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GRAVITY AS AN EMERGENT PHENOMENON*

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There are strong reasons to believe that the gravitational interaction — described in terms of a metric on a smooth space–time — is an emergent, long wavelength phenomenon, like elasticity. I describe a concrete framework for realizing this paradigm against the backdrop of several recent results. In this perspective, quantum fluctuations of the microscopic degrees of freedom of the space–time lead to residual random displacements of any null surface. The latter can be described in terms of an effective theory using an action associated with the normal displacements of the null surfaces. Extremizing this action leads to an equation *determining the background geometry.* The resulting theory is Einstein gravity at the lowest order with the Lanczos–Lovelock type quantum corrections. The metric is *not* a dynamical variable in this approach and gravity arises as a coarse-grained statistical feature of an underlying microscopic theory.

Keywords: Emergent gravity; horizon thermodynamics; black hole entropy.

In the study of ordinary solids, one can distinguish between three levels of description. At the macroscopic level, we have the theory of elasticity, which has a life of its own and can be developed purely phenomenologically. At the other extreme, the microscopic description of a solid will be in terms of the statistical mechanics of a lattice of atoms and their interaction.

Both of these are well known; but interpolating between these two limits is the thermodynamic description of a solid at finite temperature *which provides a crucial window on the existence of the corpuscular substructure of solids.* As Boltzmann taught us, heat is a form of motion and we would not have the thermodynamic layer of description if matter were a continuum all the way to the finest scales and atoms did not exist! *The mere existence of a thermodynamic layer in the description is proof enough that there are microscopic degrees of freedom.*

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Now move on from a solid to the space–time. Again, we should have three levels of description. The macroscopic level is the smooth space–time continuum with a metric tensor $g_{ab}(x^i)$, and the equations governing the metric have the same status as the phenomenological equations of elasticity. At the microscopic level, we expect a quantum description in terms of the "atoms of space–time" and some associated degrees of freedom q_A which are still elusive. But what is crucial is the existence of an interpolating layer of thermal phenomenon associated with null surfaces in the space–time. Just as a solid cannot exhibit thermal phenomenon if it does not have microstructure, *the thermal nature of horizons cannot arise without the space–time having a microstructure.* This is the key theme I will describe here.

The above paradigm[1–6] is based on a fundamental relationship between the dynamics of gravity and the thermodynamics of horizons, and the following three results are strongly supportive of the above point of view:

- There is a deep connection between the dynamical equations governing the metric and the thermodynamics of horizons. An explicit example was provided in Ref. 7, in the case of spherically symmetric horizons in four dimensions, in which it was shown that Einstein's equations can be interpreted as a thermodynamic relation, $TdS = dE + PdV$, arising out of virtual radial displacements of the horizon. Further work showed that this result is valid in *all* the cases for which explicit computation can be carried out — like in the Friedmann models[8,9] as well as for rotating and time-dependent horizons in Einstein's theory.[10]
- The Hilbert Lagrangian has the structure $L_{\rm EH} \propto R \sim (\partial g)^2 + \partial^2 g$. In the usual approach the surface term arising from $L_{\rm sur} \propto \partial^2 g$ has to be ignored or canceled to get Einstein's equations from $L_{\rm bulk} \propto (\partial g)^2$. But there is a peculiar (unexplained) relationship between $L_{\rm bulk}$ and $L_{\rm sur}$:

$$\sqrt{-g}L_{\rm sur} = -\partial_a \left(g_{ij} \frac{\partial \sqrt{-g} L_{\rm bulk}}{\partial(\partial_a g_{ij})} \right). \tag{1}$$

  This shows that the gravitational action is "holographic," with the same information being coded in both the bulk and surface terms and one of them is sufficient. One can indeed obtain Einstein's equations from an action principle which uses *only* the surface term and the virtual displacements of horizons.[11] Since the surface term has the thermodynamic interpretation as the entropy of horizons, this establishes a direct connection between space–time dynamics and horizon thermodynamics.
- Most importantly, recent work has shown that *all the above results extend far beyond Einstein's theory.* The connection between field equations and the thermodynamic relation $TdS = dE + PdV$ is not restricted to Einstein's theory (GR) alone, but is in fact true for the case of the generalized, higher derivative Lanczos–Lovelock gravitational theory in D dimensions as well.[12] The same is true[13] for the holographic structure of the action functional: the Lanczos–Lovelock action has the same structure and — again — the entropy of the horizons is related

to the surface term of the action. *These results show that the thermodynamic description is far more general than just Einstein's theory* and occurs in a wide class of theories in which the metric determines the structure of the light cones and null surfaces exist blocking the information.

The conventional approach to gravity fails to provide any clue to these results, just as Newtonian continuum mechanics — without corpuscular, discrete substructure for matter — cannot explain thermodynamic phenomena. *A natural explanation for these results requires a different approach to space–time dynamics*, which I will now outline.

Suppose that there are certain microscopic — as yet unknown — degrees of freedom q_A (which are analogous to the atoms in the case of solids) described by some microscopic action functional, $A_{\rm micro}[q_A]$. In the long wavelength limit, we describe the same system in terms of a metric. Interpolating between the two is the thermodynamic description which is closely linked to the information blocked by null surfaces. In the case of a solid, the relevant long wavelength dynamics is captured by the *displacement vector field* which occurs in the equation $x^a \to x^a + \xi^a(x)$. We see from the results mentioned earlier that, in the case of gravity, the virtual displacements of null surfaces, described by the set of all null normals ξ^a, play a key dynamical role. That is, just as the displacement vector captures the macro-description in the case of solids, the normal displacements of null surfaces capture the essential macro-description in the case of gravity. This, in turn, can be described in terms of an an effective action/entropy $S[\xi^a]$ associated with the null normals ξ^a in the space–time. (Since the Euclidean action is closely related to the entropy, we need not distinguish between an action functional and an entropy functional at this stage.) More formally, we expect the coarse-graining of microscopic degrees of freedom to lead to an effective action in the long wavelength limit:

$$\sum_{q_A} \exp(-A_{\rm micro}[q_A]) \to \exp(-S[\xi^a]). \tag{2}$$

The general form of $S[\xi^a]$ in such an effective description, at the quadratic order, will be

$$S[\xi] = \int_{\mathcal{V}} d^D x \sqrt{-g}\left(4P_{ab}{}^{cd}\nabla_c\xi^a\nabla_d\xi^b - T_{ab}\xi^a\xi^b\right), \tag{3}$$

where $P_{ab}{}^{cd}$ and T_{ab} are two tensors (and the signs, notation etc. are chosen with hindsight). There is, however, one crucial difference between the dynamics in gravity and elasticity. In the latter, extremizing the entropy function will lead to an equation *for* the displacement field and determine ξ^a. In the case of space–time, the variational principle should be valid for all null vectors ξ^a and provide a condition for the *background metric.* This imposes the following two restrictions on the structure of $P_{ab}{}^{cd}$ and T_{ab}. First, the tensor P_{abcd} should have the algebraic symmetries similar to the Riemann tensor R_{abcd} of the D-dimensional space–time.

Second, we need

$$\nabla_a P^{abcd} = 0 = \nabla_a T^{ab} \,. \tag{4}$$

In a complete theory, the form of P^{abcd} will be determined by the long wavelength limit of the microscopic theory, just as the elastic constants can — in principle — be determined from the microscopic theory of the lattice. In the absence of such a theory, we can take a cue from the renormalization group theory and expand P^{abcd} in powers of derivatives of the metric.[11,14] That is, we expect that

$$P^{abcd}(g_{ij}, R_{ijkl}) = c_1 \overset{(1)}{P}{}^{abcd}(g_{ij}) + c_2 \overset{(2)}{P}{}^{abcd}(g_{ij}, R_{ijkl}) + \cdots \,, \tag{5}$$

where $c_1, c_2, \ldots$ are coupling constants and the terms progressively probe smaller and smaller scales. The lowest order term must clearly depend only on the metric with no derivatives. The next term depends (in addition to the metric) linearly on the curvature tensor, and the next one will be quadratic in curvature, etc. It can be shown that the mth order term which satisfies our constraints is *unique* and is given by

$$\overset{(m)}{P}{}_{ab}^{\ cd} \propto \delta^{cda_3\ldots a_{2m}}_{abb_3\ldots b_{2m}} R^{b_3b_4}_{a_3a_3} \cdots R^{b_{2m-1}b_{2m}}_{a_{2m-1}a_{2m}} = \frac{\partial \mathcal{L}^{(D)}_m}{\partial R^{ab}{}_{cd}} \,, \tag{6}$$

where $\delta^{cda_3\ldots a_{2m}}_{abb_3\ldots b_{2m}}$ is the alternating tensor and the last equality shows that it can be expressed as a derivative of the mth order Lanczos–Lovelock Lagrangian[11,15–17] with respect to the curvature tensor. The lowest order term (which leads to Einstein's theory) is

$$\overset{(1)}{P}{}^{ab}_{cd} = \frac{1}{16\pi}\frac{1}{2}\delta^{a_1a_2}_{b_1b_2} = \frac{1}{32\pi}(\delta^a_c\delta^b_d - \delta^a_d\delta^b_c) \,, \tag{7}$$

and the first order term (which gives the Gauss–Bonnet correction) is

$$\overset{(2)}{P}{}^{ab}_{cd} = \frac{1}{16\pi}\frac{1}{2}\delta^{a_1a_2a_3a_4}_{b_1\,b_2\,b_3\,b_4} R^{b_3b_4}_{a_3a_4} = \frac{1}{8\pi}(R^{ab}_{cd} - G^a_c\delta^b_d + G^b_c\delta^a_d + R^a_d\delta^b_c - R^b_d\delta^a_c), \tag{8}$$

and so on.

In our paradigm based on Eq. (2), the field equations for gravity arise from extremizing S with respect to variations of the null vector field ξ^a and demanding that the resulting condition hold for *all null vector fields*. That is, the "equilibrium" configurations of the "space–time solid" are the ones in which the entropy associated with the normal displacements of *every* null vector is extremized. Direct calculation shows[14] that the resulting constraint on the background metric takes the form

$$16\pi\left[P_b^{\ ijk} R^a{}_{ijk} - \frac{1}{2}\delta^a_b \mathcal{L}^{(D)}_m\right] = 8\pi T^a_b + \Lambda\delta^a_b. \tag{9}$$

These are identical to the field equations for Lanczos–Lovelock gravity with a cosmological constant arising as an undetermined integration constant. To the lowest order, when we use Eq. (7) for $P_b^{\ ijk}$, Eq. (9) reduces to $R^a_b - (1/2)\delta^a_b R = 8\pi T^a_b + \Lambda\delta^a_b$ and reproduces Einstein's theory and allows us to identify T^a_b as the matter stress tensor. More generally, we get Einstein's equations with higher order corrections which are to be interpreted as emerging from the derivative expansion of the action functional as we probe smaller and smaller scales. *Remarkably enough, we can derive not only Einstein's theory but even Lanczos–Lovelock theory from a dual description in terms of the normal displacements of null surfaces, without varying g_{ab} in an action functional!*

The physical meaning of the functional S — which we have been calling action or entropy — becomes clear when we evaluate the value of the functional on-shell. When evaluated on-shell it correctly reproduces[14] the entropy of the horizons:

$$S|_{\mathcal{H}} = \sum_{m=1}^{K} 4\pi m c_m \int_{\mathcal{H}} d^{D-2}x_{\perp}\sqrt{\sigma}\mathcal{L}_{(m-1)}^{(D-2)} = \frac{1}{4}[\text{area}]_{\perp} + \text{corrections}, \tag{10}$$

where $x_{\perp}$ denotes the transverse coordinates on the horizon $\mathcal{H}$, σ is the determinant of the intrinsic metric on $\mathcal{H}$ and we have restored a summation over m, thereby giving the result for the most general Lanczos–Lovelock case. The expression (10) *is precisely the entropy of a general Killing horizon in Lanczos–Lovelock gravity* based on the general prescription given by Wald and others[18,19] and computed by several authors. Further, in any space–time, if we take a local Rindler frame around any event we will obtain an entropy for the locally defined Rindler horizon. In the case of GR, this entropy per unit transverse area is just 1/4, as expected.

These results suggest an attractive conceptual framework for realizing the paradigm mentioned at the beginning. The key point is that our thermodynamic description is far more general — as it should be, since it encodes part of microscopic physics — than just Einstein gravity. It is based only on the following chain of reasoning:

- The principle of equivalence requires gravity to be described by a metric and light rays to be affected by gravity.
- Every null surface in the space–time acts as a one-way membrane ("local Rindler horizon") for a suitable class of observers (i.e. a congruence of timelike curves).
- The microscopic degrees of freedom q_A in Eq. (2) are subject to quantum fluctuations which are analogous to lattice vibrations in a solid. Near the one-way boundaries in the space–time, these fluctuations lead to changes in the amount of information accessible to some observer and thus a TdS contribution at any null surface. (That a boundary can generate a macroscopic residue from virtual quantum fluctuations is well known in the context of the Caisimir effect.)
- In the long wavelength limit, the resulting features can be captured in terms of an effective theory related to the degrees of freedom contained in the normal displacements of the null surfaces. This is obvious from the previous results which established the connection between the thermodynamic description based on virtual displacements of a null surface and the field equations of the theory.[7–12]
- The theory one obtains is far more general than Einstein gravity since it incorporates some of the microscopic corrections. Einstein's equations provide the lowest

order description of the dynamics, and *calculable*, higher order corrections arise as we probe smaller scales.

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