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Why does an accelerated detector click?

T Padmanabhan

Astrophysics Group, TIFR, Homi Bhabha Road, Bombay 400 005, India

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Abstract. The conflict between the definition of particles based on (*a*) field theory formalism and (*b*) simple detector models is discussed. An improved model for the detector is constructed taking the effect of accelerating potential into account. Analysis of this model shows that the detector results are better interpreted as a radiation process.

1. Introduction

In order to define the concept of a 'particle' in quantum field theory, one requires the notion of a timelike killing vector field. In field theories which are invariant under Lorentz transformations alone, one has a natural choice for this global timelike killing vector field. Standard particle physics uses this killing vector to define the Hamiltonian and the particles.

It has been noted repeatedly in the literature (for a review see [1]) that when curvilinear coordinate transformations are allowed, there exists other possible killing vector fields which are timelike in part of the manifold. Particle states—in particular the vacuum state—can be defined with respect to these killing vector fields. In general, these definitions will not be equivalent. The Minkowski vacuum will appear to be a many particle state according to the new definition.

The question arises as to whether an observer who uses the particular curvilinear coordinate system will detect these particles in the Minkowski vacuum. The answer to this question seems to be generally believed to be in the affirmative (see [6, 7]). A simple model for the detector appears to confirm this view in one special case (see [7]).

We re-examine this question in this paper. We show in § 2 that, in general, the conventional model for detector will not see the 'particles' as defined by the field theory formalisms. (This was previously demonstrated in [2, 3]; we will only stress its importance.) Next, we shall develop a simple model for a detector taking into account the accelerating potential. We will show that the excitation energy for the detector comes directly from the accelerating potential. We will analyse this detector's performance in the inertial as well as accelerated frames and argue why it is better to consider the process as radiation. In § 3 we develop some simple preliminary results to study quantum mechanics in an accelerated frame. An expert reader may skip this section.

We believe that there are two distinct physical phenomena involved here. (i) Positive frequency modes and vacuum state can be defined in more than one way. The Minkowski vacuum will appear as a many 'particle' state with reference to a different particle definition. (ii) A system with internal degrees of freedom, when placed in an external accelerating potential, can absorb energy from the potential and make a transition to an excited state. One should *not*, in general, re-interpret (ii) as a result of detection of particles encountered in (i). This re-interpretation is valid in the special case of the uniformly accelerated trajectory but fails in general (see § 2). These two situations are physically distinct. This is the main conclusion of the paper.

2. Particles and detectors

Flat Minkowski spacetime possesses ten killing vector fields corresponding to rotation, translation and Lorentz boosts. One can construct from these linear combinations of killing vectors which will be timelike in part of the manifold. The integral curves to this timelike killing vector field can act (in that part of the manifold) as possible trajectories for observers and detectors. One can introduce the coordinate system appropriate to such an observer, with the killing vector defining the time direction.

By its very construction the metric in such a coordinate system will be stationary and one can define positive and negative frequency modes (say, ψ_i and ψ_i^*) and corresponding creation and annihilation operators.

On the other hand, one has the positive and negative energy modes defined with respect to usual Minkowski time coordinates (say φ_j and φ_j^*). In general, one can have the Bogoliubov expansion,

$$\psi_j = \sum_k (\alpha_{jk}\varphi_k + \beta_{jk}\varphi_k^*). \tag{1}$$

The non-vanishing of $|\beta|^2$ indicates the inequivalence of the vacua and signals the presence of ψ particles in the φ vacuum.

One can proceed in a different way by constructing an instrument—usually called the 'particle detector'—which is described by the following coupling to the scalar field Φ

$$L = m(\tau)\Phi[x^\mu(\tau)]. \tag{2}$$

Here $x^\mu(\tau)$ is the integral curve to the killing vector field and $m(\tau)$ is *assumed* to evolve as

$$m(\tau) = \mathrm{e}^{\mathrm{i}H\tau} m(0)\, \mathrm{e}^{-\mathrm{i}H\tau}. \tag{3}$$

By a straightforward analysis one can show that such a detector will behave as though it is immersed in a spectrum given by

$$P(\omega) = 2\pi\rho(\omega)\int_0^\infty \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\omega\tau}\langle 0|\Phi[x^\mu(\tau+s)]\Phi[x^\mu(s)]|0\rangle \tag{4}$$

where $\rho(\omega)$ is a density of states factor. The existence of particles is signalled by non-zero $P(\omega)$.

Thus we have here two definitions: (i) field theory definition of a particle through $\beta \neq 0$ in (1) and (ii) particle detector definition through $P(\omega) \neq 0$ in (4). One can demonstrate that for inertial trajectories and for uniformly accelerated trajectories these two definitions give an identical description.

We wish to stress that the agreement probably ends right there. The present author [2] and independently Letaw [3] have constructed other general examples in which $\beta = 0$ but $P(\omega) \neq 0$. One simple case corresponds to the trajectory

$$x^\mu(\tau) = (\tau + \tfrac{1}{2}g^2\tau^3, \tfrac{1}{2}g\tau^2, \tfrac{1}{6}g^2\tau^3, 0) \tag{5}$$

for which (4) will give the spectra

$$P(\omega)=\frac{\omega^2}{8\pi^2 g^2\sqrt{3}}\exp\left(-3\sqrt{2}\frac{\omega}{g}\right) \tag{6}$$

while an explicit construction of field modes ([2], § 8) will lead to the conclusion,

$$\beta=0. \tag{7}$$

Other examples can be seen in the references cited above.

It is also possible to have the reverse situation where $P(\omega)$ vanishes (the detector sees no particles) but $|\beta|^2$ does not. This occurs in the case of the uniformly accelerated frame itself when one asks the opposite question: what does an *inertial* detector see in the so-called Rindler vacuum of the accelerated frame? This question was recently attacked by Candelas and Sciama [4] who show that, at least to first order

$$P(\omega)=0. \tag{8}$$

On the other hand, one can explicitly compute the number of *Minkowski* particles in the *Rindler vacuum.* (Normally one calculates the number of Rindler particles in the Minkowski vacuum.) In fact, one can choose mode functions such that this is also a Planck spectrum. (The detailed implications of this result will be published elsewhere.) While this symmetry is quite interesting, it does conflict with the result above (8), because

$$\beta\neq 0. \tag{9}$$

Another simple situation where the definitions are inequivalent is the physically interesting case of variable acceleration, with the acceleration vanishing asymptotically. One can define asymptotic 'in' and 'out' states and show that the β's vanish between the 'in' and 'out' fields. Thus the 'in' and 'out' vacua are the same. Nevertheless $P(\omega)$ will now be a complicated time dependent function. All this goes on to suggest that the 'model detector' is not seeing the particles defined through the field theory. These 'particles' are not responsible for the clicking of the detector. The two particle definitions are different.

This leads one to ask two questions. (i) If the detector is not clicking due to the 'particles' in the Minkowski vacuum, why is it clicking at all? (ii) If the detector does not see the 'particles', are the particles 'real'?

We shall completely refrain from attempting to answer the *second* question which is ill defined and ambiguous. It is an undeniable mathematical fact that more than one (inequivalent) definition for positive frequency modes and particles exist in flat space. Calling any one of them more 'real' than the others can be justified only when additional physical criteria are given.

We shall attempt to answer the first question in the next two sections, at least partially. It is extremely difficult to tackle this question using the above model detector for two reasons

(i) It is conceivable that the 'clicking' of the detector is due to the accelerating potential. In that case we need a more detailed model for the detector including the accelerating source.

(ii) One would like to begin with an inertial detector and accelerate it explicitly by an external force. In the model above, we have *a priori* assumed the evolution of the monopole moment (3). In other words we have by fiat, constructed a detector which will only respond to positive frequency modes with respect to the time coordinate τ. Once this is done, the detection is almost tautological.

Therefore we shall develop a simple quantum mechanical model for the detector in § 4 and use this model to study the dynamics. In the next section we shall gather together some simple results needed later on.

3. Quantum mechanics in accelerated frames

Consider a particle described by the Lagrangian, (x and t are inertial coordinates)

$$L = \tfrac{1}{2} m \dot{x}^2 - V(x, t). \tag{10}$$

The Schrödinger equation for the particle reads as

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x, t)\psi. \tag{11}$$

A non-relativistic transformation to an accelerated frame may be indicated as

$$\bar{x} = x - \xi(t), \qquad \bar{t} = t, \qquad |\dot{\xi}| \ll c. \tag{12}$$

Under these transformations (which are only coordinate relabellings), $\psi(x, t)$ and $V(x, t)$ may be treated as scalars. This would mean that the Lagrangian in the accelerated frame will read as

$$\begin{aligned} L &= \tfrac{1}{2} m (\dot{\bar{x}} + \dot{\xi})^2 - V(\bar{x} + \xi, t) \\ &= \tfrac{1}{2} m \dot{\bar{x}}^2 - V(\bar{x} + \xi, t) + m \dot{\bar{x}} \dot{\xi} + \tfrac{1}{2} m \dot{\xi}^2 \end{aligned} \tag{13}$$

and the Schrödinger equation has the form,

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial \bar{t}} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial \bar{x}^2} + V\psi + \mathrm{i}\hbar \dot{\xi} \frac{\partial \psi}{\partial \bar{x}}. \tag{14}$$

This equation can be cast into a more transparent form. Notice that the Lagrangian in (13) and the following Lagrangian

$$L_1 \equiv \tfrac{1}{2} m \dot{\bar{x}}^2 - V(\bar{x} + \xi, t) - m \ddot{\xi} \bar{x} \tag{15}$$

differ only by a total time derivative of the function

$$f(\bar{x}, t) = \int^t \tfrac{1}{2} m \dot{\xi}^2 \, \mathrm{d}t + m \bar{x} \dot{\xi}(t). \tag{16}$$

Thus, classically, the physical content of L and L_1 is the same. Instead of quantising L and arriving at the Schrödinger equation (14), one can use L_1 and reach the Schrödinger equation

$$\mathrm{i}\hbar \frac{\partial \psi_1}{\partial \bar{t}}(\bar{x}, t) = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_1}{\partial \bar{x}^2} + (V + m \ddot{\xi} \bar{x}) \psi_1. \tag{17}$$

As to be expected in such canonical transformations, the wavefunctions ψ_1 and ψ in (14) are related by the simple phase change

$$\psi_1(\bar{x}, t) = \psi(\bar{x}, t) \exp - \mathrm{i}\hbar^{-1} \left(\int^t \tfrac{1}{2} m \dot{\xi}^2 \, \mathrm{d}t + m x \dot{\xi} \right). \tag{18}$$

Note that $\psi(\bar{x}, t)$ was just the original inertial wavefunction transformed as a scalar under (12). Thus we know that, given any solution in the inertial frame $\psi_I(x, t)$, one can obtain the solution to (17) by (a) transforming as a scalar to $\psi(\bar{x}, t) \equiv \psi_I(\bar{x}+\xi, t)$ and (b) making the phase change indicated in (18).

The Schrödinger equation (17), which we shall use hereafter, shows that transforming to an accelerated frame is equivalent to adding the potential $(m\ddot{\xi}x)$, which is what one expects from a naive application of the principle of equivalence.

One particularly interesting case corresponds to a free particle described in a uniformly accelerated frame. This corresponds to

$$V=0, \qquad \xi(t)=\tfrac{1}{2}gt^2. \tag{19}$$

In the inertial frame, the stationary states are the usual plane waves

$$\psi_I(x, t) = N \exp \mathrm{i}\hbar^{-1}(px - E_p t), \qquad E_p = p^2/2m. \tag{20}$$

In the accelerated frame, the Schrödinger equation reads as

$$\mathrm{i}\hbar \frac{\partial \psi_1}{\partial \bar{t}} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi_1}{\partial \bar{x}^2} + mg\bar{x}\psi_1. \tag{21}$$

The plane wave solutions in (20) can be transformed by our prescription (18) and will appear in the accelerated frame as

$$\psi_1(\bar{x}, t) = N \exp \mathrm{i}\hbar^{-1}[(p - mgt)\bar{x} - E_p t + \tfrac{1}{2}pgt^2 - \tfrac{1}{6}mg^2t^3]. \tag{22}$$

This does satisfy (21) because of the nature of its construction. Obviously these are not stationary states in the accelerated frame. Equation (21), because of time independence, does admit stationary state solutions, which can be expressed in terms of Airy functions [5] as

$$\psi_E(\bar{x}, t) = \frac{(2m)^{1/3}}{\pi(mg)^{1/6}\hbar^{2/3}} \exp\left(-\frac{\mathrm{i}E}{\hbar}t\right)\left\{\int_0^\infty \mathrm{d}u \cos\left[\tfrac{1}{3}u^3 - u\left(\frac{2m^2g}{\hbar^2}\right)^{1/3}\left(\frac{E}{mg} - \bar{x}\right)\right]\right\}. \tag{23}$$

For future reference, let us note that this rather formidable expression has a simple Fourier transform

$$a_E(p, t) = (2\pi mg\hbar)^{-1/2} \exp \frac{\mathrm{i}}{mg\hbar}\left(E_p - \frac{p^3}{6m}\right) \tag{24}$$

and also note the following result,

$$\left|\int_{-\infty}^{+\infty} \mathrm{d}x\, \mathrm{e}^{-\mathrm{i}kx}\psi^*_{E_1}(x)\psi_{E_2}(x)\right|^2 = \frac{1}{2\pi}\left(\frac{1}{g\hbar|k|}\right) \qquad (k \neq 0, g \neq 0). \tag{25}$$

It must be emphasised that the stationary states in (23) are physically distinct (they have no simple form in the inertial frame) from (transformed) 'plane wave' states of (22).

4. Quantum mechanical model for the detector

A hydrogen atom in the ground state is a good example of a detector for photons. The existence of photons will be signalled by the excitation of the electron to a higher energy orbit. With this model in mind, we shall describe our simplified detector by the following Lagrangian

$$L=\tfrac{1}{2}m_1\dot{x}_1^2+\tfrac{1}{2}m_2\dot{x}_2^2-V(x_1-x_2)+qx_1\Phi(x_1,t). \tag{26}$$

We have here two particles of mass m_1 and m_2 bound by a potential $V(x_1-x_2)$. We will assume that the potential becomes infinitely large for large $|x_1-x_2|$ so that the internal energy levels are discrete. The last term represent a coupling of particle 1 to a scalar field $\Phi(x_1, t)$. We will treat this field as second quantised with the states described by (standard) occupation numbers. The non-vanishing matrix elements are

$$\langle n_k|\Phi(x_1,t)|n_k+1\rangle=(2\omega_k)^{-1/2}(n_k+1)^{1/2}\exp[-\mathrm{i}(\omega_k t-kx_1)] \tag{27}$$

and

$$\langle n_k+1|\Phi(x_1,t)|n_k\rangle=(2\omega_k)^{-1/2}(n_k+1)^{1/2}\exp[\mathrm{i}(\omega_k t-kx_1)]. \tag{28}$$

We shall consider q to be a small parameter and will work to first order in q. The coupling of Φ to x_1 is only chosen for convenience; more complicated models do not add any new physical insight.

We are ultimately interested in the behaviour of this detector when it is accelerated. However, in order to make convenient comparisons let us first consider the inertial motion itself. Such a discussion of a quantum mechanical system interacting with a second quantised field is in all quantum mechanics textbooks and hence, we will only stress the essentials.

Introducing the centre-of-mass and relative coordinates, we can write L as

$$L=\tfrac{1}{2}M\dot{R}^2+[\tfrac{1}{2}\mu\dot{r}^2-V(r)]+q[R+m_2r/M]\Phi[R+m_2r/M,t]. \tag{29}$$

The unperturbed wavefunction describing a state with centre-of-mass energy E and 'internal' energy ε can be written as the product

$$\Psi_{E,\varepsilon}(R,r,t)=\underline{N}\exp\{-\mathrm{i}\hbar^{-1}(Et-PR)\}\varphi_\varepsilon(r)\exp(-\mathrm{i}\hbar^{-1}\varepsilon t). \tag{30}$$

Here

$$\left(-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2}+V(r)\right)\varphi_\varepsilon(r)=\varepsilon\varphi_\varepsilon(r) \tag{31}$$

and

$$E=P^2/2M. \tag{32}$$

The wavefunction in (30) describes the detector moving with uniform velocity (P/M) with the internal state described by $\varphi_\varepsilon(r)$. Let us assume that the field Φ is in the vacuum state. We have to first convince ourselves of the (admittedly trivial) fact that it is not possible for the detector to jump to an internal state of energy higher than ε. In other words, the detector will not click in the vacuum, merely because it is moving with uniform velocity.

To check this, consider the matrix element of the perturbing potential

$$\begin{aligned}V_{\text{pert}}&=q(\hat{R}+m_2\hat{r}/M)\hat{\Phi}(R+m_2r/M,t)\\&=q\hat{R}\hat{\Phi}(R+m_2r/M,t)+q(m_2/M)\hat{r}\Phi[R+m_2r/M,t]\end{aligned} \tag{33}$$

between two states, Ψ_{E_i,ε_i} and Ψ_{E_f,ε_f}. We shall restrict ourselves to the case where the wavelengths involved are large compared with the dimensions of the bound state.

Then the matrix element for the field can be approximated to

$$\langle 1_k|\Phi(R+m_2r/M,t)|0\rangle = (2\omega_k)^{-1/2}\exp[\mathrm{i}(\omega_k t - k(R+m_2 r/M))]$$
$$\simeq (2\omega_k)^{-1/2}\exp[\mathrm{i}(\omega_k t - kR)]. \tag{34}$$

The V_{pert} in (33) has two terms. However we are interested in the case where there *is* an internal excitation, i.e. when $\varepsilon_i \neq \varepsilon_f$. The first term in V_{pert} cannot produce these internal transitions (since it is independent of r, when (34) is used) and we have to consider only the second term. This leads to the matrix element, (transition amplitude)

$$\int_{-\infty}^{+\infty} \mathrm{d}t\langle \mathrm{f}|V|\mathrm{i}\rangle = \int_{-\infty}^{+\infty}\mathrm{d}R\int_{-\infty}^{+\infty}\mathrm{d}r\int_{-\infty}^{+\infty}\mathrm{d}t(2\omega_k)^{-1/2}$$
$$\times\exp[\mathrm{i}(\omega_k t - kR)]N^2\exp[(\mathrm{i}/\hbar)^{-1}(E_\mathrm{f}t - P_\mathrm{f}R)]$$
$$\times\exp[-(\mathrm{i}/\hbar)^{-1}(E_\mathrm{i}t - P_\mathrm{i}R)]\varphi^*_{\varepsilon_\mathrm{f}}(r)\varphi_{\varepsilon_\mathrm{i}}(r)q(m_2/M)r\exp[(\mathrm{i}/\hbar)(\varepsilon_\mathrm{f}-\varepsilon_\mathrm{i})t]$$
$$= \delta(P_\mathrm{i}-P_\mathrm{f}-\hbar k)\delta(E_\mathrm{i}+\varepsilon_\mathrm{i}-\hbar\omega_k - E_\mathrm{f}-\varepsilon_\mathrm{f})\cdot q\frac{m_2}{M}\int_{-\infty}^{+\infty}\mathrm{d}r\,\varphi^*_{\varepsilon_\mathrm{f}}(r)r\varphi_{\varepsilon_\mathrm{i}}(r). \tag{35}$$

The delta functions announce conservation of energy and momentum. A simple calculation will show that it is impossible to satisfy both these conservation conditions as long as we demand that $\varepsilon_\mathrm{f} > \varepsilon_\mathrm{i}$. In other words, the detector will not get excited when Φ is in the vacuum state and the motion is inertial.

We are now ready to consider the case when the detector is accelerated. The simplest way to accelerate the particles m_1 and m_2 uniformly, is to add the extra potential,

$$U_{\text{accl}} = m_1 g x_1 + m_2 g x_2, \tag{36}$$

to the Lagrangian. In the centre-of-mass coordinates, the Lagrangian will now appear as,

$$L = (\tfrac{1}{2}M\dot{R}^2 + MgR) + (\tfrac{1}{2}\mu\dot{r}^2 - V(r)) + V_{\text{pert}}. \tag{37}$$

Classically, this represents a system whose centre-of-mass is in uniformly accelerated motion with acceleration g. This is the simplest possible description of a uniformly accelerated non-relativistic, quantum mechanical detector.

The unperturbed stationary states of the system are now described by the wavefunction (we shall use the same symbol as in (30), hoping no confusion will arise),

$$\Psi_{E,\varepsilon}(R,r,t) = \eta_E(R)\varphi_\varepsilon(r)\exp[-\mathrm{i}/\hbar^{-1}(E+\varepsilon)t] \tag{38}$$

where $\eta_E(R)$ are essentially Airy functions discussed in § 3, satisfying the equation

$$\left(-\frac{\hbar^2}{2M}\frac{\mathrm{d}^2}{\mathrm{d}R^2} + MgR\right)\eta_E(R) = E\eta_E(R) \tag{39}$$

and $\varphi_\varepsilon(r)$ are the same functions encountered previously (31). Notice that $\eta_E(R)$ are energy eigenfunctions but are *not* momentum eigenfunctions. This is to be contrasted with the situation when the centre-of-mass was moving inertially. The plane wave solutions are simultaneous eigenfunctions of energy and momentum.

Let us again consider the case when the field Φ is initially in the vacuum state. We are interested in the matrix element describing the transition from one internal state $\varphi_{\varepsilon_\mathrm{i}}(r)$ to another state $\varphi_{\varepsilon_\mathrm{f}}(r)$ *with* $\varepsilon_\mathrm{f} > \varepsilon_\mathrm{i}$. The matrix element, (following an analysis similar to the one that led to (35)) is given by

$$\mathcal{M} = \int_{-\infty}^{+\infty} \mathrm{d}t\langle \mathrm{f}|V|\mathrm{i}\rangle = \int_{-\infty}^{+\infty} \mathrm{d}R \int_{-\infty}^{+\infty} \mathrm{d}r \int_{-\infty}^{+\infty} \mathrm{d}t(2\omega_k)^{-1/2} \exp[\mathrm{i}(\omega_k t - kR)]\eta^*_{E_\mathrm{f}}(R)\eta_{E_\mathrm{i}}(R)$$
$$\times \exp[\mathrm{i}/\hbar(E_\mathrm{f}-E_\mathrm{i})t]\varphi^*_{\varepsilon_\mathrm{f}}(r)(qm_2r/M)\varphi_{\varepsilon_\mathrm{i}}(r)\exp[\mathrm{i}\hbar^{-1}(\varepsilon_\mathrm{f}-\varepsilon_\mathrm{i})t]$$
$$= \frac{2\pi}{(2\omega_k)^{-1/2}}\delta(E_\mathrm{f}+\varepsilon_\mathrm{f}+\hbar\omega_k - E_\mathrm{i}-\varepsilon_\mathrm{i}) \int_{-\infty}^{+\infty} \mathrm{d}r\,\varphi^*_{\varepsilon_\mathrm{f}}(r)(qm_2r/M)\varphi_{\varepsilon_\mathrm{i}}(r)$$
$$\times \int_{-\infty}^{+\infty} \mathrm{d}R\,\eta^*_{E_\mathrm{f}}(R)\eta_{E_\mathrm{i}}(R)\exp(-\mathrm{i}kR) \tag{40}$$

using the formulae of § 3, one can show that

$$\left|\int_{-\infty}^{+\infty} \mathrm{d}R\,\eta^*_\mathrm{f}(R)\eta_\mathrm{i}(R)\exp(-\mathrm{i}kR)\right|^2 = (2\pi)^{-1}(g\hbar|k|)^{-1}. \tag{41}$$

Therefore the probability per second for the transition from $|\mathrm{i}\rangle$ to $|\mathrm{f}\rangle$ is given by

$$|\mathcal{M}|^2 = \left(\frac{\pi}{g\hbar\omega_k^2}\right)q^2\left(\frac{m_2}{M}\right)^2\langle r_\mathrm{fi}\rangle^2\delta(E_\mathrm{i}+\varepsilon_\mathrm{i}-E_\mathrm{f}-\varepsilon_\mathrm{f}-\hbar\omega_k). \tag{42}$$

(We have, as usual, interpreted the square of the delta function as giving the rate of transition.) Quite clearly, transition from ε_i to ε_f is possible *even when* $\varepsilon_\mathrm{f} > \varepsilon_\mathrm{i}$. There will be a corresponding transition (in the downward direction) in the centre-of-mass energy maintaining overall conservation of energy. In the absence of the accelerating potential, one has to conserve *both* energy and momentum which forbids transitions with $\varepsilon_\mathrm{f} > \varepsilon_\mathrm{i}$. The externally applied, coordinate dependent potential, absorbs the recoil momentum in this case making this transition possible.

The delta function in (42) clearly shows that the energy released in the change of centre-of-mass motion (namely $E_\mathrm{i} - E_\mathrm{f} > 0$) is (a) partly emitted as a scalar field energy $\hbar\omega$ and (b) partly used to excite the internal mode from ε_i to ε_f. In other words, the scalar field introduces an indirect coupling between the internal mode and the centre-of-mass motion. The process is completely energy conserving.

At this stage one may raise the following question. The above analysis crucially depends on the MgR term—the accelerating potential—in the Lagrangian in (37). It is this term which changes the plane wave dependence of the centre-of-mass wavefunction to the $\eta_E(R)$ functions. Nevertheless, we know from the analysis in § 3 that one can make a transformation to an accelerated frame, thereby completely eliminating the MgR coupling. How can one explain the transition in this frame?

The transformation in question is given by the equations, (the bar denotes accelerated frame coordinates)

$$x_1 = \bar{x}_1 + \tfrac{1}{2}gt^2, \qquad x_2 = \bar{x}_2 + \tfrac{1}{2}gt^2 \tag{43}$$

in other words, r remains unchanged and

$$R = \bar{R} + \tfrac{1}{2}gt^2. \tag{44}$$

The Lagrangian becomes

$$\bar{L} = \tfrac{1}{2}M\dot{\bar{R}}^2 + (\tfrac{1}{2}\mu\dot{r}^2 - V(r)) + q(\bar{R} + \tfrac{1}{2}gt^2 + (m_2/M)r)\Phi(\bar{R} + \tfrac{1}{2}gt^2, t). \tag{45}$$

The form of the last term is the key to the whole issue. The part containing $(\bar{R} + \tfrac{1}{2}gt^2)\Phi$

does not cause internal transition. However notice that the relevant perturbation,

$$V_{\text{pert}} = q(m_2/M)\hat{r}\hat{\Phi}(\bar{R}+\tfrac{1}{2}gt^2, t) \tag{46}$$

has the following matrix element, (omitting the centre-of-mass wavefunction)

$$\begin{aligned}\langle 1_k;\mathrm{f}|V_{\text{pert}}|0;\mathrm{i}\rangle &= q(m_2/M)\langle \mathrm{f}|\hat{r}|\mathrm{i}\rangle\langle 1_k|\Phi(\bar{R}+\tfrac{1}{2}gt^2, t)|0\rangle \\ &= q(m_2/M)r_{\mathrm{fi}}\exp[\mathrm{i}\hbar^{-1}(\varepsilon_{\mathrm{f}}-\varepsilon_{\mathrm{i}})t](2\omega_k)^{-1/2}\exp[\mathrm{i}(\omega_k t-\tfrac{1}{2}kgt^2-k\bar{R})]. \end{aligned}\tag{47}$$

The time dependence of Φ has picked up a t^2 term. In other words, the transition element will now conserve momentum but not energy. Let us evaluate the transition amplitude when the centre-of-mass term changes from E_{i} to E_{f} along with the transition in (47). We get

$$\begin{aligned}\mathscr{A} = \int_{-\infty}^{+\infty} \mathrm{d}t\langle \mathrm{f}|V_{\text{pert}}|\mathrm{i}\rangle &= \int_{-\infty}^{+\infty}\mathrm{d}R\int_{-\infty}^{+\infty}\mathrm{d}t\exp[\mathrm{i}\hbar^{-1}(E_{\mathrm{f}}-E_{\mathrm{i}})t] \\ &\quad\times\exp[\mathrm{i}\hbar^{-1}(\varepsilon_{\mathrm{f}}-\varepsilon_{\mathrm{i}})t]\exp[\mathrm{i}(\omega_k t-\tfrac{1}{2}kgt^2)] \\ &\quad\times qr_{\mathrm{fi}}\left(\frac{m_2}{M}\right)\frac{1}{\sqrt{2\omega_k}}\exp[\mathrm{i}\hbar^{-1}(P_{\mathrm{i}}-P_{\mathrm{f}}-\hbar k)R]\frac{1}{2\pi\hbar} \\ &= q\left(\frac{m_2}{M}\right)r_{\mathrm{fi}}\frac{1}{\sqrt{2\omega_k}}\delta(P_{\mathrm{i}}-P_{\mathrm{f}}-\hbar k)\left(\frac{2\pi}{\mathrm{i}kg}\right)^{1/2} \\ &\quad\times\exp\left(\frac{\mathrm{i}}{\hbar}\frac{1}{2kg}(E_{\mathrm{i}}+\varepsilon_{\mathrm{i}}-E_{\mathrm{f}}-\varepsilon_{\mathrm{f}}-\hbar\omega_k)^2\right).\end{aligned}\tag{48}$$

The transition probability for unit time reads as

$$|\mathscr{A}|^2 = q^2\left(\frac{m_2}{M}\right)^2\langle r_{\mathrm{fi}}\rangle^2\frac{\pi}{\hbar g\omega_k^2}\delta(P_{\mathrm{i}}-P_{\mathrm{f}}-\hbar k). \tag{49}$$

The process conserves momentum but not energy. This is to be expected because, in the accelerated frame, the scalar field Φ has a non-trivial time dependence.

In summary, the excitation of the internal detector modes are possible when there is an external accelerating potential. This potential supplies the necessary energy or momentum to the centre-of-mass motion which is required for the excitation as well as the emission of scalar quanta.

One also notices that the expressions in (42) and (49) are different, from the physical point of view. In (42) the transition matrix is evaluated between two stationary states in the presence of MgR potential. As noted in § 3, these states do *not* transform to the plane wave states in the rest frame. In (49) we have evaluated the matrix between the plane wave states which are physically different from the states used in (42). (Since we are transforming everything as scalars, it is clear that matrix elements between the same physical states will have the same numerical value. This is a trivial and uninteresting result.)

5. Conclusion

We have demonstrated that the energy for the excitation of the detector comes from the accelerating potential. This was often conjectured in the literature, though—as far as the author knows—has never been explicitly proved. From this point of view, the delta functions that occur in (42) and (49) are the main results of the paper.

It should be noted that there is a fundamental distinction between choosing a Killing vector field for which the integral curves are geodesics and choosing a field for which the integral curves are not geodesics. The analysis in this paper shows that the accelerated detector can click only if the wavefunction describing the centre-of-mass motion of the detector is influenced by the non-geodesic nature of the path and passes on this influence to the internal state of the detector through the perturbing potential. This is a feature which is lacking in simple 'one-particle' detectors, discussed in earlier literature (e.g. [3, 4]), and is the main advantage of using the 'two-particle' model to interpret the detection process partly as a radiation process.

Notice that the excitation of the detector is accompanied, in both the inertial *and* accelerated frames, by the emission of scalar quanta. In other words, the final Hilbert space state for the scalar field is a one-particle state $|1_k\rangle$ in both the frames. It seems reasonable to interpret the whole process as an emission of scalar quanta with the simultaneous excitation of the internal mode (the energy being supplied by the accelerating potential) rather than as any 'detection' of particles in the Minkowski vacuum. It is an undeniable mathematical result that there exists more than one (inequivalent) definition for positive frequency in the Minkowski frame. However, we had already shown, in § 2, that there is *no* one-to-one correspondence between what the 'model detector' sees and these inequivalent field modes. We believe that these two aspects are quite different and the results about the detector should be better interpreted as a radiation process.

Many possible extensions of the present formalism are possible. One may attempt a more rigorous analysis by omitting many approximations made in the discussion. More important, one should consider the case in which the acceleration is time dependent and vanishes asymptotically. These questions are under investigation.

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