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Singularity free inhomogeneous viscous fluid cosmological models

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Abstract

We obtain two new classes of singularity free inhomogeneous viscous fluid cosmological solutions, which reduce to the Senovilla [1] class when viscosity is switched off. It is interesting to note that inclusion of viscosity does not disturb the singularity free character and general behaviour of the models remains qualitatively the same.

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In 1990 Senovilla [1] made a remarkable discovery of finding a new class of exact solutions of Einstein equations describing inhomogeneous perfect fluid cosmological models free of the big-bang singularity. This was the first instance of singularity free cosmological model satisfying the energy conditions as well as all other physical requirements. The question then arose how do these solutions bypass the singularity theorems [2]. Clearly one of the assumptions of the theorems has to break down and it was subsequently shown [3] that the existence of "trapped surface" does not remain valid for this class of models. In contrast to the physical imperativeness of energy conditions, regularity and well behavedness of physical and geometrical invariants, one is not sure of how imperative is the requirement of "trapped surface" for cosmology.

The Senovilla solutions can be interpreted as representing a cylindrical universe filled with inhomogeneous perfect fluid. The matter parameters like density and pressure as well as curvature invariants remain finite, regular and smooth for all values of t and r - exhibiting the singularity free character. Spatially inhomogeneous models are of interest firstly because our Universe is not perfectly homogeneous at all scales and secondly no special initial conditions be prescribed for the Universe. We have elsewhere argued [4, 5] that in early stages of evolution of the Universe neither homogeneity and isotropy of space nor a particular form of matter can be assumed with certainty. To accommodate these features easily we further opt for cylindrical symmetry in place of traditional spherical symmetry. It has also been argued [6] that cylindrical symmetry seems to be very relevant for avoidance of singularities.

In this letter, we wish to introduce viscosity in the Senovilla framework and obtain two new classes of exact singularity free cylindrically symmetric cosmological solutions. Viscous effects in fluid flow are ordinarily negligible but they may not remain so far exotic and dense form of the matter in the early stages. The role of viscosity in cosmology has been considered by several authors [7 - 12]. Our primary aim is to examine how does inclusion of viscosity affect the most important property (the singularity free character) of this class of solutions. We establish that this property is retained for certain classes of solutions. Our solutions reduce to the Senovilla [1] singularity free solutions when viscosity is switched off.

We begin by writing the energy momentum tensor for a viscous fluid,

$$T_{ik} = (\bar{p} + \rho)\, u_i u_k - \bar{p} g_{ik} - \eta \mu_{ik} \tag{1}$$

$$\mu_{ik} = 2\left(u_{(i;k)} - u_{(i} f_{k)}\right) \tag{2}$$

$$\bar{p} = p - \left(\zeta - \frac{2}{3}\right)\eta\theta \tag{3}$$

where $u_i u^i = 1$, $f_i = u_{i;k}u^k$, $\theta = u^i_{;i}$, and ζ and η are coefficients of bulk and shear viscosity.

The shear scalar σ of the velocity field is defined by

$$\sigma^2 = \sigma_{ik}\sigma^{ik} \tag{4}$$

where

$$\sigma_{ik} = \frac{1}{2}\mu_{ik} - \frac{1}{3}\theta(g_{ik} - u_i u_k). \tag{5}$$

For the Einstein equation

$$R_{ik} - \frac{1}{2}Rg_{ik} = -8\pi T_{ik} \tag{6}$$

we obtain the following two classes of solutions

$$ds^2 = C^4(at)C^{-6c}(3ar)(dt^2 - dr^2) - C^{-2}(at)C^{2c}(3ar)dz^2 - C^4(at) C^{-2(1+2c)}(3ar)S^2(3ar)d\phi^2 \tag{7}$$

and

$$ds^2 = C^{-4b}(at)C^2(3ar)(dt^2 - dr^2) - C^{2b}(at)C^{-2/3}(3ar)dz^2 - C^{-4b}(at)C^{-2/3}(3ar)S^2(3ar)d\phi^2 \tag{8}$$

where a, b and c are constants. We shall denote $C(x) = coshx, S(x) = sinhx, \quad T(x) = tanhx, andSe(x) = sechx$. Clearly the metrics are singularity free and well behaved for the entire range of the coordinates, $-\infty < t, z < \infty, o < r < \infty, o < \phi < 2\pi$. These are globally regular cylindrical symmetric spacetimes.

The physical and kinematic parameters of the solutions are given as follows :

For the solution (7) :

$$8\pi\rho = \frac{9a^2(2 + c)}{A^2}Se^2(3ar) \tag{9.1}$$

$$8\pi\bar{p} = \frac{3a^2}{A^2}\left[-5(4c + 1) + (4 + 20c + 9c^2)T^2(3ar)\right] \tag{9.2}$$

$$16\pi\eta\theta = 3\frac{a^2(3c + 1)}{A^2}\left[7 - (c + 2)T^2(3ar)\right] \tag{9.3}$$

$$\theta = \frac{3a}{A}T(at) \tag{9.4}$$

$$f_r = \frac{9ac}{A}T(3ar) \tag{9.5}$$

$$\sigma_{zz} = -2\sigma_{rr} = -2\sigma_{oo} = \frac{2aT(at)}{A} \tag{9.6}$$

where $A = C^2(at)C^{-3c}(3ar)$.

For the class (8) :

$$8\pi\rho = \frac{15a^2}{B^2} Se^2(3ar) \tag{10.1}$$

$$8\pi\bar{p} = \frac{3a^2}{B^2}\left[(b+1)(1-bT^2(at)) + \frac{5}{3}Se^2(3ar)\right] \tag{10.2}$$

$$16\pi\eta\theta = -3a^2\frac{(b+1)}{B^2}\,(1-bT^2(at)) \tag{10.3}$$

$$\theta = -\frac{3ab}{B}T(at) \tag{10.4}$$

$$f_r = -3\frac{aT}{B}(3ar) \tag{10.5}$$

$$\sigma_{zz} = -2\sigma_{rr} = -2\sigma_{\phi\phi} = \frac{2ab}{B}T(at) \tag{10.6}$$

where $B = C^{-2b}(at)C(3ar)$.

Two classes (7) and (8) reduce to the Senovilla class of solutions for $c = -\frac{1}{3}$ and $b = -1$ respectively. This is when viscosity vanishes. Thus our solutions are the viscous fluid generalization of the Senovilla class [1]. All the above parameters are finite and well behaved for the entire range of t and r. Note that physically relevant quantity is $\eta\theta$ as it appears in eqn.(3) and it always remains finite and well-behaved. For specification of bulk viscosity we have to prescribe an equation of state for the fluid, $p = k\rho$, say. Then from (3), we can write $\zeta\theta = k\rho + \frac{2}{3}\eta\theta - \bar{p}$

On Macsyma computing package, we have verified that components of curvature and weyl tensors as well as curvature invariants remain regular and finite over the whole spacetime. No singularity is encountered in physical as well as geometrical invariants at $t = 0$ and/or $r = 0$.

The energy conditions [2] means $R_{ik}u^iu^k \leq 0$ implying

$$\rho + 3\bar{p} + 2\eta\theta \geq 0. \tag{11}$$

For the solution (7), this condition together with positivity of the physical parameters prescribes $-\frac{1}{3} \leq c \leq -\frac{1}{18}$ while for the solution (8), $-1 \leq b < 0$. Equality at the lower limit makes viscosity to vanish and the solutions go over to the Senovilla class. The upper limit is due to the energy condition in (11).

The evolution behaviour of the models is similar to that of Senovilla. At $t = 0$, density, radial acceleration and the parameter $\eta\theta$ attain the maximum values at each r while expansion and shear vanish. For $t \to \pm\infty$, density vanishes and the metrics tend to become flat. The radial acceleration vanishes at $r = 0$. The maximum value of density that occurs at $t = 0 = r$ is essentially represented by the constants a, b, c, which can be chosen as large as we please. The physical parameters decrease as $|t|$ and r increase and vanish as $|t|, r \to \infty$.

Thus we have generalized the Senovilla singularity free inhomogeneous models to include viscosity. The evolution as well as the singularity structure of the models remain qualitatively the same. It is interesting to note that cylindrical symmetry seems to play a very important role in avoidance of the big-bang singularity as well as for description of the Universe in the early stages [3-6, 13].

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