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Localized Branes and Black Holes

Sumati Surya
I.U.C.A.A., Post Bag 4, Ganeshkhind, Pune 411007, India

Donald Marolf
Physics Department, Syracuse University, Syracuse, New York 13244
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Abstract

We address the delocalization of low dimensional D-branes and NS-branes when they are a part of a higher dimensional BPS black brane, and the homogeneity of the resulting horizon. We show that the effective delocalization of such branes is a classical effect that occurs when localized branes are brought together. Thus, the fact that the few known solutions with inhomogeneous horizons are highly singular need not indicate a singularity of generic D- and NS-brane states. Rather, these singular solutions are likely to be unphysical as they cannot be constructed from localized branes which are brought together from a finite separation.

I. INTRODUCTION

The construction of BPS black brane and other solutions [1–8] from D-branes and NS-branes has been invaluable in studying black hole entropy [9–20], dualities [8,21,22], and other phenomena within string/M-theory [23,24]. Such black p branes are typically constructed from several types of D- or NS-branes, including branes of dimension less than p. Thus, from the perspective of these lower dimensional branes, the black p brane has transverse internal directions. It is an interesting fact that all known smooth solutions have horizons which are translationally invariant in the internal transverse directions [25–28]. Thus, all features of the black brane, including the distribution of D- and NS- branes as well as any waves that may be present (at least outside the horizon [28]) will share this invariance. This is in contrast to the fact that, as long as the branes remain separated (and properly oriented), one can construct static solutions describing such D- and NS- branes distributed in an arbitrary manner with respect to the internal directions. Indeed, any solution in which only a finite number of lower dimensional branes are present will not be translationally invariant in any internal transverse direction. Much the same feature arises in the construction of intersecting brane solutions [29–32] at various angles, as those branes also appear in a delocalized form.

It is important to note that related solutions *can* be constructed in which this internal transverse symmetry is broken either by the distribution of branes themselves or by various 'waves' that are associated with the branes. However, such solutions are highly singular [27,28]. Since a black brane is thought to correspond to an ensemble of D-brane states, the assumption that a generic bound state of D-branes would not have this internal transverse symmetry leads naturally to the conclusion that such singular states will dominate the physics near the horizon of a black brane [33–35]. It is therefore important to understand if this assumption is correct.

We address this issue below by analyzing in detail classical BPS solutions corresponding to collections of branes that are localized, but separated from one another. In particular, we consider the construction of BPS black fivebranes in ten dimensional string theory from onebranes and fivebranes, as is familiar from the study of black hole entropy for five dimensional (three charge) black holes [11].. The distributions of onebranes and waves are described by a collection of moments around an internal transverse torus. We study what happens when the separation between the onebranes and fivebranes goes to zero while the intrinsic moments that describe the distributions are held fixed. One special case of this process is when a fixed number of localized onebranes is brought into contact with the fivebranes. We find that the result is *not* a singular solution. Instead, as the infinite throat of the incipient black hole forms and the onebranes move farther and farther inside it, the various moments that describe the inhomogeneity of the solution are 'screened' from a distant observer. As measured from the external region, the multipole moments of the corresponding fields go to zero and the solution goes over to the well known homogeneous one. Thus, even if the solution is constructed by moving a single localized onebrane onto a fivebrane, the result is a smooth black hole solution with an exact translational symmetry in the internal transverse directions[1].

To see intuitively how such a result can arise, it is useful to consider an analogy with flat space electrostatics. Suppose that we have a very deep cylindrical well with conducting walls. We put some charge in a bucket at the top and begin to lower the bucket down into the well. The bucket and winch are arranged so that, instead of lowering the bucket down the center of the well, the bucket is lowered against one wall at some angular coordinate θ around the cylindrical well. In this case, the electric field emerging from the mouth of the well is not rotationally invariant, but encodes the angular position θ of the charged bucket. However, as the bucket is lowered into the well, this information will fade from view and the field at the mouth will become approximately rotationally symmetric. The well here is in analogy with the infinite throat of the incipient extremal black hole mentioned above. The detailed effects of the singular fields that arise in the brane case are studied below.

We will see that, in addition to the fivebrane case mentioned above, this same feature also arises in the construction of BPS black six-branes in ten dimensional string theory from Kaluza-Klein monopoles, fivebranes, and onebranes which is familiar from the study of black hole entropy for four dimensional (four charge) black holes. We discuss both cases below and look at potential inhomogeneities in a number of charges. For each case, this includes the distribution of onebranes and longitudinal momentum (in the terminology of [26–28]) along the onebranes. We also study the distribution of transverse momentum along the onebranes. The black fivebrane is addressed in section II and the black sixbrane is addressed in section III. We conclude with a few comments in section IV.

II. BUILDING A THREE CHARGE BLACK FIVEBRANE

Here we examine a class of BPS saturated black hole solutions of string theory compactified on $T^5 \times R^5$. In particular, we will find it convenient to study "chiral null models" so that we may use the results of [4]. Our solutions will be generalizations of the models studied in section 2-4 of [7]. Chiral null models, first discussed in [36], are exact solutions of string theory (type IIA, type IIB and heterotic) in which the only nonzero fields are the metric, the dilaton Φ, and a Neveu-Schwarz antisymmetric tensor field H_{ABC}. As a result, our BPS black branes (and related solutions) will be constructed entirely from Neveu-Schwarz objects. However, the associated results for D-branes follow directly by using S-duality.

We now briefly introduce these models since we will use them explicitly in our study of both the black fivebrane and black sixbrane solutions. The 2-dimensional σ-model Lagrangian coupled to the above fields is given by

$$L = (G_{AB} + B_{AB})(X)\partial X^A \bar{\partial} X^B + R\Phi(X), \tag{2.1}$$

[1] At least outside the horizon. It is less clear how to construct an interior solution by such methods.

where $G_{AB}(X)$ is the metric, $B_{AB}(X)$ the axion field, $\Phi(X)$ the dilaton field and R is related to the string worldsheet metric γ and its scalar curvature ${}^{(2)}R$ by $R = \frac{1}{4}{}^{(2)}R$. In the chiral null model the σ-model Lagrangian (2.1) is restricted to take the particular form [4],

$$L = F(x)\partial u[\bar{\partial} v + K(x,u)\bar{\partial} u + 2A_M(x,u)\bar{\partial} x^M] \\ +(G_{MN} + B_{MN})(x)\partial x^M \bar{\partial} x^N + R\Phi(x), \tag{2.2}$$

where u, v are the light-cone coordinates and x^M are coordinates in the other directions. We take the topology of the u, v directions to be $S^1 \times \mathbb{R}$, where this S^1 is the factor of the T^5 mentioned above, around which the onebranes will be wrapped. We will refer to the remaining T^4 as the "transverse internal torus" below. Note that these models have $B_{uv} = G_{uv}$ and $B_{uM} = G_{uM}$, and that $\frac{\partial}{\partial v}$ is is a null Killing vector. One of the requirements [4] for this Lagrangian to be conformally invariant to all orders in α' is that the functions $F(x), K(x,u), A_i(x,u), A_a(x,u)$ and $\Phi(x)$ satisfy

$$-\frac{1}{2}\nabla^2 F^{-1} + \partial^M \psi \partial_M F^{-1} = 0 \tag{2.3}$$

$$-\frac{1}{2}\nabla^2 K + \partial^M \psi \partial_M K + \partial_u \nabla_M A^M = 0 \tag{2.4}$$

$$-\frac{1}{2}\hat{\nabla}_M \mathcal{F}^{MN} + \partial_M \psi \mathcal{F}^{MN} = 0, \tag{2.5}$$

where

$$\hat{\nabla} \equiv \nabla(\hat{\Gamma}) \quad ; \quad \hat{\Gamma}^M_{NP} = \Gamma^M_{NP} + \frac{1}{2}H^M{}_{NP},$$
$$\mathcal{F}_{MN} = \partial_M A_N - \partial_N A_M \quad ; \quad H_{MNP} = 3\partial_{[M} B_{NP]}$$
$$\Phi(x) = \psi(x) + \frac{1}{2}\ln F(x),$$

with ∇_M being the connection compatible with the transverse metric G_{MN}. The other requirement is that the lower dimensional σ-model $(G_{MN} + B_{MN})\partial x^M \bar{\partial} x^N$ be conformal when supplemented by the dilaton coupling $\psi(x)$.

Such models can be used to describe BPS black fivebranes and related solutions by breaking the coordinates x^M into two sets (x^i, y^a) where $x^i (i = 1,2,3,4)$ are coordinates in the asymptotically flat directions transverse to all of the branes and $y^a (a = 1,2,3,4)$ are coordinates on the transverse internal T^4. For simplicity, we will consider static (i.e., u-independent) solutions which describe Neveu-Schwarz fivebranes wrapped around the internal directions (including u, v) and Neveu-Schwarz onebranes wrapped only around the u, v directions. We allow the branes to carry longitudinal momentum (momentum in the u, v directions), transverse internal momentum (in the y^a directions), external linear momentum (in the x^i directions) and angular momentum associated with the spherical symmetry in the x^i coordinates. The various charges need not reside on the fivebrane and need not be distributed homogeneously around the T^4. Specifically, we consider solutions of the form:

$$ds^2 = H_1(x,y)^{1/4}H_5(x)^{3/4}[\frac{du}{H_1(x,y)H_5(x)}\left(-dv + K(x,y)du + 2A_i(x,y)dx^i + 2A_a(x,y)dx^a\right)$$
$$+\frac{dy_a dy^a}{H_5(x)} + dx_i dx^i] \qquad (2.6)$$

$$e^{-2\Phi(x,y)} = \frac{H_1(x,y)}{H_5(x)} \qquad (2.7)$$

$$H_{iuv} = H_1^{-2}\partial_i H_1 \qquad H_{auv} = H_1^{-2}\partial_a H_1, \qquad H_{ijk} = -\epsilon_{ijkl}\partial^l H_5 \qquad (2.8)$$

in the Einstein frame. When $A_i = 0$ (so that the angular and external momentum vanishes), these are just the S-dual of the u-independent special case of the solutions constructed in [28].Related solutions were also discussed in [7].

The corresponding σ-model Lagrangian is

$$L = \frac{1}{H_1(x,y)}\partial u[\bar\partial v + K(x,y)\bar\partial u + 2A_i(x,y)\bar\partial x^i + 2A_a(x,y)dy^a]$$
$$+H_5(x)\partial x_i\bar\partial x^i + \partial y_a\bar\partial y^a + B_{ij}(x)\partial x^i\bar\partial x^j + \frac{1}{2}R\ln H_5(x), \qquad (2.9)$$

where

$$H_{ijk} = 3\partial_{[i}B_{jk]}(x) = -\epsilon_{ijkl}\partial^l H_5(x),$$
$$H_{abc} = H_{abi} = 0 = H_{aij} = 0.$$

Comparing with (2.2), $F(x,y) = H_1^{-1}(x,y)$, $G_{ij} = H_5(x)\delta_{ij}$, $G_{ia} = 0$, $G_{ab} = \delta_{ab}$. Substituting into (2.5), we get the set of equations,

$$\partial_i^2 H_1(x,y) + H_5(x)\partial_a^2 H_1(x,y) = 0 \qquad (2.10)$$
$$\partial_i^2 K(x,y) + H_5(x)\partial_a^2 K(x,y) = 0\ . \qquad (2.11)$$
$$\partial_j^2 A_i(x,y) + H_5(x)\partial_a^2 A_i(x,y) - \partial_i\partial_j A_j(x,y) - H_5(x)\partial_i\partial_a A_a(x,y)$$
$$-\frac{1}{2H_5(x)}[\partial_k H_5(\partial_k A_i(x,y) - \partial_i A_k(x,y)) - \epsilon_{kjil}\partial_l H_5(x)(\partial_j A_k(x,y)$$
$$-\partial_k A_j(x,y))] = 0 \qquad (2.12)$$
$$\partial_i^2 A_a(x,y) + H_5(x)\partial_b^2 A_a(x,y) - \partial_a\partial_i A_i - H_5(x,y)\partial_a\partial_b A_b = 0. \qquad (2.13)$$

Here, the repeated indices are summed even if they are both covariant and $\partial_i^2 = \partial_i\partial_i$. That is, we make implicit use of the metrics δ_{ij} and δ_{ab}.

By comparison with, for example, [28], we know that the lower dimensional σ-model defined by G_{MN}, B_{MN}, and ψ will be conformally invariant if we impose

$$\partial_i^2 H_5(x) = 0. \qquad (2.14)$$

We will assume that the fivebranes are all localized at the origin of the asymptotically flat coordinates and take $H_5 = 1 + \frac{r_5^2}{r^2}$ where $r^2 = \sum_i x^i x^i$.

Note that the field H_1 appears only in equation (2.10) above. Thus, having fixed H_5, we may study H_1 separately from the remaining fields. Equation (2.10) will govern H_1 away from any sources, but we would like to include the sources by considering a properly normalized Green's function for this equation. The normalization may be checked by computing the total abelian electric charge carried by the anti-symmetric tensor field. The result shows that the Green's function for a unit charge satisfies

$$\partial_i^2 G(\vec{r},\vec{y};\vec{r}_0,\vec{y}_0) + (1+\frac{r_5^2}{r^2})\partial_a^2 G(\vec{r},\vec{y};\vec{r}_0,\vec{y}_0) = -\delta(\vec{r}-\vec{r}_0)\delta(\vec{y}-\vec{y}_0), \tag{2.15}$$

with

$$\partial_i^2 = \frac{1}{r^3}\partial_r(r^3\partial_r) + \frac{1}{r^2}\hat{L}^2(\psi,\theta,\phi), \tag{2.16}$$

$\hat{L}(\psi,\theta,\phi)$ being the 4 dimensional angular momentum operator. By "the Green's function for a unit charge," we mean that, if the charge above were smeared out so as to become uniform in the internal T^4 directions, then the solution would be just $H_1 = \frac{1}{2r^2}$.

The solutions of (2.15) may be studied by expanding $G(\vec{r},\vec{y};\vec{r}_0,\vec{y}_0)$ in terms of the 3-dimensional spherical harmonics $D^j_{mn}(\psi,\theta,\phi)$ and the plane wave modes $e^{i\vec{q}\cdot(\vec{y}-\vec{y}_0)}$ in the T^4 directions:

$$G(\vec{r},\vec{y};\vec{r}_0,\vec{y}_0) = \frac{1}{V}\sum_{\vec{q},j,m,n} G_{j\vec{q}}(r;r_0) D^{*j}_{mn}(\psi_0,\theta_0,\phi_0) D^j_{mn}(\psi,\theta,\phi) e^{i\vec{q}\cdot(\vec{y}-\vec{y}_0)}, \tag{2.17}$$

where $\hat{L}^2(\psi,\theta,\phi)D^j_{mn}(\psi,\theta,\phi) = -j(j+2)D^j_{mn}(\psi,\theta,\phi)$ and V is the volume of the T^4. This reduces (2.15) to

$$\frac{1}{r^3}\partial_r(r^3\partial_r G_{j\vec{q}}(r;r_0)) - \frac{1}{r^2}j(j+2)G_{j\vec{q}}(r;r_0) - q^2(1+\frac{r_5^2}{r^2})G_{j\vec{q}}(r;r_0) = -\frac{1}{r^3}\delta(r-r_0). \tag{2.18}$$

Putting $G_{j\vec{q}}(r;r_0) = \frac{A(r_0)}{r}Z_\mu(r)$, the homogeneous equation resembles the modified Bessel equation, when $\mu^2 = 1 + j(j+2) + q^2 r_5^2$. In other words, we find that

$$G_{j\vec{q}}(r;r_0) = \frac{qI_\mu(qr_0)K_\mu(qr)}{rr_0} \quad \text{for } r > r_0 \tag{2.19}$$

$$G_{j\vec{q}}(r;r_0) = \frac{qI_\mu(qr)K_\mu(qr_0)}{rr_0} \quad \text{for } r < r_0, \tag{2.20}$$

for $q = |\vec{q}| \neq 0$, and

$$G_{j0} = \frac{1}{2(j+1)}\frac{r_0^j}{r^{j+2}} \qquad r > r_0 \tag{2.21}$$

$$G_{j0} = \frac{1}{2(j+1)}\frac{r^j}{r_0^{j+2}} \qquad r < r_0, \tag{2.22}$$

for $|\vec{q}| = 0$.

Suppose that we examine this solution from the external region $r > r_0$. Then, as the onebrane source is brought close to the origin ($r_0 \to 0$), we have, for $|\vec{q}| \neq 0$,

$$G_{j\vec{q}}(r; r_0) \approx q \left(\frac{q}{2}\right)^{\mu} \frac{r_0^{\mu-1}}{\Gamma(\mu+1)} \frac{K_\mu(qr)}{r} \tag{2.23}$$

so that all the $G_{j\vec{q}}$ vanish except when $|\vec{q}| = 0$ *and* $j = 0$. This means that the only term that survives in the expansion (2.17) is the fully homogeneous one. Note that, so long as the multipole moments of the source do not increase exponentially, the series (2.17) converges geometrically for $r > r_0$ and it is sufficient to consider each term individually. Thus, we conclude that the full solution for $r > 0$ becomes homogeneous in the limit. Although it was clear from the structure of the equations that the angular dependence would die out as the onebrane approached $r = 0$, it is surprising to find that, even though there is a non-trivial transverse internal T^4 at the origin, the inhomogeneity in that direction disappears as well.

From (2.11) above, it is evident that the equation for the longitudinal momentum (carried by the K-field) is similar, so this charge exhibits the same effect. Indeed, if we simplify the discussion by taking both A_i and A_a to be divergence free[2] ($\partial_i A_i = 0$ and $\partial_a A_A = 0$), then the equation (2.13) satisfied by A_a (which carries the transverse momentum) also takes the form (2.15) and will possess the same feature. However, because of the more complicated form of (2.12), we have not been able to exhibit a corresponding property of the angular momentum distribution.

III. BUILDING A FOUR CHARGE BLACK SIXBRANE

Here we consider the compactification of the 10-dimensional theory on $T^6 \times R^4$. Corresponding black hole solutions may be built from Neveu-Schwarz fivebranes and onebranes together with Kaluza-Klein monopoles. We consider 10-dimensional solutions, inspired by the 6-dimensional solutions of [4,7], in which the fivebrane and monopole charges are located at $r = 0$ while the other charges may be distributed arbitrarily. Our notation is chosen to match that of section II as closely as possible, rather than matching that of [4]. The fields H_1, H_5, and K are related to the distribution of onebranes, fivebranes, and momentum as above and the field H_M is related to the Kaluza-Klein monopoles. In the Einstein frame,

$$\begin{aligned} ds^2 = H_1^{\frac{1}{4}}(x,y) H_5^{\frac{3}{4}}(x) \Bigg[&\frac{du}{H_1(x,y)H_5(x)} du \Big(dv + K(x,y)du + 2A_w(x,y)a_i(x)dx^i \\ &+2A_a(x,y)dy^a + 2A_w(x,y)dw \Big) + \frac{dw^2}{H_M(x)} + \frac{dy_a dy^a}{H_5(x)} + 2\frac{a_i(x)}{H_M(x)} dw dx^i \\ &+ \left(H_M^{-1}(x) a_i(x) a_j(x) + H_M(x)\delta_{ij} \right) dx^i dx^j \Bigg], \end{aligned} \tag{3.1}$$

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[2] Note that such a restriction does not prevent the solution from carrying angular momentum.

$$e^{-2\Phi(x,y)} = \frac{H_1(x,y)}{H_5(x)}, \qquad H_{wij} = \epsilon_{ijl}\partial^l H_5, \tag{3.2}$$

where the monopole is in the w direction and the labels $a = 1,2,3,4$ span the transverse internal T^4 directions while $i = 1,2,3$, span the spatial coordinates. The associated σ-model Lagrangian is

$$\begin{aligned} L = {} & H_1^{-1}(x,y)\partial u(\bar\partial v + K(x,y)\bar\partial u + 2A_w(x,y)[\bar\partial w + a_i(x)\bar\partial x^i] + 2A_a(x,y)\bar\partial y^a) \\ & + \frac{1}{2}R\ln H_1^{-1}(x,y) + L_\perp, \end{aligned} \tag{3.3}$$

with

$$\begin{aligned} L_\perp = {} & \partial y_a\bar\partial y^a + H_5(x)H_M^{-1}(x)[\partial w + a_i(x)\partial x^i][\bar\partial w + a_j(x)\bar\partial x^j] \\ & + H_5(x)H_M(x)\partial x^i\bar\partial x^i + b_i(x)[\partial w\bar\partial x^i - \bar\partial w\partial x^i] + R\psi(x). \end{aligned} \tag{3.4}$$

Note that there is an additional translational Killing vector $\frac{\partial}{\partial w}$ in the monopole direction as well. Moreover, we follow [4] in taking the functions $H_5(x), H_M(x), b_i(x)$ and $a_i(x)$ to satisfy

$$\partial_i^2 H_5(x) = 0 \;;\; \partial_i^2 H_M(x) = 0 \tag{3.5}$$

$$\partial_i b_j(x) - \partial_j b_i(x) = -\epsilon_{ijl}\partial^l H_5(x) \;;\; \partial_i a_j(x) - \partial_j a_i(x) = -\epsilon_{ijl}\partial^l H_M(x) \tag{3.6}$$

$$\psi(x) = \frac{1}{2}\ln H_5(x) \tag{3.7}$$

so that $L_\perp$ is conformally invariant. The conditions (2.5) of the chiral null model reduce to[3]

$$\partial_i^2 H_1 + H_5 H_M \partial_a^2 H_1 = 0 \tag{3.8}$$

$$\partial_i^2 K + H_5 H_M \partial_a^2 K = 0 \tag{3.9}$$

$$\begin{aligned} & A_w[H_M^{-2}a_i a_j\partial_i H_5\partial_j H_M^{-1} + \partial_i H_5\partial_i H_M^{-1} + H_5 H_M^{-1}\epsilon_{ijl}\partial_i a_l\partial_j H_M^{-1} \\ & + H_M^{-1}\epsilon_{ijl}a_i\partial_j H_5\partial_l H_M^{-1}] + H_M^{-2}\epsilon_{ijl}a_i\partial_j H_5\partial_l A_w + H_M^{-1}\partial_i H_5\partial_i A_w \\ & + H_5\partial_i H_M^{-1}\partial_i A_w + H_5 H_M^{-1}\epsilon_{ijl}a_i\partial_j H_M^{-1}\partial_l A_w - H_5 H_M^{-1}\partial_i^2 A_w \\ & - H_5^2 H_M^{-2}a_i\partial_i\partial_a A_a - H_5^2\partial_a^2 A_w = 0 \end{aligned} \tag{3.10}$$

$$-H_M^{-1}\partial_i^2 A_a - H_5\partial_b^2 A_a + H_5\partial_a\partial_b A_b = 0 \tag{3.11}$$

$$\begin{aligned} & A_w[\epsilon_{ijl}\partial_j H_M^{-1}\partial_l H_5 - H_M^{-1}a_j\partial_j H_5\partial_i H_M^{-1}] - H_M^{-1}\epsilon_{ijl}\partial_j H_5\partial_l A_w \\ & - H_5\epsilon_{ijl}\partial_j H_M^{-1}\partial_l A_w + H_5^2 H_M^{-1}\partial_i\partial_a A_a = 0. \end{aligned} \tag{3.12}$$

Here the index b refers to the T^4 internal directions and is summed over. Since the fivebrane and monopole charges are to reside at $r = 0$, using (3.5), we take $H_5 = 1 + r_5/r$ and $H_M = 1 + r_M/r$. As before, we will focus on the field associated with the onebrane distribution (H_1). This time, however, we will only be able to discuss the limiting case in which the

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[3]This computation was done using MathTensor.

onebrane charge is located near $r = 0$. Then, to leading order the Green's function for equation (3.8) satisfies

$$(\partial_i^2 + \frac{r_5 r_M}{r^2}\partial_a^2)G(\vec{r},\vec{y};\vec{r}_0,\vec{y}_0) \approx -\delta(\vec{r}-\vec{r}_0)\delta(\vec{y}-\vec{y}_0). \tag{3.13}$$

Again, the norm can be checked by computing the total electric charge for such a solution. Expanding $G(\vec{r},\vec{y};\vec{r}_0,\vec{y}_0)$ as in (2.17), we get

$$(\partial_r(r^2\partial_r) - l(l+1) - q^2 r_5 r_M)G_{\vec{q}l}(r;r_0) \approx -\delta(r-r_0), \tag{3.14}$$

which has solutions,

$$\begin{aligned} G_{\vec{q}l} &= \frac{1}{\sqrt{1+s^2}\sqrt{rr_0}}\left(\frac{r}{r_0}\right)^{\frac{\sqrt{1+s^2}}{2}} \quad \text{for } r < r_0 \\ G_{\vec{q}l} &= \frac{1}{\sqrt{1+s^2}\sqrt{rr_0}}\left(\frac{r_0}{r}\right)^{\frac{\sqrt{1+s^2}}{2}} \quad \text{for } r > r_0, \end{aligned} \tag{3.15}$$

where $s^2 = 4(l(l+1)+q^2)$. Examining the solution from the exterior region, we see that as $r_0 \to 0$, the $G_{\vec{q}l}(r;r_0)$ vanish except when $s^2 = 0$, which is only satisfied if $l = 0$ *and* $|\vec{q}| = 0$. A similar analysis follows from (3.9) for the longitudinal momentum carried by the field K. Here then, is another case in which the onebrane charge and the longitudinal momentum distributions become homogeneous when the charge is moved to $r = 0$.

A similar examination of the monopole charge and the w-momentum is, on the other hand, not so straightforward as we can see from (3.10), (3.11) and (3.12). However, if we set the w-momentum to zero ($A_w = 0$) and impose a divergence free condition $\partial_a A_a = 0$, we find that (3.10), (3.11), and (3.12) reduce to a single equation of the form studied above:

$$\partial_i^2 A_a + H_5 H_M \partial_b^2 A_a = 0. \tag{3.16}$$

Thus, the transverse internal momentum also becomes translationally invariant when a fixed distribution of sources is moved to $r = 0$.

IV. DISCUSSION

We have seen that, when collections of branes are brought close together, the information about the distribution of charges in the internal transverse directions will be obscured from an observer who remains a fixed distance away. So long as the intrinsic moments of the charge distributions are held fixed, the multipole moments that are measured from infinity will vanish in the limit as the branes are brought together. We have studied this in detail for the case of bringing onebranes with longitudinal and transverse internal momentum together with a fivebrane in string theory (type IIA, IIB and heterotic), and for the case of bringing such onebranes together with an already bound state of Kaluza-Klein monopoles and fivebranes in string theory. Although they are of a slightly different form, we also note that the solutions of [37] with strings localized on fivebranes cannot be formed from a finite collection of branes. This can be seen from the fact that for such solutions, even after compactifying the translationally invariant directions, the integral of $H_1(x, y) - 1$ around a sphere at infinity (i.e., the total onebrane charge) diverges. The solutions studied in the present paper were static, but from the results of [28] we see that, at least for the fivebrane case when $A_i = 0$, a trivial extension allows an arbitrary u dependence in the longitudinal and transverse waves so long as the transverse wave is divergence free. Having seen this effect in these contexts, it is natural to assume that it occurs more generally. In principle, an analysis of equations (2.12) and (2.13) would determine if angular momentum on a BPS black fivebrane and other waves associated with the asymptotically flat directions also behave in this way.

A moment should be taken to comment on our bringing the charges together "while holding the intrinsic charge distribution fixed." Clearly, the distribution need only be fixed in the limit as the charges are brought together. However, the reader may wonder just which limiting distributions are allowed and which are not. Recall that our analysis in sections II and III worked from the multipole moments of the charge distribution around the transverse internal T^4. All that we require is that these moments increase less than exponentially with the mode number q so that the convergence of the series (2.17) can be controlled. As a result, a δ-function (which would describe, for example, a single localized onebrane) and in fact all tempered distributions are included in our treatment. Thus, the only way that an inhomogeneous solution might be obtained when the charges are brought together is if they approach an arrangement more singular than any tempered distribution.

Although our result may seem surprising at first, in retrospect it should perhaps have been expected. This can be seen from an analogy with Einstein-Maxwell theory. Consider what happens when we bring a fixed distribution of electric charge near a Schwarzschild black hole. Let us suppose that the charge distribution is localized on a scale much smaller than the black hole horizon. If we allow the electric charge to fall into the black hole then, due to the "no hair" theorems [38] the electric field at any value of the radial coordinate outside the horizon rapidly becomes spherically symmetric even though the charge does not cross the horizon until $t = \infty$ in Schwarzschild coordinates. The case of the BPS branes is, however, slightly different due to the existence of static solutions in which the onebranes remain at a finite separation from the fivebrane. In the Einstein-Maxwell example above, in order to create a static solution one would need to add some extra stress-energy representing, for example, a string to hold the charge up and keep it from falling into the black hole. In a sequence of static solutions in which the charge is lowered to the horizon, this stress energy would diverge with the increasing proper acceleration of the charge. Thus, the limit of a sequence of static solutions would in fact have a singular horizon.

A better analogy would be to consider the case of a small extremal black hole approaching a large extremal black hole. In this case, no extra forces are needed and the electric field rapidly becomes spherically symmetric as the horizons approach each other. This can be seen directly from the Majumdar-Papapetrou solutions [39,40]. The same spherical symmetry is also broken when our branes (from sections II and III) are separated, and it is restored in the same way when they come together. However, because isolated branes are point objects in the asymptotically flat directions, it comes as no surprise that spherical symmetry is restored when the two objects merge.

Exactly the same argument can be applied to the singularities discussed in [33] associated with the nonspherical longitudinal waves. Suppose that we attempt to assemble a nonspherical wave by bringing together in a spherically asymmetric manner a number of objects (either localized branes or black branes), each of which individually is a spherically symmetric[4]. Then, despite our efforts, the wave carried by the final merged object will be spherically symmetric. Mathematically, this case is exactly equivalent to the merger of extremal black holes just discussed in the context of the Majumdar-Papapetrou solutions.

In addition to the black hole cases already considered above, it is plausible that a similar feature arises for the various intersecting brane solutions (such as those in [29–32]). While such cases do not form an infinitely deep throat, the singular fields near the branes may well produce a similar effect. All of these cases seem worthy of investigation.

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[4]One can make this precise by considering black branes and requiring the horizon of each to be symmetric under the appropriate $SO(n)$ group.

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