

# Galactic dynamo action in presence of stochastic alpha and shear

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## ABSTRACT

Using a one-dimensional  $\alpha\omega$ -dynamo model appropriate to galaxies, we study the possibility of dynamo action driven by a stochastic alpha effect and shear. To determine the field evolution, one needs to examine a large number of different realizations of the stochastic component of  $\alpha$ . The net growth or decay of the field depends not only on the dynamo parameters but also on the particular realization, the correlation time of the stochastic  $\alpha$  compared to turbulent diffusion timescale and the time over which the system is evolved. For dynamos where both a coherent and fluctuating  $\alpha$  are present, the stochasticity of  $\alpha$  can help alleviate catastrophic dynamo quenching, even in the absence of helicity fluxes. One can obtain final field strengths up to a fraction  $\sim 0.01$  of the equipartition field  $B_{eq}$  for dynamo numbers  $|D| \sim 40$ , while fields comparable to  $B_{eq}$  require much larger degree of  $\alpha$  fluctuations or shear. This type of dynamo may be particularly useful for amplifying fields in the central regions of disk galaxies.

**Key words:** magnetic fields – turbulence – galaxies: magnetic fields

## 1 INTRODUCTION

Large-scale magnetic fields in stars and galaxies are thought to be generated and maintained by a mean-field turbulent dynamo (Moffatt 1978; Krause & Rädler 1980). The potential driver of such mean-field dynamos is the  $\alpha$ -effect, arising whenever one has rotation and stratification in a turbulent flow. Mean-field dynamo (MFD) models using a coherent  $\alpha$ -effect and shear have been invoked to explain large-scale fields observed in disk galaxies (Ruzmaikin et al. 1988).

The possibility of efficient dynamo action arising from random fluctuations in the  $\alpha$ -effect in combination with shear was first pointed out by Vishniac & Brandenburg (1997). They investigated a reduced mean-field dynamo model appropriate to accretion disks and showed that growth can occur for large enough random fluctuations in alpha. Several authors have since elaborated various aspects of this stochastic alpha-shear dynamos (Sokoloff 1997; Silantev 2000; Fedotov et al. 2006; Proctor 2007; Kleeorin & Rogachevskii 2008). In particular, Sokoloff (1997) examined a model of a disk dynamo with a fluctuating alpha antisymmetric in space but which changes sign randomly with equal probability. He argued that intermittent large-scale magnetic fields can grow. The role of a stochastic  $\alpha$  has also been analyzed in the context of solar dynamos (Proctor 2007; Brandenburg & Spiegel 2008; Moss et al. 2008).

The exact origin of such an incoherent  $\alpha$ -effect is as yet unclear. In any large Reynolds number system, many degrees of freedom exist, and hence there could always be a stochastic component of the mean turbulent electromotive force (emf). This could lead to additive or a multiplicative noise in the MFD equations. Additive noise provides a seed field for the dynamo, whereas mul-

tiplicative noise in say the  $\alpha$  effect, combined with shear, can lead to exponential growth of the mean field. In the solar context, Hoyng (1993) argued for  $\alpha$  fluctuations  $\sim u_0/\sqrt{M}$ , where  $u_0$  is the turbulent velocity and  $M$  is the number of cells being averaged over in defining the mean field. In principle this can be larger than any coherent  $\alpha$ -effect. Multiplicative noise is also seen in simulations which measure the  $\alpha$ -effect both in the kinematic regime (Sur et al. 2008) and in the nonlinear regime (Cattaneo & Hughes 2006; Brandenburg et al. 2008) and also in direct simulations of the galactic dynamo (Gressel et al. 2008). In fact Brandenburg et al. (2008) measure an incoherent  $\alpha$ -effect, with a Gaussian probability density function (PDF), even in turbulence driven with a non-helical forcing, where one does not expect a coherent  $\alpha$ -effect. Combined with shear, such systems show large-scale dynamo action (Brandenburg et al. 2008; Yousef et al. 2008). Here, we simply examine, in the context of galactic dynamos, the consequence of having an incoherent alpha effect, without considering in detail its exact origin. The growth of the mean field varies significantly from one realization of the stochastic process to another, as also pointed out in Sokoloff (1997). It is therefore necessary to examine a large number of realizations of the stochastic  $\alpha$  to determine the efficiency of the stochastic  $\alpha\omega$ -galactic dynamo.

We outline in section 2, the basics of a one-dimensional stochastic  $\alpha\omega$ -dynamo model appropriate to galaxies. We present numerical solutions of the above model in section 3, with two different PDF's for the stochastic alpha; the first as considered in Sokoloff (1997) and the second where the stochastic alpha has a gaussian PDF. In Section 4, we explore the possibility of alleviating catastrophic  $\alpha$ -quenching in absence of helicity fluxes by including the effects of a stochastic  $\alpha$ . Section 5 summarizes our results and the implications of a stochastic  $\alpha\omega$ -dynamo for galaxies.

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## 2 THE STOCHASTIC ALPHA-SHEAR DYNAMO

In MFD theory, one starts by splitting the relevant physical quantities into mean and fluctuating parts, for example  $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$  for the magnetic field and  $\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}$  for the velocity field. The over-bars denote a suitable averaging procedure with  $\overline{\mathbf{b}} = \overline{\mathbf{u}} = 0$ . This results in the standard mean field dynamo equation

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathcal{E}} - \eta_t \nabla \times \overline{\mathbf{B}}), \quad \nabla \cdot \overline{\mathbf{B}} = 0. \quad (1)$$

The averaged equation now has a new term, the mean electromotive force (emf)  $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$ , which crucially depends on the statistical properties of the small-scale velocity and magnetic fields,  $\mathbf{u}$  and  $\mathbf{b}$ , respectively.  $\overline{\mathbf{U}}$  is the mean fluid velocity. Assuming that  $\overline{\mathbf{B}}$  is spatially smooth,  $\overline{\mathcal{E}}$  can be expressed in terms of  $\overline{\mathbf{B}}$  and its derivative,

$$\overline{\mathcal{E}} \equiv \overline{\mathbf{u} \times \mathbf{b}} = \alpha \overline{\mathbf{B}} - \eta_t \overline{\mathbf{J}} \quad (2)$$

Here  $\overline{\mathbf{J}} = \nabla \times \overline{\mathbf{B}} / \mu_0$  (we assume  $\mu_0 = 1$  hereafter) and  $\alpha$  and  $\eta_t$  are turbulent transport coefficients that can be expressed in terms of the statistical properties of the flow. In the kinematic regime, and assuming isotropic turbulence, one has  $\alpha = \alpha_k = -\frac{1}{3} \tau \mathbf{u} \cdot \nabla \times \mathbf{u}$ , and the turbulent diffusion coefficient  $\eta_t = \frac{1}{3} \tau u^2$ . Here,  $\tau$ , the correlation time of the turbulent velocity  $\mathbf{u}$ , is assumed to be short.

Since galactic disks are thin, it often suffices to consider a one dimensional model, where only  $z$  derivatives of physical variables are retained (Ruzmaikin et al. 1988). For the stochastic dynamo which we examine here, we also modify the  $\alpha$ -effect to be of the form:  $\alpha = \alpha_k = \overline{\alpha}(z) + \alpha_1(z, t)$ . Here  $\overline{\alpha}(z)$  is the average  $\alpha_k$ , while  $\alpha_1(z, t)$  is the stochastic  $\alpha$  term. Therefore, the total  $\alpha$  is a sum of the standard kinetic alpha  $\overline{\alpha}$  and a stochastic component  $\alpha_1$ . Further, we consider a mean flow consisting of only a differential rotation such that  $\overline{\mathbf{U}} = (0, r\Omega(r), 0)$ . Then, going to dimensionless variables, Eqn. (1) gives evolution equations for the azimuthal ( $\overline{B}_\phi$ ) and radial ( $\overline{B}_r$ ) fields, (see also Vishniac & Brandenburg (1997))

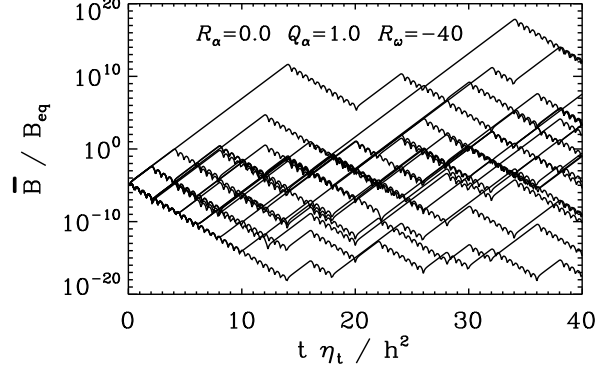
$$\frac{\partial \overline{B}_r}{\partial t} = -\frac{\partial}{\partial z} (R_\alpha g(z) \overline{B}_\phi + Q_\alpha f(z) N \overline{B}_\phi) + \frac{\partial^2 \overline{B}_r}{\partial z^2}, \quad (3)$$

$$\frac{\partial \overline{B}_\phi}{\partial t} = R_\omega \overline{B}_r + \frac{\partial}{\partial z} (R_\alpha g(z) \overline{B}_r + Q_\alpha f(z) N \overline{B}_r) + \frac{\partial^2 \overline{B}_\phi}{\partial z^2}. \quad (4)$$

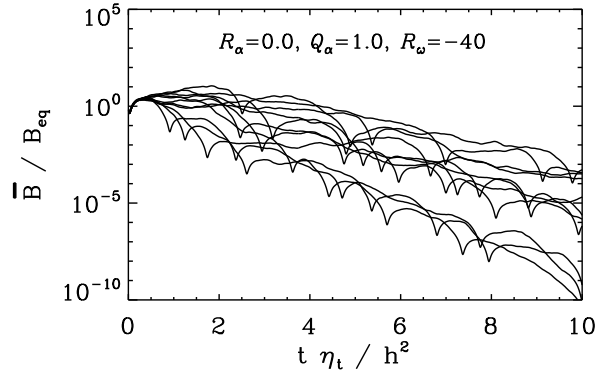
Here the length and time units are  $h$  and  $t_d = h^2 / \eta_t$  respectively, with  $h$  the semi-thickness of the disk. We adopt  $\overline{\alpha} = \alpha_0 g(z)$ , and  $\alpha_1 = \alpha_s f(z) N(t)$  where  $f(z) = g(z) = \sin(\pi z)$  takes care of the symmetry condition.  $N(t)$  is a stochastic function. In our numerical solutions we adopt the following procedure: We split  $t$  into equally spaced intervals  $[n\tau_c, (n+1)\tau_c]$ , where  $\tau_c$  is the correlation time of the stochastic alpha, and  $n = 0, 1, 2, \dots$  are integers. And in any such time interval  $N$  is a random number chosen from a Gaussian (or some other) probability distribution, with unit variance. The relevant dynamo control parameters are  $R_\alpha$ ,  $Q_\alpha$  and  $R_\omega$  defined as

$$R_\alpha = \frac{\alpha_0 h}{\eta_t}, \quad Q_\alpha = \frac{\alpha_s h}{\eta_t}, \quad R_\omega = \frac{G h^2}{\eta_t}, \quad (5)$$

Here  $G = r d\Omega / dr = -\Omega$ , for a flat rotation curve. From Krause's formula,  $\alpha_0 \simeq l_0^2 \Omega / h$ , where  $l_0$  is the integral scale of interstellar turbulence. Then  $R_\alpha \sim 3\Omega t_{ed}$ , assuming  $\eta_t \sim l_0 u_0 / 3$  and  $\tau \sim t_{ed} = l_0 / u_0$ , the eddy turnover time. Typical values of the dynamo control parameters in the solar neighborhood of the Milky Way could be  $R_\alpha \sim 1.0$ , and  $|R_\omega| \sim 10 - 15$ , corresponding to a "dynamo number"  $D = R_\alpha R_\omega \sim -10$  to  $D \sim -15$  (Ruzmaikin et al. 1988). The strength of  $Q_\alpha \sim 3(h/l_0)M^{-1/2}$  if one uses the estimate of Hoyng (1993). A horizontal average over



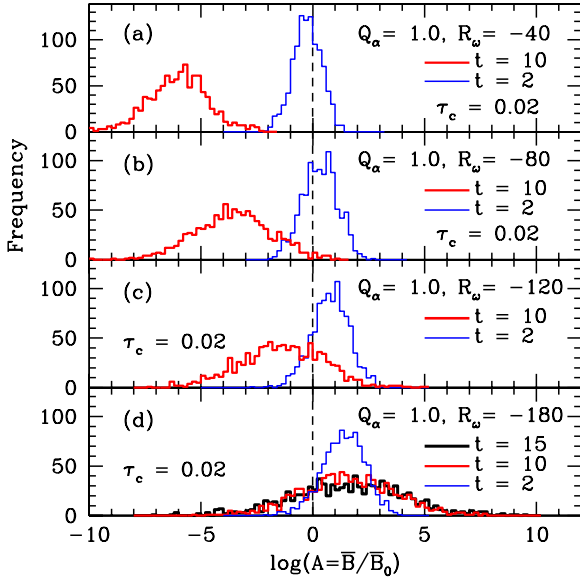
**Figure 1.** Time evolution of the rms mean magnetic field in the Sokoloff 1997 model for different realizations using a long correlation time for  $\alpha_1(t)$ . Parameter values used are  $R_\alpha = 0.0$ ,  $Q_\alpha = 1.0$ , and  $R_\omega = -40$ .



**Figure 2.** Time evolution of the rms mean magnetic field in different realizations using a short correlation time for  $\alpha_1(t)$  and a Gaussian PDF. Plots are obtained for parameter values  $R_\alpha = 0.0$ ,  $Q_\alpha = 1.0$ , and  $R_\omega = -40$ .

a scale  $h$  to define  $\overline{B}$  (cf. Brandenburg et al. (2008)) would suggest  $M \sim (h/l_0)^2$  and hence  $Q_\alpha \sim 3$ . However since the exact origin of such fluctuations is as yet unclear, we will vary  $Q_\alpha$  around these values. Thus in general, we will have  $|R_\omega| \gg R_\alpha, Q_\alpha$  so that one can make the standard  $\alpha\omega$ -dynamo approximation, where one neglects the terms with co-efficients  $R_\alpha$  and  $Q_\alpha$  in Eqn. (4).

Note that  $\Omega \propto 1/r$ , and thus one can have larger dynamo parameters towards the disk centre, depending also on how  $h$  and  $l_0$  behave there. The disk height could be smaller, but  $l_0$  could also be smaller in the denser inner galactic regions, where supernovae are more confined. This could lead to a net increase in  $R_\omega \propto \Omega h^2 / l_0$ . Any increase in  $R_\alpha$  depends on how much  $l_0$  decreases compared to the increase in  $\Omega$ . Changes in  $Q_\alpha$  depend on the origin of the  $\alpha$  fluctuations. For example, if  $h$  decreases by factor 2 and  $l_0$  decreases a factor 5 in the inner galaxy,  $R_\omega$  would increase by a factor  $6.25(r/2\text{kpc})^{-1}$  and  $R_\alpha$  or  $Q_\alpha$  would remain about the same, compared to the solar neighborhood. Overall larger dynamo numbers can be expected in the inner regions of disk galaxies. We now turn to the solution of the stochastic  $\alpha\omega$ -dynamo equations.



**Figure 3.** Frequency distribution of the dynamo amplification  $A = \overline{B}/\overline{B}_0$  at  $t = 2$  (blue/thin),  $t = 10$  (red/normal) and also at  $t = 15$  (black/thick) (in panel d), for 1000 realizations of  $\alpha_1(t)$  in each histogram. Here  $R_\alpha = 0$ ,  $Q_\alpha = 1.0$ , and  $R_\omega = -40, -80, -120$  and  $-180$ . Bin size is 0.2.

### 3 NUMERICAL SOLUTIONS

Our primary interest is in a scenario where large-scale dynamo action is possible in presence of stochastic alpha and shear. Thus we first seek numerical solutions to Eqs. (3) and (4), in the  $\alpha\omega$ -dynamo approximation, with the coherent part of the  $\alpha$ -effect taken to be zero; that is with  $R_\alpha = 0$ . The code uses a 6th order explicit finite difference scheme for the space-derivatives and 3rd order accurate time-stepping scheme; see Brandenburg (2003) for details. We use vacuum boundary conditions for the fields

$$\overline{B}_r = \overline{B}_\phi = 0 \quad \text{at } z = \pm h \quad (6)$$

As a test case, we numerically implemented the Sokoloff (1997) model with  $\bar{\alpha} = 0$  and  $\alpha_1 = \alpha_s f(z)N(t)$ ;  $N$  being either  $+1$  or  $-1$  with equal probability in any time interval  $n\tau_c < t < (n+1)\tau_c$ . For  $N = 1$ , the system behaves as a standard  $\alpha\omega$ -dynamo with growing solutions, while for  $N = -1$  we have decaying oscillations. So if the system is evolved over a finite time interval, there would be random instances of growth and decay. If  $\gamma$  is the growth rate of the growing solutions and  $-\zeta$  that of the decaying ones, the ensemble averaged growth rate is  $\Gamma = (\gamma - \zeta)/2$  (Sokoloff 1997). Thus when  $\gamma > \zeta$ , one obtains  $\Gamma > 0$  resulting in an overall growth above a critical dynamo number  $D_c$ . To estimate  $D_c$ , we use the perturbation solutions discussed in Sur et al. (2007). This gives  $\gamma \approx -\pi^2/4 + \sqrt{\pi}|D|/2$  and  $\zeta \approx \pi^2/4$ , and thus  $D_c \approx -\pi^3$  for the Sokoloff (1997) model. This is somewhat larger in magnitude than the critical dynamo number  $\sim \pi^3/4$ , which obtains for the coherent  $\alpha\omega$  dynamo (by demanding  $\gamma > 0$ ).

These features are illustrated in Fig. (1). Here we have chosen  $\tau_c = 2t_d$  so that one can clearly see both the growing and decaying phases and their net effect. Starting with random seed fields  $\overline{B}_r, \overline{B}_\phi \sim 10^{-6}$ , we find a number of growing as well as decaying realizations for moderate dynamo number  $D = -40$ . It is evident from Fig. (1), that any given realization has periods,  $N_+\tau_c$ , of steady growth (when  $N = 1$ ) and periods,  $N_-\tau_c$  of oscillatory

decay (when  $N = -1$ ). One gets a net growth of the field in about 65% of the realizations, as roughly expected from the above arguments for  $|D| > |D_c|$ . For a larger magnitude of the dynamo number  $|D|$  one gets a greater probability for growth. We have also examined the opposite limit when  $\tau_c < t_d$ , and find that the dynamo becomes less efficient (see below). These solutions clearly demonstrate the basic idea behind the incoherent  $\alpha\omega$  dynamo as discussed in Sokoloff (1997), that one needs to consider many realizations of the stochastic process. Just solving a double averaged version of the MFD equations need not be representative of the actual evolution of the dynamo for a given realization.

Of course the PDF of the stochastic alpha is not expected to be as described in Sokoloff (1997); for example Brandenburg et al. (2008) found it can be approximated as a Gaussian. Also in general we expect  $\tau_c < t_d$ . We now present the results obtained by solving Eqs. (3) and (4) with the random number  $N$  for  $\alpha_1$  chosen from a gaussian PDF, and adopting  $\tau_c = 0.02t_d$ . This is about  $1.5t_{ed}$ , assuming  $h = 500$  pc,  $\eta_t = 10^{26}$  cm<sup>2</sup> s<sup>-1</sup>,  $l_0 = 100$  pc, and  $u_0 \sim 10$  km s<sup>-1</sup>. The initial seed fields are random with amplitudes of  $O(1)$ . The MFD equations were evolved up to 10 turbulent diffusion time scales,  $t = 10$ , for dynamo numbers  $D = -40, -80$  and  $-120$  and upto  $t = 15$  for  $D = -180$ . We also considered 1000 realizations of  $\alpha_1(t)$  for each  $D$  so as to obtain good statistics. Note that to probe the PDF of the dynamo amplification upto a  $3\sigma$  level one needs about these many realizations.

Fig. (2) shows the time evolution of the RMS (large scale) magnetic field,  $\overline{B}$ , for a subset of realizations with  $R_\alpha = 0.0, Q_\alpha = 1.0$  and  $R_\omega = -40$ . There is an initial decay of  $\overline{B}$ , while the system discovers the proper eigenfunction. Further evolution then occurred on the diffusion time-scale  $t_d$ . In all realizations,  $\overline{B}$  shows an oscillatory decay, even though a significant number of realizations showed growth of  $\overline{B}$  up to  $t = 2$ . For higher dynamo numbers, growth is sustained for a longer time and for a larger number of realizations. We find that the Sokoloff (1997) model also shows similar features for short correlation times. Thus having  $\tau_c \ll t_d$ , qualitatively changes the behavior of the dynamo and leaves an imprint of  $t_d$  in the system evolution rather than  $\tau_c$ . In order to have a quantitative measure of how many realizations show net growth, we show in Fig. 3 the frequency distribution of the dynamo amplification  $A = \overline{B}/\overline{B}_0$  at  $t = 2, t = 10$  and also at  $t = 15$  in panel (d) for 1000 realizations of  $\alpha_1(t)$ , at dynamo numbers,  $D = -40, -80, -120$  and  $-180$ . Here  $\overline{B}_0 = 0.32 - 0.35$  is roughly the value to which  $\overline{B}$  initially decays in all the realizations. For  $D = -40, -80$  and  $-120$ , we obtain  $A > 1$ , for respectively 34%, 65% and 82.8% of realizations at  $t = 2$ . However at a later time  $t = 10$ , this percentage decreases to 0%, 18% and 24% respectively. This is evident in the gradual shift of the histogram to the left. For  $|D| = 160 - 180$  the PDF of  $|A|$  remains stationary at late times; see panel (d). Above this range, the mean amplification secularly increases with time. Thus, our results show that a stochastic  $\alpha\omega$ -dynamo is reasonably efficient over a few  $t_d$  even at  $R_\omega = -40$ , but requires much larger dynamo numbers, as plausible towards galactic centres, to sustain fields for long periods.

### 4 DYNAMICAL ALPHA QUENCHING OF THE STOCHASTIC DYNAMO

Conservation of magnetic helicity is regarded as a key constraint in the evolution of large-scale magnetic fields (see Brandenburg & Subramanian (2005) for a review). A consequence of helicity conservation is the production of equal and opposite

amounts of magnetic helicity in  $\overline{\mathbf{B}}$  and  $\mathbf{b}$  by the turbulent emf  $\overline{\mathcal{E}}$ . Closure models then imply a suppression of dynamo action due to the growing current helicity associated with  $\mathbf{b}$  (Pouquet et al. 1976; Kleeorin & Ruzmaikin 1982; Blackman & Field 2002). The effect of the small-scale magnetic field on the total  $\alpha$ -effect is described by the addition of a magnetic alpha to the kinetic alpha,  $\alpha = \alpha_k + \alpha_m$ . Here  $\alpha_k$  represents the kinetic  $\alpha$ -effect and  $\alpha_m = \frac{1}{3}\rho^{-1}\tau\overline{\mathbf{j}\cdot\mathbf{b}}$  is the magnetic contribution to the  $\alpha$ -effect, with  $\rho$  the fluid density. Specifically, the growth of the magnetic alpha ( $\alpha_m$ ) to cancel the kinetic alpha ( $\alpha_k$ ) results in a suppression of the total  $\alpha$ -effect. This suppression can be catastrophic in the sense that the large-scale field is quenched in an  $R_m$  dependent manner. Helicity fluxes across the boundaries of the disk have been identified as a possible mechanism to shed small-scale magnetic helicity, and prevent such quenching (Blackman & Field 2000; Kleeorin et al. 2000; Blackman & Field 2001; Vishniac & Cho 2001; Brandenburg 2005; Shukurov et al. 2006; Sur et al. 2007).

This situation could change in the presence of a stochastic component, as the kinetic alpha can undergo frequent sign reversals. Hence by the time the  $\alpha_m$  grows to cancel  $\alpha_k$ , the kinetic alpha itself might have changed sign. It is then of interest to ask whether addition of a stochastic component to the kinetic alpha can stem the catastrophic quenching. This would then naturally provide a mechanism for healthy dynamo action even in the absence of helicity fluxes. The numerical analysis of the previous section is therefore extended by including an  $\alpha_m$  contribution to  $\alpha$  in equations (3) and (4) supplemented with an evolution equation for  $\alpha_m$ . This can readily be motivated by considering the helicity conservation equation written in terms of the helicity density  $\chi$  of the small-scale magnetic field (Subramanian & Brandenburg 2006),

$$\frac{\partial\chi}{\partial t} + \nabla \cdot \mathbf{F} = -2\overline{\mathcal{E}} \cdot \overline{\mathbf{B}} - 2\eta\overline{\mathbf{j}\cdot\mathbf{b}}. \quad (7)$$

Here  $\mathbf{F}$  is the helicity flux density. Retaining only the  $z$ -derivatives and using the fact that the main contribution to  $\alpha_m$  comes from the integral scale of turbulence (Shukurov et al. 2006), so that  $\overline{\mathbf{j}\cdot\mathbf{b}} \simeq k_0^2 \overline{\mathbf{a}\cdot\mathbf{b}}$

$$\alpha_m \simeq \frac{1}{3}\tau k_0^2 \frac{\chi}{\rho}. \quad (8)$$

Eqn. (7) can be expressed in dimensionless form, by measuring  $\alpha$  in units of  $\alpha_0$  and the magnetic field in units of  $B_{eq}$ , where  $B_{eq}^2 = \rho u^2$ . In the absence of helicity fluxes, i.e  $\mathbf{F} = 0$  we have,

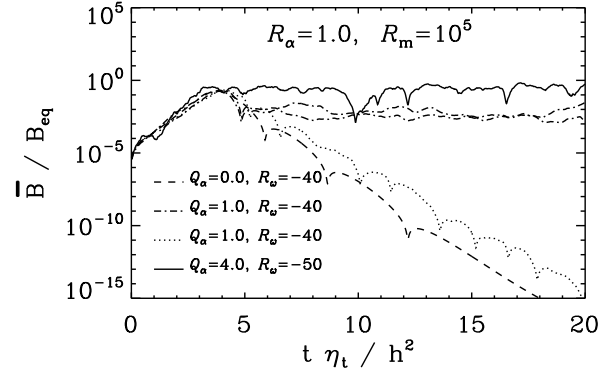
$$\frac{\partial\alpha_m}{\partial t} = -C \left( \left( g + \frac{Q_\alpha}{R_\alpha} fN + \alpha_m \right) \overline{\mathbf{B}}^2 - \frac{\overline{\mathbf{J}\cdot\mathbf{B}}}{R_\alpha} + \frac{\alpha_m}{R_m} \right) \quad (9)$$

where  $R_m = \eta_t/\eta$ ,  $C = 2\pi^2(k_0/k_1)^2$ ,  $k_1 = \pi/h$  and we take  $k_1/k_0 = 5$ . Further,  $\overline{\mathbf{J}\cdot\mathbf{B}}$  is the current helicity density of the large-scale field and is given by

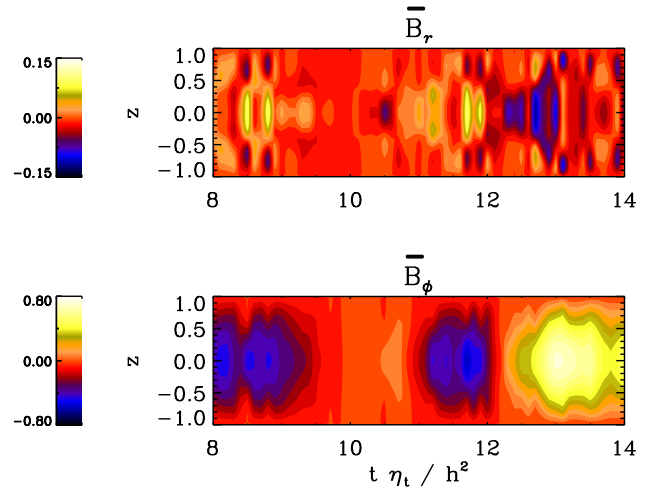
$$\overline{\mathbf{J}\cdot\mathbf{B}} = \overline{B_\phi} \frac{\partial\overline{B_r}}{\partial z} - \overline{B_r} \frac{\partial\overline{B_\phi}}{\partial z}. \quad (10)$$

We adopt  $\alpha_m = 0$  at  $t = 0$  and random initial fields of  $O(10^{-6})$ . The system of equations (3), (4) and (9) are then solved numerically in the  $\alpha\omega$ -dynamo approximation. Note that there is an extra term  $-\partial(R_\alpha\alpha_m\overline{B_\phi})/\partial z$  in Eq. 3 and no helicity fluxes are added to the r.h.s of Eqn. (9).

Fig. (4) shows the time evolution of the RMS large-scale field in a number of realizations with  $R_\alpha = 1.0$ ,  $Q_\alpha = 0.0 - 4.0$ ,  $|R_\omega| = 40 - 50$  and  $R_m = 10^5$ . Note that for  $Q_\alpha = 0.0$ ,  $R_\alpha = 1.0$  and  $R_\omega = -40$  (shown by dashed lines), we recover the



**Figure 4.** Time evolution of the rms mean magnetic field for different realizations in the dynamical  $\alpha$ -quenching model with parameter values  $R_\alpha = 1.0$ ,  $Q_\alpha = 0.0 - 4.0$ ,  $|R_\omega| = 40 - 50$  and  $R_m = 10^5$ .



**Figure 5.** Space-time diagrams of the radial and azimuthal components of the large-scale field for a realization with parameter values  $R_\alpha = 1.0$ ,  $Q_\alpha = 4.0$  and  $|R_\omega| = 50$ . The color bars on the left panel shows the magnitude of the field.

standard result that the magnetic field is catastrophically quenched to very low values. The catastrophic quenching still obtains in some realizations for a moderate value of  $Q_\alpha = 1.0$  (shown in dotted line in the figure). But in other realizations (shown by dash-dotted lines in the figure), a stochastic kinetic alpha alleviates this quenching to obtain fields of order  $0.01 - 0.001 B_{eq}$ . A detailed analysis shows that, for these dynamo parameters,  $\overline{\mathbf{B}}$  has a net growth in about 13% of all the realizations, even till  $t = 20$ . In fact, stronger values of  $Q_\alpha$  and  $R_\omega$  can even amplify the field to near equipartition values. Such an example, adopting  $Q_\alpha = 4.0$  and  $|R_\omega| = 50$  is shown by the solid line in the above figure. A space-time diagram for this realization, between times  $t = 8 - 14$ , is shown in Fig. 5. Both the radial and azimuthal fields have quadrupolar symmetry and show several reversals in sign during this period. We recall that high values of the dynamo control parameters are plausible towards the central regions of a galaxy. Therefore a stochastic  $\alpha\omega$ -dynamo is more likely to grow coherent magnetic fields efficiently towards the central regions of disk galaxies.

## 5 CONCLUSIONS

We have examined here how a stochastic  $\alpha$ -effect in association with shear can lead to the generation of large-scale galactic magnetic fields. To determine the field evolution, one needs to examine a large number of different realizations of the stochastic  $\alpha_1(t)$ . The net growth or decay of the field depends on the particular realization, the correlation time of the stochastic  $\alpha$  compared to turbulent diffusion timescale and the time over which the system is evolved.

The results are illustrated first with the simple model of Sokoloff (1997) in Fig. 1. Here the magnitude of  $\alpha_1$ , takes randomly a value  $+\alpha_s g(z)$  or  $-\alpha_s g(z)$  over any time interval  $\tau_c$ , with equal probability. Any given realization of  $\alpha_1(t)$  will have  $N_+$  periods of steady growth (when  $N = 1$ ) and  $N_-$  periods of oscillatory decay (when  $N = -1$ ). But, since the growth rate ( $\gamma$ ) and decay rates ( $\zeta$ ) are different, this could lead to a net growth or decay as pointed out by Sokoloff (1997). The critical dynamo number for getting growth for say 50% of realizations is  $|D_c| \sim 30$ , moderately larger than  $|D_c| \sim 10$  required for the coherent  $\alpha\omega$ -dynamo. Our numerical solutions confirm the applicability of this picture for long correlation times  $\tau_c = 2t_d$ , while for  $\tau_c < t_d$ , the picture changes qualitatively, and the dynamo becomes less efficient.

We then examined more realistic MFD models with a short correlation time ( $\tau_c = 1.5t_{ed}$ ), for a stochastic  $\alpha_1(t)$  chosen from a gaussian PDF. Our results are given in Fig. 2 and Fig. 3. In this case as well, and for a stochastic  $\alpha\omega$ -dynamo number  $D = -40$ , about 34% of realizations showed growth of  $\overline{B}$  till  $t \sim 2$ . However subsequently  $\overline{B}$  decays to negligible values by  $t \sim 10$ . For higher dynamo numbers, growth is sustained for a longer time and for a larger number of realizations (cf. Fig. 3). One requires  $D \sim -120$  to obtain long term growth (till  $t \sim 10$ ), in a significant number ( $\approx 24\%$ ) of realizations,  $|D| \approx 160 - 180$  for the PDF of  $|A|$  to remain stationary and a larger  $|D|$  for secular growth at late times. Note that having an additional coherent  $\alpha$ , with  $|R_\alpha R_\omega| > 10$ , would ensure growth of  $\overline{B}$ . However, the quenching imposed by helicity conservation still needs to be alleviated.

In the usual  $\alpha\omega$ -dynamo, such helicity conservation leads to a growth of the magnetic  $\alpha_m$ , which tends to cancel the kinetic  $\alpha_k$ , so as to catastrophically quench the dynamo. This problem can be alleviated by having fluxes of magnetic helicity. In contrast, for a stochastic  $\alpha\omega$ -dynamo, by the time  $\alpha_m$  has grown,  $\alpha_k$  could have changed sign. This raises the possibility of alleviating catastrophic  $\alpha$ -quenching without helicity fluxes. To examine this possibility, we solved the stochastic  $\alpha\omega$ -dynamo equations along with the dynamical  $\alpha$ -quenching equation. We included both a coherent and incoherent  $\alpha$ -effect. In general the radial and azimuthal fields are again quadrupolar and show occasional reversals in time (see Fig. 5). When the coherent and stochastic components of  $\alpha$  are comparable, and even without any helicity flux, we find that steady large-scale magnetic field of strengths of about  $0.01B_{eq}$  could be obtained in some realizations even for  $D \sim -40$ . Field strengths above  $0.3B_{eq}$  are obtained for stronger amplitude of random  $\alpha$ -fluctuations in association with strong shear (Fig. 4). Therefore, a stochastic  $\alpha\omega$ -dynamo model is more likely to grow large-scale magnetic fields efficiently towards the central regions of a galaxy and even in the absence of helicity fluxes.

We have focussed on the application of a stochastic  $\alpha\omega$ -dynamo model to galaxies. However, our emphasis on examining a large number of realizations and our results on alleviating  $\alpha$ -quenching with a random  $\alpha$  could be applicable to other astrophysical dynamos as well. More work is needed to elucidate the

origin of the incoherent  $\alpha$  and also study the influence of spatial decorrelation on the dynamo.

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