

A&A manuscript no.
(will be inserted by hand later)

Your thesaurus codes are:
12 (12.07.1; 12.04.3; 11.03.1; 08.19.4)

Observing high-redshift Supernovae in lensed galaxies

Tarun Deep Saini¹, Somak Raychaudhury¹, and Yuri A. Shchekinov^{2,3} *

¹ Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411007, India

² Department of Physics, Rostov University, Sorge, 5, 344090 Rostov on Don, Russia

³ Osservatorio Astrofisico di Arcetri, Largo E. Fermi, 5, 50125 Firenze, Italy

Received ; accepted

Abstract. Supernovae in distant galaxies that are gravitationally lensed by foreground galaxy clusters make excellent cosmological candles for measuring quantities like the density of the Universe in its various components and the Hubble constant. Distant supernovae will be more easily detectable since foreground cluster lenses would magnify such supernovae by up to 3–4 magnitudes. We show that in the case of the lens cluster Abell 2218, the detectability of high-redshift supernovae is significantly enhanced due to the lensing effects of the cluster. Since lensed supernovae will remain point images even when their host galaxies are stretched into arcs, the signal-to-noise ratio for their observation will be further enhanced, typically by an order of magnitude. We recommend monitoring well-modelled clusters with several known arclets for the detection of cosmologically useful SNe around $z = 1$ and beyond.

Key words: Cosmology: gravitational lensing — Cosmology: distance scale — Galaxies: clusters: general— Stars: supernovae: general

1. Introduction

Observations of distant sources with known absolute luminosity (cosmological standard candles) are of primary importance to modern cosmology, since the relation between the apparent magnitude, luminosity and redshift of distant galaxies can be used to determine the Hubble constant H_0 , the deceleration (or density) parameter q_0 , and the cosmological constant Λ . Observations of standard candles beyond $z = 0.05$ (where peculiar velocities are small) can yield the value of H_0 with reasonably small uncertainty (*e.g.*, Filippenko 1996, Hamuy *et al.* 1996). Observations of standard candles at $z > 0.3$ are being used for the determination of the fraction of the total energy of the Universe in matter Ω_M and in some hitherto unknown form Ω_Λ (Riess *et al.* 1998b, Perlmutter *et al.* 1999,

Saini *et al.* 2000). The study of the gravitational magnification of standard candles at even higher redshift will put tighter constraints on dark matter models of cosmogony (*e.g.*, Kolatt & Bartelmann 1998, Marri & Ferrara 1998, Holz 1998, Metcalf 1999, Porciani & Madau 2000).

The work of Riess *et al.* (1998b) and Perlmutter *et al.* (1999) has shown that, if detected significantly earlier than the epoch of their peak luminosity, type Ia supernovae (SNe Ia) would be the most useful among cosmological candles at high redshift. However, the required integration times for good photometry and for obtaining spectra of such supernovae at redshift $z \sim 1$ are estimated to be tens of hours on a 10m telescope for $0''.75$ seeing (Goobar & Perlmutter 1995). These observations would clearly be more favourable if these supernovae occur in galaxies magnified by gravitational lensing.

The magnification due to lensing can be significant enough to make possible the detection of supernovae (SNe) in galaxies at high redshifts ($z \gtrsim 1$). Narasimha & Chitre (1988) first pointed out that such events in giant luminous arcs (as in the A370 system) can be used as a test of the lens models. In the case of multiply imaged supernovae, Kovner & Paczyński (1988) deduce simple relations between the magnification of such a SN, the separation of images, and the differences between the arrival times of the event in different images.

Indeed, such SNe would serve as a unique probe for not only the distribution of matter in the clusters, but also for studying the source galaxies themselves. Due to the increased flux produced by the magnification of the images, photometric and spectroscopic studies of very distant galaxies can become possible. This would enable us to obtain information, which would be otherwise unavailable, about the star formation process in the young galaxies (Mellier *et al.* 1991, Yee & de Robertis 1991), the evolutionary status of AGN (Stickel *et al.* 1991), and even the morphology of distant galaxies (Colley, Tyson & Turner 1996). Indeed, one of the farthest known galaxies (at $z=4.92$, Franx *et al.* 1997) would not have been detected had it not been for the 10-fold magnification by the cluster CL1358+62 at $z = 0.33$.

Send offprint requests to: Tarun Deep Saini

* saini, somak@iucaa.ernet.in; yus@rsuss1.rnd.runnet.ru

In this paper we address the feasibility of detecting lensed SN events in high redshift galaxies which would be useful in the measurement of cosmological parameters. From a qualitative point of view such a study seems worthwhile for several reasons. For a typical magnification of 3–4 mag (Kovner & Paczyński 1988) the study of lensed SNe stretches the usefulness of using them to characterize the distance ladder to further distances by a factor of 4–6, or, equivalently, results in a considerable decrease in the required duration of observation. Furthermore, although galaxies lensed into arcs are resolved in one direction due to stretching, a supernova in such a galaxy will remain a point source, hence the signal-to-noise ratio (SNR) of a lensed supernova in an arc will be superior to that of one in an unlensed galaxy. Finally, a cluster lens typically produces multiple images with time delays between them being up to several months, thereby making it possible to observe the same SN again, and measuring its light curve more accurately, particularly in its pre-peak phase.

In the searches we propose, we do not have to be confined to one lensed SN at a time. In many known cases of gravitational lensing of background objects by galaxy clusters, several arcs and arclets can be found in an area of the sky typically imaged by a single CCD frame. In the case of Abell 2218, for instance, there are 30 observed arclets (Ebbels *et al.* 1998, Bézecourt *et al.* 1998) with $R \leq 23.5$ and $\mu_R \leq 25.5$ between $z = 0.5$ to 1.5, so clearly a lot of galaxies can be simultaneously monitored. This is also true of the cluster Abell 2390 at $z = 0.23$, in which, in addition to the famous “straight arc” (triple image of a galaxy at $z = 0.913$), there are at least 12 arclets ($R < 21$) between $z = 0.4$ –1.3 in an area of 2.7×2.7 arcmin² around it (Bézecourt & Soucail 1997). In the same area, in the magnitude range $21 < R < 23.5$, there are four images of two galaxies at $z = 4.05$ (Pelló *et al.* 1998, Frye and Broadhurst 1998) lensed by the same cluster.

Similarly, in a single HST WFPC field, which covers much less area (~ 5 arcmin²) than most CCD cameras on terrestrial telescopes, one finds 20 arclets brighter than $m_{F675W}=24$ (corresponding to at least 15 independent galaxies) beyond $z=0.7$ in the cluster Abell 370 ($z=0.37$, Bézecourt *et al.* 1999), and 14 arclets corresponding to 5 galaxies between $z = 0.5$ –1.5 in the cluster MS0440+0204 at $z=0.197$ (Gioia *et al.* 1998). Not all of these, of course, would be magnified to the same degree, but on average they would be magnified, making them easier to be observed than in an unlensed case.

Hereafter, we briefly review the usefulness of SN Ia and SN IIL in the determination of cosmological parameters, and in §3 quantify the effect of a cluster lens on the detectability of high- z SNe. We illustrate our case with the cluster lens Abell 2218. In §4, we show that, in addition to this, for SNe detected in giant arcs, the signal-to-noise ratio for the detection of SNe is enhanced by an order of

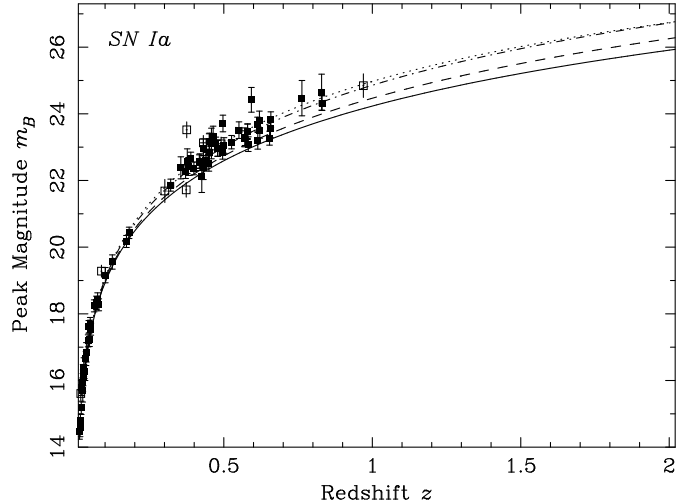


Fig. 1. The corrected apparent peak B magnitudes of 79 published Type Ia supernovae (Perlmutter *et al.* 1999, Riess *et al.* 1998, Hamuy *et al.* 1996) are plotted against redshift. The filled boxes represent SNe that have been used elsewhere (Perlmutter *et al.* 1999, Saini *et al.* 2000) to determine cosmological parameters, while the open boxes indicate SNe that are generally left out (including SN1997ck at $z = 0.97$ which has not been spectroscopically confirmed to be type Ia). The lines represent models with $(\Omega_M, \Omega_\Lambda)$: (a) Standard Cosmology (1.0, 0.0), full line, (b) Open matter-only Universe (0.3, 0.0), dashed line (c) Perlmutter *et al.* best flat model: (0.28, 0.72), dot-dashed line, and (d) Perlmutter *et al.* best general model: (0.73, 1.32), dotted line.

magnitude due to lensing. We summarize in §5 in the light of ongoing SN searches.

2. Standard Bombs: Supernovae of Type Ia & IIL

Two subgroups of SNe seem to be relevant for cosmological use: SNe Ia and SNe IIL. SNe II are more frequent (by a factor of ~ 4 , van den Bergh & Tammann 1991) in late spirals (Sbc-Sd), which are the most numerous among field galaxies. It is more likely that supernovae at $z > 1$, where the dependence of the luminosity distance on models is the most sensitive (Fig. 1), will be of type-II (*e.g.*, Madau *et al.* 1998). Though SNe II maxima in general are known to have a large spread in luminosities, about half of them (those that have “linear” light curves, SNe IIL, Young & Branch 1989, Cappellaro *et al.* 1997) represent very good standard bombs (Gaskell 1992), since they have a small intrinsic scatter ($\sigma = 0.3$ mag) around their peak magnitude ($\langle M_B \rangle = -17.05$, Miller & Branch 1990, $H_0 = 75$).

However, SNe Ia are much more luminous at their peak ($\langle M_B \rangle = -18.95$ for $H_0 = 75$), with a smaller scatter in peak magnitude, if corrected for the slope of their light curve (Perlmutter *et al.* 1999, Riess *et al.* 1998b).

The major problem in the use of high redshift SNe as standard candles lies in the identification of the type of the SN, and its photometric calibration. The identification of the type of the SN depends much on the shape of the light curve, which at high redshifts will be time dilated, making it easier to determine its shape. The magnified flux will render it easier to obtain a spectroscopic identification as well. It is therefore obvious that due to the considerable magnification, lensed SNe will be far more suitable candidates than unlensed SNe at the same redshift.

3. The Effect of Lens Magnification on detectability

3.1. Gravitational magnification

We summarize briefly the essential results needed for this paper. Excellent reviews of gravitational lensing can be found elsewhere (*e.g.*, Blandford & Narayan 1986, Schneider *et al.* 1992).

The basic equation which relates the angular coordinates of the source (β_1, β_2) to those of the image (θ_1, θ_2) is

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \nabla\psi(\boldsymbol{\theta}). \quad (1)$$

The dimensionless relativistic lens potential satisfies the two dimensional Poisson equation $\nabla^2\psi(\boldsymbol{\theta}) = 2\kappa(\boldsymbol{\theta})$, the convergence $\kappa(\boldsymbol{\theta}) = \Sigma(\boldsymbol{\theta})/\Sigma_{cr}$, $\Sigma(\boldsymbol{\theta})$ being the two dimensional surface mass density of the lens, and $\Sigma_{cr} = (c^2/4\pi G)(d_s/d_l d_{ls})$ is the critical density. Here the distance between the observer and the source, that between the observer and the lens and that between the lens and the source are d_s , d_l and d_{ls} respectively.

Gravitational lensing preserves the surface brightness of the light rays. The flux of light received by an observer is directly proportional to the solid angle subtended by the image at the observer. Since the solid angle of the image after lensing is, in general, different from that of the source the observer can receive more (or less) flux than in the unlensed situation. Thus galaxy clusters can act as gravitational telescopes by collecting light from the distant galaxies over a large area and sending it in our direction.

The shape and size of the image are related to that of the source by the transformation matrix $M_{ij}^{-1} = \partial\beta_i/\partial\theta_j$. This matrix is generally written in the form

$$M^{-1} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}, \quad (2)$$

where κ is the usual convergence and $\gamma_1 = 1/2(\partial^2\psi/\partial\theta_1^2 - \partial^2\psi/\partial\theta_2^2)$, $\gamma_2 = \partial^2\psi/\partial\theta_1\partial\theta_2$ are the components of the shear. The magnification for a point source is given by the Jacobian of the inverse mapping $f: \boldsymbol{\beta} \mapsto \boldsymbol{\theta}$, which is, in general, one to many. From equation (2) we find that the magnification

$$\mu \equiv \det[M] = 1/((1 - \kappa)^2 - \gamma^2), \quad (3)$$

where $\gamma^2 = \gamma_1^2 + \gamma_2^2$, and is therefore different for different images. The set of all the points where $\det[M^{-1}]$ vanishes (singular points) in the source plane is called the caustic set, and the images of the caustic set are called the critical curves. A source of finite size (*i.e.*, a galaxy) close to the caustic produces magnified images near the critical curves. Any point source within these galaxies will also be substantially magnified.

For well-studied cluster lenses which have $\gtrsim 10$ arclets, the model parameters of the lens can be constrained well enough so that the magnification of a SN occurring at any point on an arclet can be estimated reliably to an accuracy of $\lesssim 0.5$ mag. In this paper, we suggest the monitoring of such well-modelled lenses to look for cosmologically significant high- z SNe.

3.2. The enhancement of detectable events

A SN event at a redshift $z \sim 1$ is expected to have an apparent blue magnitude $m_B \sim 25$. If we are monitoring a system of arclets with an observing setup (telescope and detector) of limiting magnitude of detection m_{lim} (given an acceptable value of S/N), then we would like to estimate the probability that the SNe might be magnified by an amount $\Delta m = m_B - m_{lim}$. Given this probability and the SNe rates, one can obtain an estimate of the expected number of detectable events.

Consider an area of the sky of a few arcmin² around the centre of a cluster at redshift z_L being monitored with an array of CCDs. Within this region, let the number of SNe occurring per year in galaxies between redshifts z_s and $z_s + dz_s$ be

$$dN = N_0 \mathcal{N}(z_s) dz_s \quad (4)$$

where

$$\int_{z_L}^{z_{max}} \mathcal{N}(z_s) dz_s = 1.$$

If m_{lim} is the limiting magnitude, then the threshold magnification at a redshift z_s for the source of unlensed magnitude m_s to be detected will be $\mu_0(z_s) = 10^{(m_s - m_{lim})/2.5}$. The number of these SNe being magnified by a ratio $\mu > \mu_0(z_s)$ is

$$dN' = N_0 \mathcal{N}(z_s) P(\mu > \mu_0(z_s)) dz_s. \quad (5)$$

where

$$P(\mu > \mu_0(z_s)) = \frac{1}{\pi\beta_0^2} \int \frac{\Theta[|\mu(\boldsymbol{\theta})| - \mu_0(z_s)] \Theta[\beta_0^2 - \beta(\boldsymbol{\theta})^2]}{|\mu(\boldsymbol{\theta})|} d^2\theta, \quad (6)$$

for a source at redshift z_s . Here Θ is the Heaviside step function, $\beta_0(z_s)$ is the radius of the source (assumed circular) and the integral is performed over the field of the observed image. Since a single source can produce more

than one image, the value of this quantity, for a given threshold magnification μ_0 , can be greater than one.

Here we would quantify the enhancement in the detection of distant SNe by

$$\Phi_L(z) = \int_z^{z_{\max}} \mathcal{N}(z_s) P(\mu > \mu_0(z_s)) dz_s. \quad (7)$$

This function represents the cumulative fraction of SNe that are observed, given the limiting magnitude m_{lim} of the observational setup, of the total number of SNe that occur between z and z_{\max} in the area of the sky that is being monitored. This depends upon the number density and redshift distribution of the host galaxies and the frequency of SN as a function of redshift. This should be compared with the quantity

$$\Phi_U(z) = \int_z^{z_{\max}} \mathcal{N}(z_s) \Theta(m_{\text{lim}} - m(z_s)) dz_s \quad (8)$$

which is the corresponding fraction that would be observed in the absence of the lens. For instance, since we assume the peak magnitude of a Type Ia SN to be $m_B = 25$ at $z = 1$, if $m_{\text{lim}} = 25$, the value of $\Phi_U(z > 1)$ will be zero whereas due to lensing $\Phi_L(z)$ can be finite till $z = z_{\max}$.

3.3. Example: the case of Abell 2218

To estimate the typical fraction of SNe which would be seen behind a cluster with a certain magnification, we consider the case of the well-studied cluster lens Abell 2218 ($z = 0.175$), for which good published mass models exist. Here we use the model of Kneib *et al.* (1996), where the bimodal mass distribution is represented by two cluster-scale clumps of dark matter centred on the two brightest elliptical galaxies, their potentials modelled by the difference of two pseudo-isothermal elliptical mass distributions (PIEMDs), with an external truncation radius. In addition, the small-scale mass structure is represented by galaxy-sized lenses corresponding to the 34 brightest galaxies belonging to the cluster, modelled by similar functions with the appropriate parameters (velocity dispersion, core radius and truncation radius) scaled to the observed luminosities of the galaxies.

There are 258 background galaxies in the magnitude range $21.5 < R < 25$, detected in the HST WFPC image of total area of 4.7 arcmin^2 analyzed by Kneib *et al.* Of these, 35 spectroscopic redshifts are known, and another 18 redshifts are estimated in Ebbels *et al.* (1998). For other galaxies, random redshifts were chosen in the range $0.175 < z < 2.5$, from a distribution that conserves the number of galaxies per unit comoving volume in a flat, matter dominated universe.

The redshift dependence of the frequencies $\mathcal{N}(z_s)$ of both SN Ia and SN II are taken from Madau *et al.* (1998), where the evolution of cosmic supernova rates with redshift is computed from estimates of the global history of

star formation compiled from multi-wavelength observations of faint galaxies. We assume that redshift dependence of Type III frequencies to be the same as that of Type II SNe taken as a whole. Here the absolute value of the frequency is not important, since we are interested in comparing the number of SNe that would be detectable in the presence of a lens to that if the lens were not there.

We take each of the background galaxies in our list, assuming their intrinsic sizes to be 10 Kpc, and map them back to the source plane, by means of the lens model and their measured/assumed redshift. The integrals (6) is performed in the image plane by summing over a fine grid, since the presence of the two Θ functions in the integrand makes it difficult to evaluate them using Gaussian quadrature.

We present the curves of $\Phi_L(z)$ for the Abell 2218 HST field for both SN Ia and SN II in Figures 2 and 3 respectively, where we consider SNe in lensed galaxies in the redshift range $0.175 < z_s < 2.5$. For comparison we also give the corresponding values $\Phi_U(z)$ for the unlensed case, to show the dramatic difference the presence of the lens makes. For example, for a limiting magnitude of $B = 25$, 20% of all SNe beyond $z > 1$ in the field of the cluster A2218 will be detected, none of which would have been detected if the lens were not present.

4. Signal to noise enhancement in giant arcs

One of the uncertainties in the accurate photometry of a SN comes from the correction for emission from the host galaxy at the site of the SN. The signal-to-noise ratio (SNR) related to this uncertainty for SNe in lensed galaxies which form significantly elongated arcs or arclets will be better than the SNR in the unlensed galaxies. An enhanced SNR will in turn favour the detection of a SN and the measurement of its characteristics.

For a seeing of 0.5–1 arcsec, a galaxy at $z = 1$ would occupy (referring to the area enclosing 90% of the light) typically ~ 20 pixels. On the other hand, a supernova in the galaxy being a point object would occupy the number of pixels covered by the seeing disk, *i.e.*, 3–4 pixels. Here we calculate the enhancement factor $\eta = SNR_{\text{lensed}}/SNR_{\text{unlensed}}$ for the SN in the galaxy image.

For the sake of simplicity, we assume that in the unlensed case the SN is not resolved. The total flux F_{tot} that we would receive is the sum of flux from the SN and the galaxy, so the SNR for the detection of the unlensed SN is given by

$$SNR \propto \frac{F_{\text{tot}} - F_{\text{gal}}}{\sqrt{F_{\text{tot}} + F_{\text{gal}}}}, \quad (9)$$

where F_{gal} is measured long after the occurrence of the SN, so that the latter no longer contributes to the flux from the galaxy.

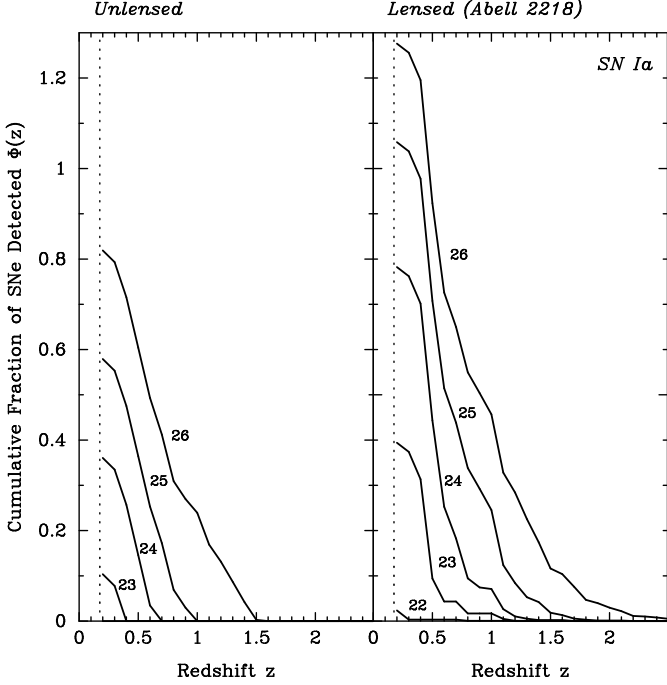


Fig. 2. The functions Φ (as defined in eqs 7 & 8), representing the cumulative fraction of detected SNe Ia as a function of redshift, is plotted for five different limiting B magnitudes of the observational setup. The *right panel* shows the function Φ_L (defined in eq 7) in the field of the lensing cluster A2218. Here we use the Kneib *et al.* (1996) model of the lens, which comprises of a bimodal distribution of dark matter and 34 of the brightest galaxies in the cluster. The *left panel* shows the corresponding function Φ_U (defined in eq 8), which represents the function in the absence of the cluster lens, for the same values of the limiting magnitudes. The z -dependence of the frequency of SNe Ia is taken from Madau *et al.* (1998). The peak magnitude of SN Ia at $z = 1$ is assumed to be $B = 25$. In the lensed case, Φ can be > 1 since sources can be mapped into multiple images.

In the lensed case the fluxes should be multiplied by an appropriate average magnification factor $\langle \mu \rangle$. The lensed galaxy is stretched into an arc in one direction, allowing us to define a stretch factor s , which is the ratio of the angular size of the seeing disk to that of the arc. The amount of galaxy light contaminating the SN flux is s times the unlensed value. Hence the *SNR* enhancement η in the lensed case is given by

$$\eta = \sqrt{\langle \mu \rangle} \left(\frac{\mathcal{F}_{tot} - sF_{gal}}{F_{tot} - F_{gal}} \right) \frac{\sqrt{F_{tot} + F_{gal}}}{\sqrt{\mathcal{F}_{tot} + sF_{gal}}} \quad (10)$$

where $\mathcal{F}_{tot} = sF_{gal} + F_{SN}$.

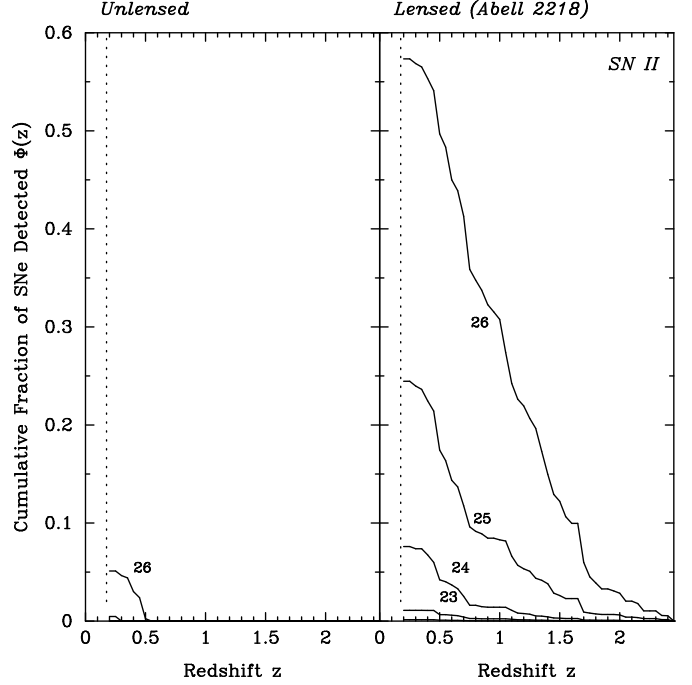


Fig. 3. The same as Figure 2, but for SN IIL. The z -dependence of the frequency of SNe II is taken from Madau *et al.* (1998) and is assumed to be the same for SN IIL. Here the peak magnitude at $z = 1$ is assumed to be $B = 27.5$.

The quantity in brackets is unity, since both numerator and the denominator are equal to F_{SN} . Denoting $r = F_{SN}/F_{gal}$ then the above formula simplifies to

$$\eta = \sqrt{\langle \mu \rangle} \frac{\sqrt{r+2}}{\sqrt{r+2s}}. \quad (11)$$

For a typical case considered above, $r \sim 0.2$, $s \sim 0.1$, $\langle \mu \rangle \sim 40$, which would give $\eta \simeq 15$. This shows that an order-of-magnitude enhancement in S/N ratio is achievable in lensed SNe that appear in giant arcs.

5. Conclusions

A considerable fraction of SNe in high redshift galaxies can be magnified by foreground galaxy clusters. This pushes the equivalent observational distance to SNe further by a factor of 2–3, and thus allows measurements of magnitudes and light-curves of high redshift SNe in significantly shorter observational periods.

Even a small telescope with a limiting magnitude of $m_{lim} = 24$ will detect SNe Ia up to $z \sim 1.4$ in the field of a lens like A2218, while in the absence of such a lens, the same setup would not be able to detect SN Ia beyond $z = 0.7$ (Figure 2). SNe of Type II will never be detected by such a setup beyond $z = 0.5$ for $m_{lim} < 26$, while in the presence of a A2218-like lens, they could be detected up to $z \sim 2$.

SNe occurring in giant arcs will have an additional gain of signal-to-noise of ~ 15 , making it easier to observe them. The magnification for lensed SNe does not change dramatically for sources between $z = 1-2$, while the surface brightness of the host galaxy decreases as $(1+z)^{-4}$, which in turn further improves the signal-to-noise ratio. Such SNe will also be multiply imaged, further constraining mass models from time-delay measurements.

A number of projects have been searching for SNe at moderate and high redshifts. Neither of the high- z SN search projects (Perlmutter *et al.* 1999, Schmidt *et al.* 1998) has yet discovered a SN with a favourable geometry behind a cluster. The low- z Abell cluster search (Riess *et al.* 1998a) considers clusters out to $z = 0.08$, again not ideal for these searches.

In order to find high- z SNe without having to wait for the commissioning of NGST, one needs to continually monitor clusters with known arcs with a favourable geometry (*e.g.*, Abell 2218, Abell 370, CL0024+17) in the spirit of the SN Cosmology searches. Once a SN event is observed, one needs to calculate the magnification (weakly model-dependent) from the location of the SN in the arc. This will yield the intrinsic apparent magnitude of the SN from the light curve, which will enable us to calculate the distance to the supernova independent of its redshift. For a SN in the redshift range $z = 1-2$, this method can even yield a reliable value of q_0 , because we would be measuring H_0 in the relativistic regime. As Fig. 1 shows, in this redshift range, SNe Ia with measured distances can distinguish, for example, between $\Lambda = 0$ and $\Lambda \neq 0$ models even if the lens model does not allow the estimation of the magnification to better than 0.5 mag.

Acknowledgements. We thank J.-P. Kneib for sending us the data for his observation and model of Abell 2218 in electronic form, and Rajaram Nityananda for useful conversations. YS acknowledges the hospitality of IUCAA, where this work began, and the hospitality of Astronomisches Institut, Ruhr Universität Bochum, where he was at the final stages of the work. He received partial financial support from the German Science Foundation (DFG) within Sonderforschungsbereich, a travel grant from the IAU, and a NATO Guest Fellowship grant (Ann. #219.29) from the Italian Consiglio Nazionale delle Ricerche. TDS thanks the University Grants Commission, India for providing the financial support (#2-5/93(II)-E.U. II) without which this work would not have been possible. This research has made use of NASA's Astrophysics Data System Abstract Service.

References

- Bézecourt J., Soucail G., 1997, A&A, 317, 661
 Bézecourt J., Pelló R., Soucail G., 1998, A&A, 330, 399
 Bézecourt J., Kneib J.-P., Soucail G., *et al.*, 1999, A&A, 347, 21
 Blandford R., Narayan R., 1986, ApJ, 310, 568
 Cappellaro E., Turatto M., Tsvetkov D.Yu., *et al.*, 1997, A&A, 322, 431
 Colley W.N., Tyson J.A., Turner E.L., 1996, ApJ, 461, L83
 Ebbels T., Ellis R., Kneib J.-P., *et al.*, 1998, MNRAS, 295, 75
 Filippenko A.V., 1996, In Thermonuclear Supernovae, Canal R., Ruiz-Lapuente P., Isern J., (eds.), Kluwer, Dordrecht
 Franx, M., Illingworth, G.D., Kelson, D.D., *et al.*, 1997, ApJ, 486, L75
 Frye, B., Broadhurst, T., 1998, ApJ, 499, L115
 Gaskell C.M., 1992, ApJ, 389, L17
 Gioia I.M., Shaya E.J., Le Fevre O., *et al.*, 1998, ApJ, 497, 573
 Goobar A., Perlmutter S., 1995, ApJ, 450, 14
 Hamuy M., Phillips M.M., Suntzeff N.B., *et al.*, 1996, AJ, 112, 2391
 Holz, D. E., 1998, ApJ, 506, L1
 Kneib J.-P., Ellis R. S., Smail, I., *et al.*, 1996, ApJ, 471, 643
 Kolatt T.S., Bartelmann M., 1998, MNRAS, 296, 763
 Kovner I., Paczyński B., 1988, ApJ, 335, L9
 Madau, P., Della Valle, M., Panagia, N., 1998, MNRAS, 297, L17
 Marri, S., Ferrara, A., 1998, ApJ, 509, 43
 Metcalf, R. B., 1999, MNRAS, 305, 746
 Mellier Y., Fort B., Soucail G., *et al.*, 1991, ApJ, 380, 334
 Miller D.L., Branch D., 1990, AJ, 100, 530
 Narasimha D., Chitre S.M., 1988, ApJ, 332, 75
 Pelló R., Kneib J.-P., Le Borgne J. F., *et al.*, 1999, A&A, 346, 359
 Perlmutter, S., Aldering, G., Goldhaber, G., *et al.*, 1999, ApJ, 517, 565
 Porciani, C., Madau, P., 2000, to appear in ApJ, astro-ph/9810403
 Riess D.J., Germany L.M., Schmidt B.P., *et al.*, 1998, AJ, 115, 26
 Riess, A., Filippenko, A. V., Challis, P., *et al.*, 1998, AJ, 116, 1009
 Saini, T. D., Raychaudhury, S., Sahni, V., *et al.*, 2000, PRL, submitted (astro-ph/9910231)
 Schmidt, B.P., Suntzeff, N.B., Phillips, M. -M., *et al.*, 1998, ApJ, 507, 46
 Schneider P., Ehlers J., Falco E.E., 1992, Gravitational Lenses. Springer Verlag, Berlin
 Stickel M., Padovani P., Urry C.M., *et al.*, 1991, ApJ, 374, 431
 van den Bergh S., Tammann G., 1991, ARA&A, 29, 363
 Yee H. K. C., de Robertis M. M., 1991, ApJ, 381, 386
 Young T. R., Branch D., 1989, ApJ, 342, L79