GS^2}, \quad (13)$$

$$p_{\text{tot}} = -\frac{(B+k)}{8\pi GS^2}, \quad (14)$$

which give the equation of state of the *perfect fluid* constituting the total matter of the universe as

$$p_{\text{tot}}^3 = K(\rho_{\text{tot}} + 3p_{\text{tot}})^2, \quad (15)$$

where $K = -(B+k)^3/(8\pi G)^3 A^2$. This is a physically reasonable equation of state since $dp_{\text{tot}}/d\rho_{\text{tot}} = 2p_{\text{tot}}/3(\rho_{\text{tot}} + p_{\text{tot}})$ indicating that $dp_{\text{tot}}/d\rho_{\text{tot}} \leq 1/3$ for $\rho_{\text{tot}} \geq p_{\text{tot}}$ (provided $(\rho_{\text{tot}} + p_{\text{tot}}) > 0$). It may be noted that the equation of state (15) breaks down for $p_{\text{tot}} = 0$ (in the same way as does the usual barotropic equation of state for a perfect fluid for $p = 0$), in which case

equations (13) and (14) may be treated as the parametric equations of state. Consequences of the resulting models for the case $\Lambda = 0$ have been discussed elsewhere by Abdussattar and Vishwakarma [9] where the models obviously get constrained by $A \geq 0$ and $(B + k) \leq 0$ and by the additional constraint $S \leq 8\pi GA/3 |B + k|$ when $(B + k) < 0$.

When $\Lambda \neq 0$, the equation of state (15) does not supply information on the ordinary matter source. If the ordinary matter is baryonic with its usual barotropic equation of state

$$p = w\rho, \quad 0 \leq w \leq 1, \quad (16)$$

this simply implies that Λ cannot just remain constant. The reason is obvious. We now have 4 independent equations (12)-(14) and (16) in 3 unknowns S , ρ and p . This over-determinacy can be compensated by allowing at least one of the remaining parameters to vary. The only such parameter of interest is Λ if we keep G as a constant (the cases with variable G have been discussed elsewhere [10]). It is obvious that we would have reached the same conclusion, had we considered any other assumption in place of (2). However, this symmetry, as we have seen, has its own significance. Equations (13), (14) and (16) thus yield

$$\Lambda = \frac{8\pi G}{(1+w)} \left[\frac{wA}{S^3} + \frac{(1+3w)(B+k)}{8\pi GS^2} \right], \quad (17)$$

$$\rho = \frac{1}{(1+w)} \left[\frac{A}{S^3} + \frac{(B+k)}{4\pi GS^2} \right]. \quad (18)$$

It may be mentioned that Λ varying as S^{-2} , which the present model has in the pressure-less phase of evolution, has also been considered by several authors to explain the present small value of Λ [11, 12]. The ansatz is primarily due to Chen and Wu [11] who postulated it through dimensional arguments made in the spirit of quantum cosmology. Recently Jafarizadeh et al [16] have calculated the tunneling rate with a cosmological constant decaying as S^{-m} and concluded that the most probable cosmological term with the highest tunneling rate occurred at $m = 2$. However, the present model differs from the above-mentioned models in the sense that contrary to the $\Lambda \sim S^{-2}$ throughout the evolution (as has been assumed in these models), Λ varies differently in the different phases of evolution in the present model as is clear from equation (17).

While comparing the model with SN Ia data, we shall be interested in the effects which occurred at redshift < 1 . We, therefore, neglect radiation and consider $w = 0$ in the following. (The parameters in the early phase of evolution can be calculated by following [8]). With this in view, we recast equations (8) and (9) in the following forms to give the relative contributions of the different cosmological parameters at the present epoch:

$$2[q_0 + \Omega_{\Lambda 0}] = \Omega_0, \quad (19)$$

$$1 + \Omega_{k0} = \Omega_0 + \Omega_{\Lambda 0}, \quad (20)$$

where $\Omega \equiv 8\pi G\rho/3H^2$, $\Omega_k \equiv k/S^2H^2$ and $\Omega_\Lambda \equiv \Lambda/3H^2$ are, respectively, the dimensionless forms of the density, the curvature and the cosmological constant parameters. and the subscript 0 characterizes the value of the quantity at the present epoch. Equations (10) and (12) may be used to give the values of the constants A and B as

$$A = (\Omega_0 - 2\Omega_{\Lambda 0}) \frac{3S_0^3 H_0^2}{8\pi G}, \quad (21)$$

$$B = (2\Omega_{\Lambda 0} - \Omega_0 + 1) H_0^2 S_0^2. \quad (22)$$

3. MAGNITUDE-REDSHIFT RELATION IN THE MODEL

The cosmic distance measures, like the *luminosity distance* and the *angular size distance*, depend sensitively on the spatial curvature and the expansion dynamics of the models and consequently on the present densities of the various energy components and their equations of state. For this reason, the magnitude-redshift relation for distant *standard candles* and the angular size-redshift relation for distant *standard measuring rods* have been proposed as potential tests for cosmological models and play crucial role in determining cosmological parameters.

Let us consider that the observer at $r = 0$ and $t = t_0$ receives light emitted at $t = t_1$ from a source of absolute luminosity L located at a radial distance r_1 . The cosmological redshift z of the source is related with t_1 and t_0 by $1 + z = S(t_0)/S(t_1)$. If the (apparent) luminosity of the source measured by the observer is l , the *luminosity distance* d_L of the source, defined by

$$l \equiv \frac{L}{4\pi d_L^2}, \quad (23)$$

is then given by

$$d_L = (1+z)S_0 r_1. \quad (24)$$

For historical reasons, the observed and absolute luminosities l and L are defined, respectively, in terms of the K-corrected observed and absolute magnitudes m and M as $l = 10^{-2m/5} \times 2.52 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1}$ and $L = 10^{-2M/5} \times 3.02 \times 10^{35} \text{ erg s}^{-1}$ [17]. When written in terms of m and M , equation (23) yields

$$m(z; \mathcal{M}, \Omega_0, \Omega_{\Lambda 0}) = \mathcal{M} + 5 \log_{10} \{ \mathcal{D}_L(z; \Omega_0, \Omega_{\Lambda 0}) \}, \quad (25)$$

where $\mathcal{M} = M - 5 \log_{10} H_0 + 25$ and $\mathcal{D}_L(z; \Omega_0, \Omega_{\Lambda 0}) \equiv H_0 d_L(z; \Omega_0, \Omega_{\Lambda 0}, H_0)$ is the dimensionless *luminosity distance*. Here d_L is measured in Mpc. By using equation (7), the coordinate distance r_1 , appearing in equation (24), yields

$$\psi(r_1) = \int_{S_0/(1+z)}^{S_0} \frac{dS}{S\dot{S}} \quad (26)$$

with

$$\begin{aligned} \psi(r_1) &= \sin^{-1} r_1, & k &= 1 \\ &= r_1, & k &= 0 \\ &= \sinh^{-1} r_1, & k &= -1. \end{aligned} \quad (27)$$

By the use of (12), (21) and (22), equation (26) yields

$$\psi(r_1) = \frac{1}{S_0 H_0} \int_0^z [(2\Omega_{\Lambda 0} - \Omega_0 + 1)(1+z')^2 - (2\Omega_{\Lambda 0} - \Omega_0)(1+z')^3]^{-1/2} dz'. \quad (28)$$

Equations (24), (27) and (28) can also be combined into a single compact equation as

$$\mathcal{D}_L(z; \Omega_0, \Omega_{\Lambda 0}) = \frac{(1+z)}{\sqrt{\mathcal{K}}} \xi \left(\sqrt{\mathcal{K}} \int_0^z [(2\Omega_{\Lambda 0} - \Omega_0 + 1)(1+z')^2 - (2\Omega_{\Lambda 0} - \Omega_0)(1+z')^3]^{-1/2} dz' \right), \quad (29)$$

where

$$\begin{aligned} \xi(x) &= \sin(x) & \text{with } \mathcal{K} &= \Omega_{k0} & \text{when } \Omega_{k0} > 0, \\ \xi(x) &= \sinh(x) & \text{with } \mathcal{K} &= -\Omega_{k0} & \text{when } \Omega_{k0} < 0, \\ \xi(x) &= x & \text{with } \mathcal{K} &= 1 & \text{when } \Omega_{k0} = 0. \end{aligned}$$

Thus for given \mathcal{M} , Ω_0 and $\Omega_{\Lambda 0}$, equations (25) and (29) give the predicted value of $m(z)$ at a given z . By using the K-corrected effective magnitudes m_i^{eff} , which have also been corrected for the light-curve width-luminosity relation and galactic extinction, and using the same standard errors $\sigma_{z,i}$ and $\sigma_{m_i^{\text{eff}}}$ of the i th supernova with redshift z_i as used by Perlmutter et al, we compute χ^2 according to

$$\chi^2 = \sum_i \frac{[m_i^{\text{eff}} - m(z_i)]^2}{(\sigma_{z,i}^2 + \sigma_{m_i^{\text{eff}}}^2)}. \quad (30)$$

The best fit parameters are obtained by minimising this equation. We note that equation (29) is sensitive to Ω_0 and $\Omega_{\Lambda 0}$ for distant sources only. For the nearby sources (in the low-redshift limit), equations (25) and (29) reduce to

$$m(z) = \mathcal{M} + 5 \log_{10} z, \quad (31)$$

which can be used to measure \mathcal{M} by using low-redshift supernovae-measurements that are far enough into the Hubble flow so that their peculiar velocities do not contribute significantly to their redshifts.

4. NUMERICAL RESULTS

We consider the low-redshift data on m^{eff} and z from the Calan-Tololo sample of 16 supernovae (excluding 2 outliers from the full sample of 18 supernovae) to estimate \mathcal{M} from equations [30] and (31). This gives $\mathcal{M} = 24.03$ (in units with $c = 1$). This value is used in equation (25) to estimate Ω_0 and $\Omega_{\Lambda 0}$ from the high-redshift measurements. For this purpose we consider the data set on m^{eff} and z of 38 supernovae from the Supernova Cosmology

Project (excluding 2 outliers and 2 likely reddened ones from the full sample of 42 supernovae). We perform a two-parameter fit of this data set by following the fitting procedure of Perlmutter et al³ considered in their Fit M (see Fig. 5 (f) in their paper [2]).

We find that the data shows a very good fit to the model giving the best-fitting flat model ($\Omega + \Omega_\Lambda = 1$) as $\Omega_0 = 0.54$ with $\chi^2 = 45.34$, which shows that the fit is almost as good as to the constant Λ -flat FLRW model: $\Omega_0 = 0.40$ with $\chi^2 = 44.92$.

The minimisation process gives the global best-fitting solution (calculated by giving free rein to Ω_0 and $\Omega_{\Lambda 0}$) as $\Omega_0 = 1.76$ and $\Omega_{\Lambda 0} = 1.34$ with $\chi^2 = 44.78$ which shows a rather high value of Ω_0 and does not seem realistic in view of the small observed value of Ω_0 . One can see that the data predicts an accelerating expansion ($q_0 < 0$) of the universe as in the constant Λ -FLRW model.

The fit of the flat model to the actual data points has been shown in Figure 1, where we have compared it with the simplest $\Lambda = 0$ model, i.e., the Einstein-de Sitter model ($\Omega = 1$), which is ruled out by the data ($\chi^2 = 115.81$).

It may be mentioned that elsewhere [7], we have used this data in the model, with a different fitting procedure, by fitting the low- and high-redshift measurements simultaneously to equation (30). The best-fitting solutions so obtained are in good agreement with those obtained here.

It would also be worth while to mention that the present model is consistent not only with the supernovae-data but it also fits very well the data on the angular size and redshift of the ultracompact radio sources compiled by Jackson and Dodgson [19] as well as the updated and modified data on compact radio sources from Gurvits et al [7].

³ It may be mentioned that Perlmutter et al [2] have also fitted the data for only 3 parameters \mathcal{M} , Ω_0 and $\Omega_{\Lambda 0}$ and not for the 4 parameters α , \mathcal{M} , Ω_0 and $\Omega_{\Lambda 0}$ as mentioned in their paper (from a personal discussion with Professor R. S. Ellis, one of the authors of the paper). A self-consistent 4-parameter fit has been done by Efstathiou et al[18].

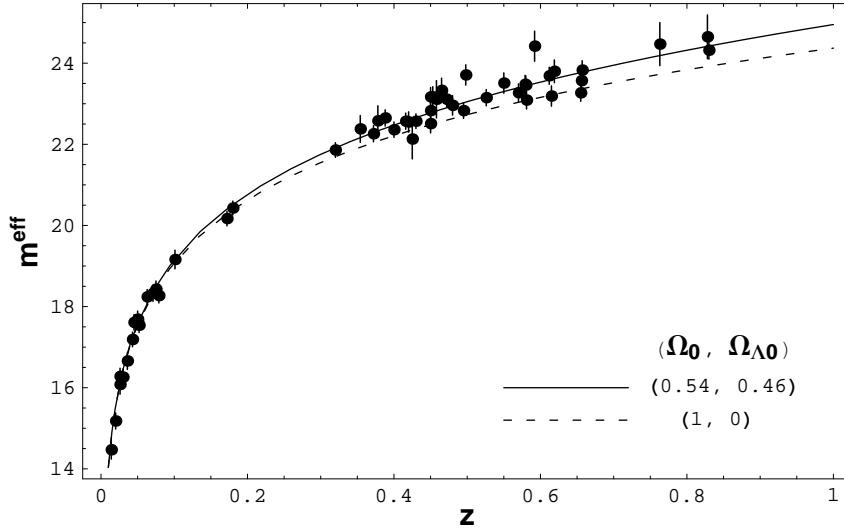


Figure 1. Hubble diagram for 38 high-redshift and 16 low-redshift supernovae: The solid curve represents the best-fitting flat model ($\Omega_0 = 0.54, \Omega_{\Lambda 0} = 0.46$). For comparison, the canonical Einstein-de Sitter model ($\Omega_k = 0, \Omega_{\Lambda} = 0$) has also been plotted (dashed curve).

4. CONCLUSION

In order to solve the cosmological constant problem, which has been made even more acute by the consequences of the current observations of CMB and type Ia supernovae, several authors have invoked a variable cosmological term $\Lambda(t)$. Instead of considering any ad-hoc assumption for the variation of Λ , as has been mainly done by the authors, we have considered a particular symmetry of the homogeneous, isotropic spacetime which is the contracted Ricci-collineation along the fluid flow. It has been shown that the incorporation of this additional symmetry into the RW model filled with baryonic matter renders the cosmological term Λ a decaying function of the scale factor. This helps in solving the cosmological constant problem. This new symmetry in the model leads to the conservation of the *total active gravitational mass* of the universe.

The resulting model fits the SN Ia data from Perlmutter et al very well and requires an accelerated expansion of the universe with a non-zero positive cosmological term. The goodness of fit of the data to the model is almost the same as in the case of the constant Λ -FLRW model. The estimates of the parameters for the best-fitting flat model are obtained as $\Omega_0 = 0.54$ and $\Omega_{\Lambda 0} = 0.46$. However, the global best-fitting solution, $\Omega_0 = 1.76$ with $\Omega_{\Lambda 0} = 1.34$, does not seem realistic (as is the case with the constant Λ -FLRW model; see reference [2]) in view of the small observed value of Ω_0 . It is also noted that the estimates of the density parameter Ω_0 for the present variable Λ -model are found a bit higher than those for the constant Λ -FLRW model.

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