

Dynamo Models: where do we stand?

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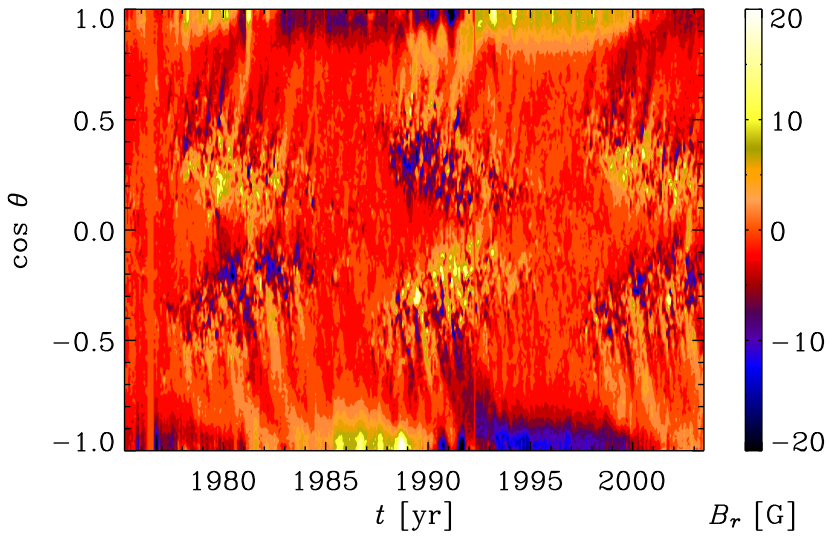
Plan

- **Solar Magnetism**
- **Turbulent dynamo paradigm: small and large-scale**
- **Large scale dynamos: The solar cycle**
- **The large scale dynamo in presence of fluctuations**
- **Magnetic Helicity constraints**
- **Where do we stand?**

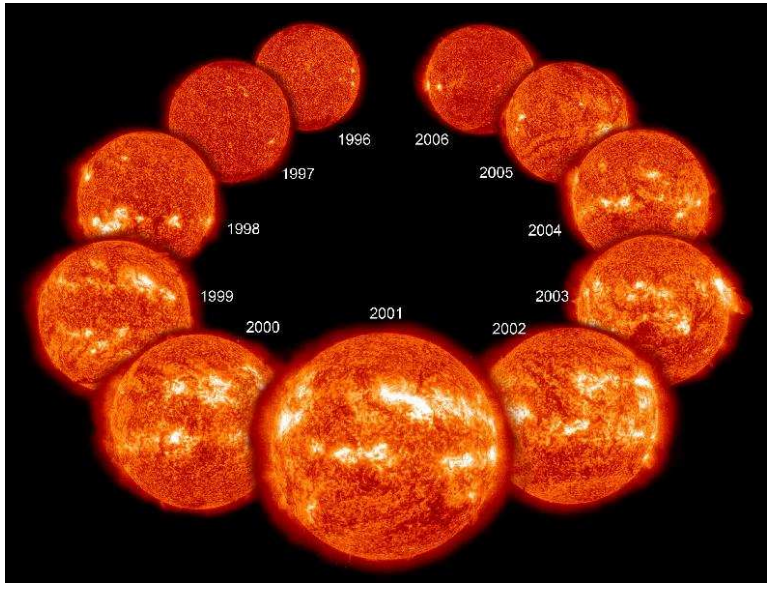
A. Brandenburg & K. Subramanian, Physics Reports, 417, 1-205 (2005)
A. Brandenburg, D. D. Sokoloff & K. Subramanian, Space Science Reviews, 2012
P. Charbonneau, Ann. Rev. Astr. Astrop., 52:251-90 (2014)
Arnab Rai Choudhuri, JAA, arXiv:1408.3968 (2014); IAU Symp. 294, arXiv:1211.0520

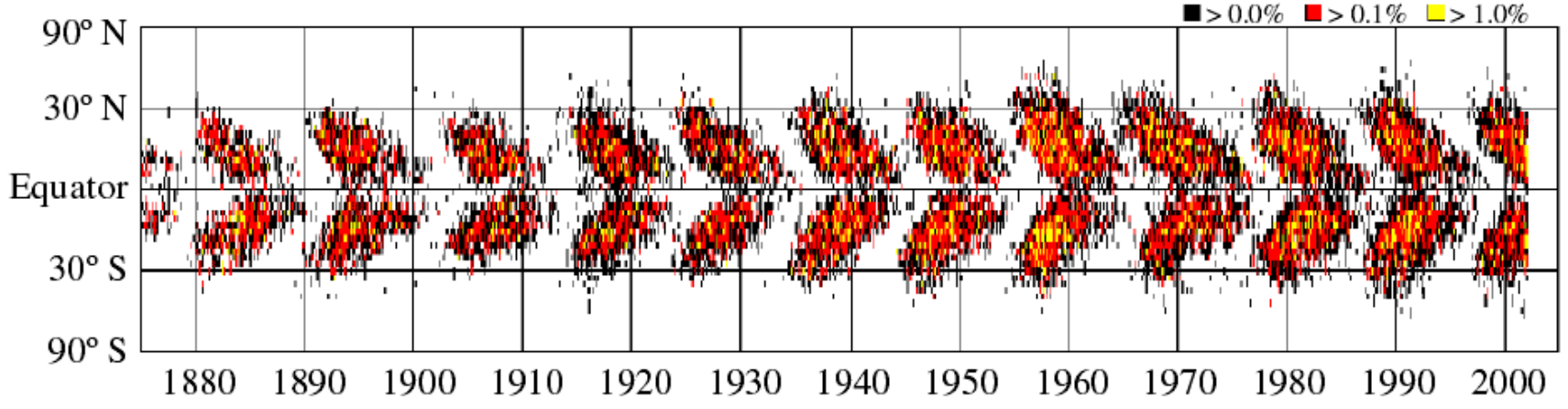
The Solar Cycle with reversals every 11 yr

B_r changes sign with t and across equator (Knaack)



Cycle in 304 $\mathring{A}$ extreme UV

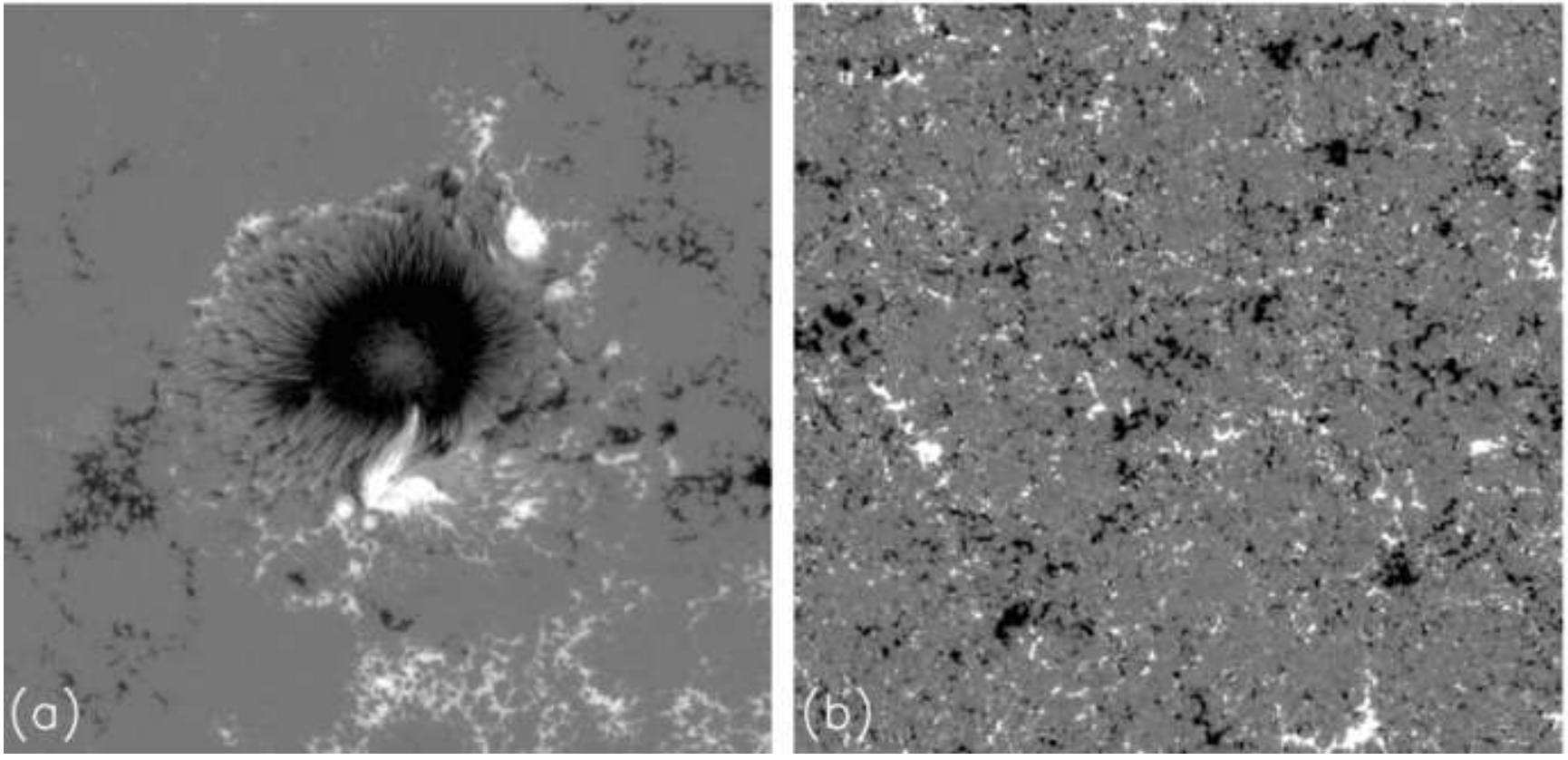




Space-time evolution of sunspot location (Hathaway) $B_\phi \sim 10^3 G$, $B_P \sim G$

The small scale B: magnetic carpet?

Parnell et al., ApJ, 698, 75, 2009



- **Positive/Negative magnetic features (a) around sunspot (b) quiet Sun**
- **Stokes V of 6302 $\AA$ absorption, made using Hinode/SOT, area:** $(100Mm)^2$
- **A power law** $P(\Phi) \propto \Phi^{-1.85}$ **for** $2 \times 10^{17} < \Phi < 10^{23}$ **Mx**

Magnetic fields origin: Dynmaos

- **Magnetic fields decay if not maintained by dynamos: Lorentz force $\rightarrow$ motions $\rightarrow$ heat**

- **EM induction by Motions in B field induces electric field Whose curl can maintain magnetic fields**

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \frac{\boldsymbol{J}}{\sigma}, \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\boldsymbol{V} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}.$$

- **Magnetic and Fluid Reynolds number: $R_m = VL/\eta$, $R_e = VL/\nu$**
  Sun's turbulent CZ (Upper/lower): $R_m \sim 10^6 - 10^9$, $R_e \sim 10^{13}$, $L \sim 0.1 - 10$ Mm, $V \sim 10^2 - 10^3$ m s^{-1}, $\tau \sim 10^3 - 10^5$ s $\ll 11$ yr

- **Turbulent convection $\rightarrow$ Random stretching $\rightarrow$ fluctuation dynamo $\rightarrow$ B field growth (FD Bound state problem, Kazantsev1967)**
  ($BA =$ constant and $\rho AL =$ constant $\rightarrow B/\rho \propto L$, and $A \propto 1/(\rho L)$)

- **Need helicity (parity breaking) to produce "mean" fields**
  Helicity of turbulence allows 'tunneling' to larger scales than L
  (Subramanian, PRL, 1999; Brandenburg, Subramanian, A&A Lett, 2000)

- **At large R_m: Can MFD operate in Presence of FD? Helicity conservation?**

Turbulent Mean-Field Dynamo: LSD

- $\mathbf{V} = \overline{\mathbf{V}} + \mathbf{v}$, $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$: **Mean + Stochastic fields**
- **Mean satisfies DYNAMO equation, with $\overline{\mathcal{E}} = \overline{\mathbf{v} \times \mathbf{b}}$:**

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{V}} \times \overline{\mathbf{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta (\nabla \times \overline{\mathbf{B}}) \right) ;$$

- **The stochastic small-scale field satisfies:**

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\overline{\boldsymbol{V}} \times \boldsymbol{b} + \boldsymbol{v} \times \overline{\boldsymbol{B}} - \eta \nabla \times \boldsymbol{b}) + \boldsymbol{G}$$

- **Here G is the "pain in neck" nonlinear term in v and b.**

$$\boldsymbol{G} = \nabla \times (\boldsymbol{v} \times \boldsymbol{b})' = \nabla \times [\boldsymbol{v} \times \boldsymbol{b} - \overline{\boldsymbol{v} \times \boldsymbol{b}}]$$

- **Finding $\overline{\mathcal{E}} = \overline{\mathbf{v} \times \mathbf{b}}$ is a closure problem:** $\overline{\boldsymbol{\mathcal{E}}} = (\alpha_K + \alpha_M)\overline{\mathbf{B}} - \eta_{turb}(\nabla \times \overline{\mathbf{B}})$

$$\underbrace{\alpha_K = -(\tau/3)\langle \mathbf{v} \cdot \omega \rangle}_{\text{kinetic alpha-effect}} ; \quad \underbrace{\alpha_M = +(\tau/3\rho)\langle \mathbf{j} \cdot \mathbf{b} \rangle}_{\text{magnetic alpha-effect}} ; \quad \underbrace{\eta_{turb} = (\tau/3)\langle \mathbf{v}^2 \rangle}_{\text{Turbulent diffusion}}$$

Turbulent Mean-Field Dynamo: Solar

- **Mean field satisfies DYNAMO equation**

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta (\nabla \times \overline{\mathbf{B}}) \right) ;$$

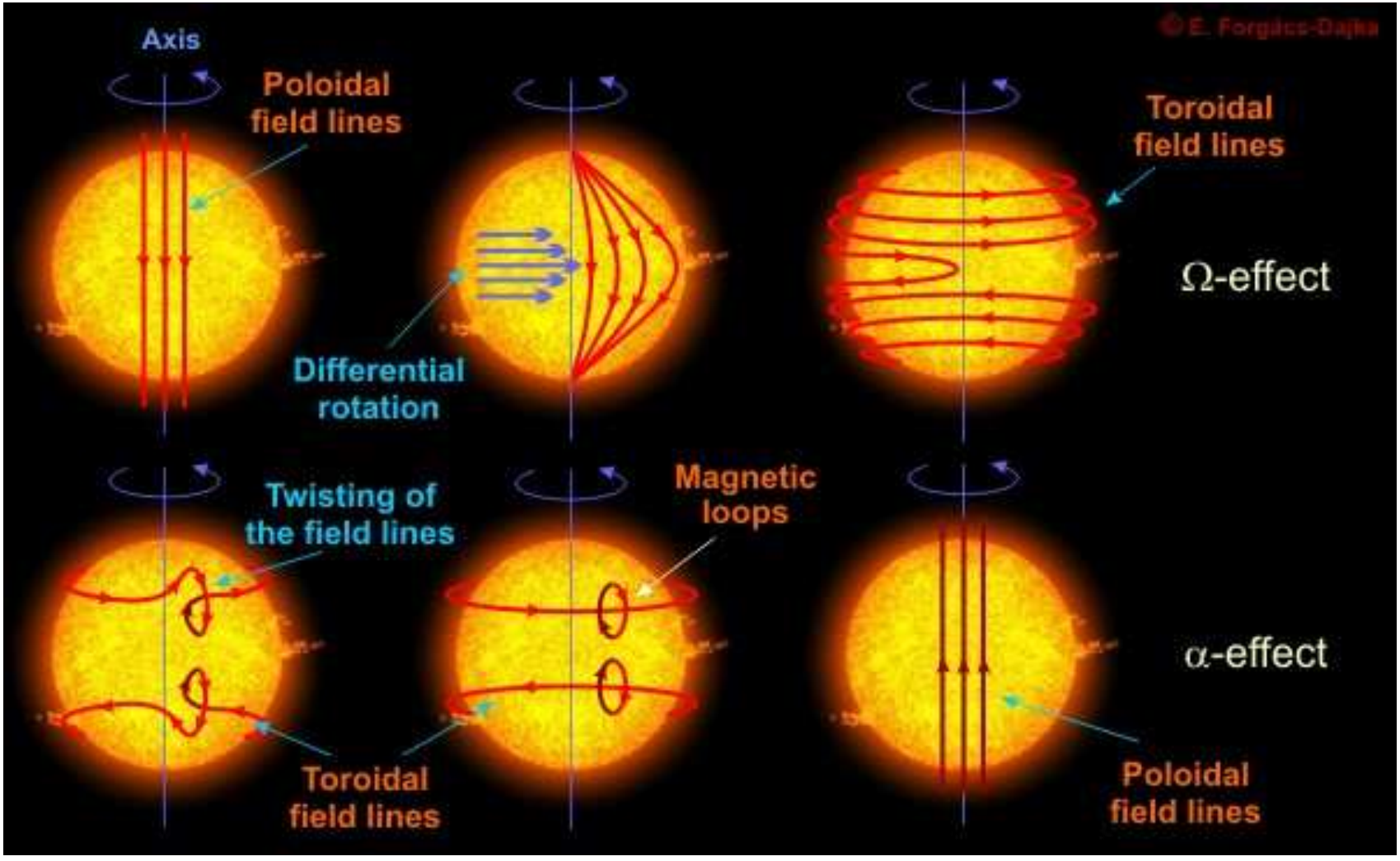
$$\overline{\boldsymbol{\mathcal{E}}}_i = \alpha_{ij} \overline{B}_j - \eta_{ij} \overline{J}_j + (\gamma \times \overline{\boldsymbol{B}})_i + (\delta \times \overline{\boldsymbol{J}})_i + \kappa_{ijk} \overline{B}_{j,k},$$

$$\gamma = -\tfrac{1}{6} \tau_0 \nabla \left(\overline{\boldsymbol{v}^{(0)2}} - \overline{\boldsymbol{b}^2} \right) - \tfrac{1}{6} \tau_0^2 \boldsymbol{\Omega} \times \nabla \left(\overline{\boldsymbol{v}^{(0)2}} + \overline{\boldsymbol{b}^2} \right), \quad (AB, KS\ 2005)$$

- **Sun's differential rotation generates B_ϕ from $B_{poloidal}$**
- **Convective turbulence becomes HELICAL (Due to Rotation + Stratification)**
- **Helical motions generate $B_{poloidal}$ from B_ϕ**
- **Or decay of tilted sunspot pairs: in FTD**
  - **Direct simulation of convection and dynamo action**
  - **Kinematic models: Flux transport dynamos**
  - **Helicity conserving dynamical quenching models**

The α-Ω Solar dynamo

http://www.konkoly.hu/solstart/

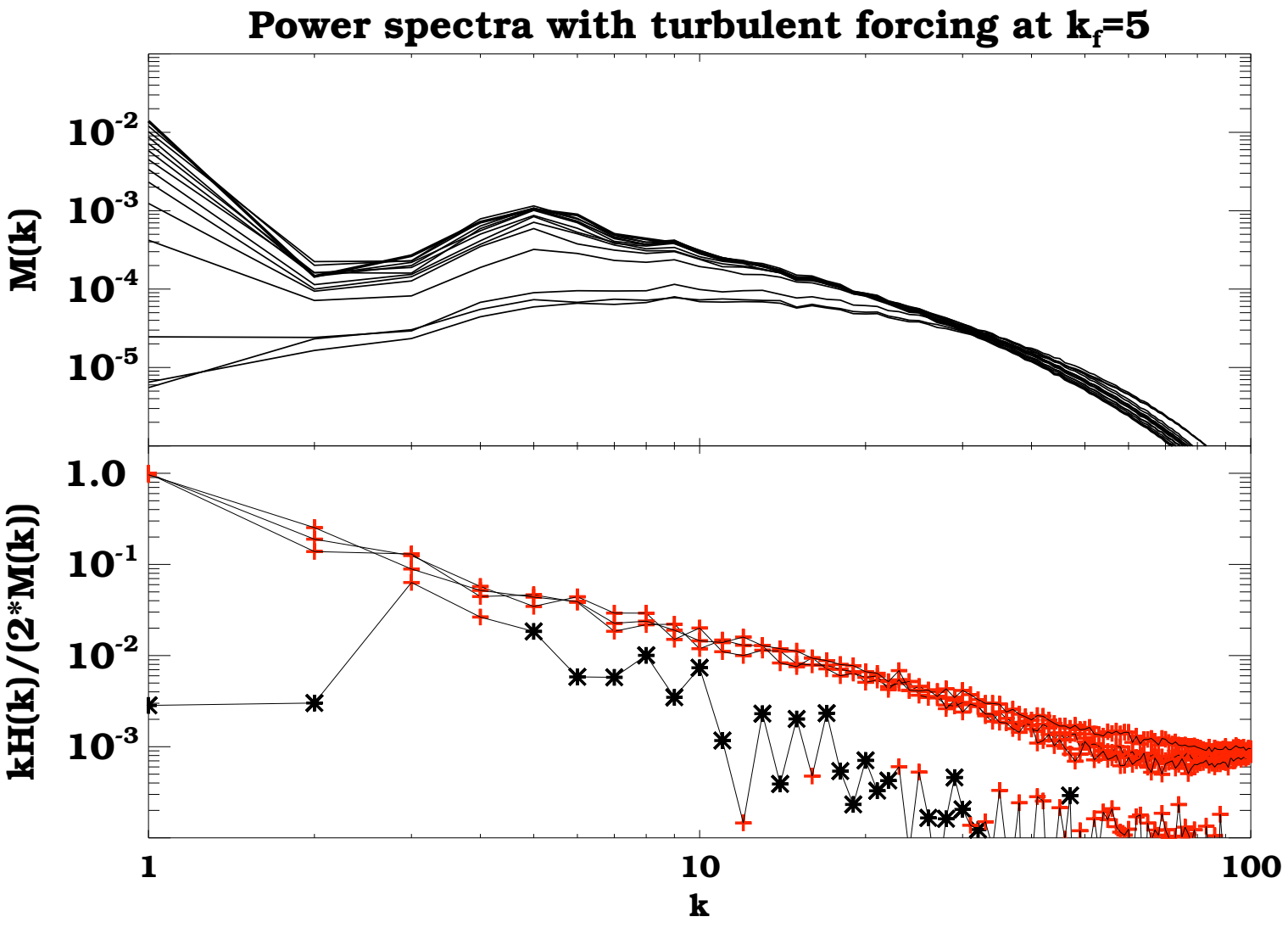


Mean vs fluctuations? Helicity (links) conservation?

Proof for magnetic alpha?

Helical fields decay on slow resistive rate: maintained by α_M

(EB,KS, 2013; Pallavi Bhat, EB, KS, MNRAS, 2014)



Solve full MHD in direct simulations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

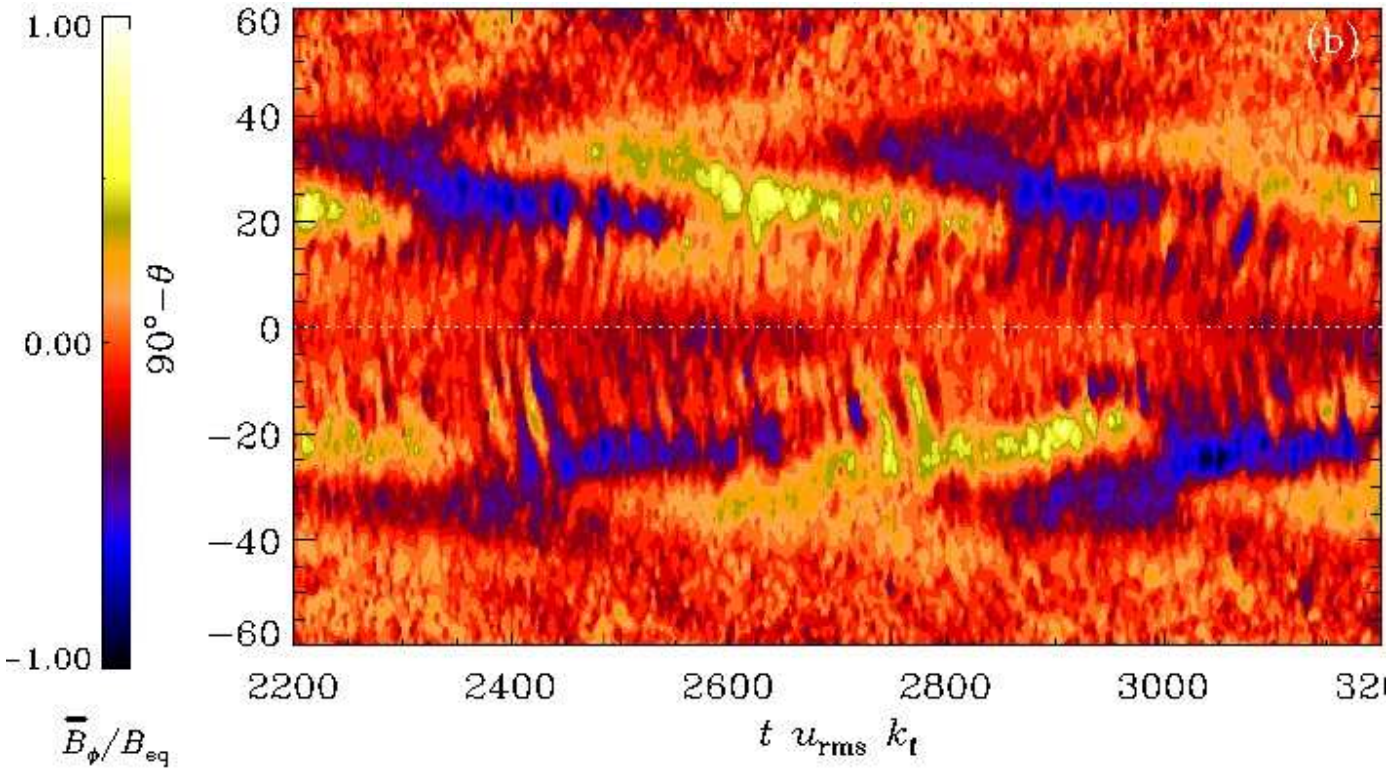
$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{\nabla p}{\rho} - 2\Omega \times \mathbf{V} + \mathbf{g} + \frac{\mathbf{F}_{\text{viscous}}}{\rho} + \frac{\mathbf{J} \times \boldsymbol{B}}{\rho c}$$

$$\rho \frac{d\epsilon}{dt} + p \nabla \cdot \mathbf{V} = -\nabla \cdot (\kappa \nabla \mathbf{T}) + \Lambda_{viscous} + \Lambda_{resistive}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) .$$

Dynamo wave from simulations

Kapyla, Mantere, Brandenburg, Ap.J. 755, L22 (2012)



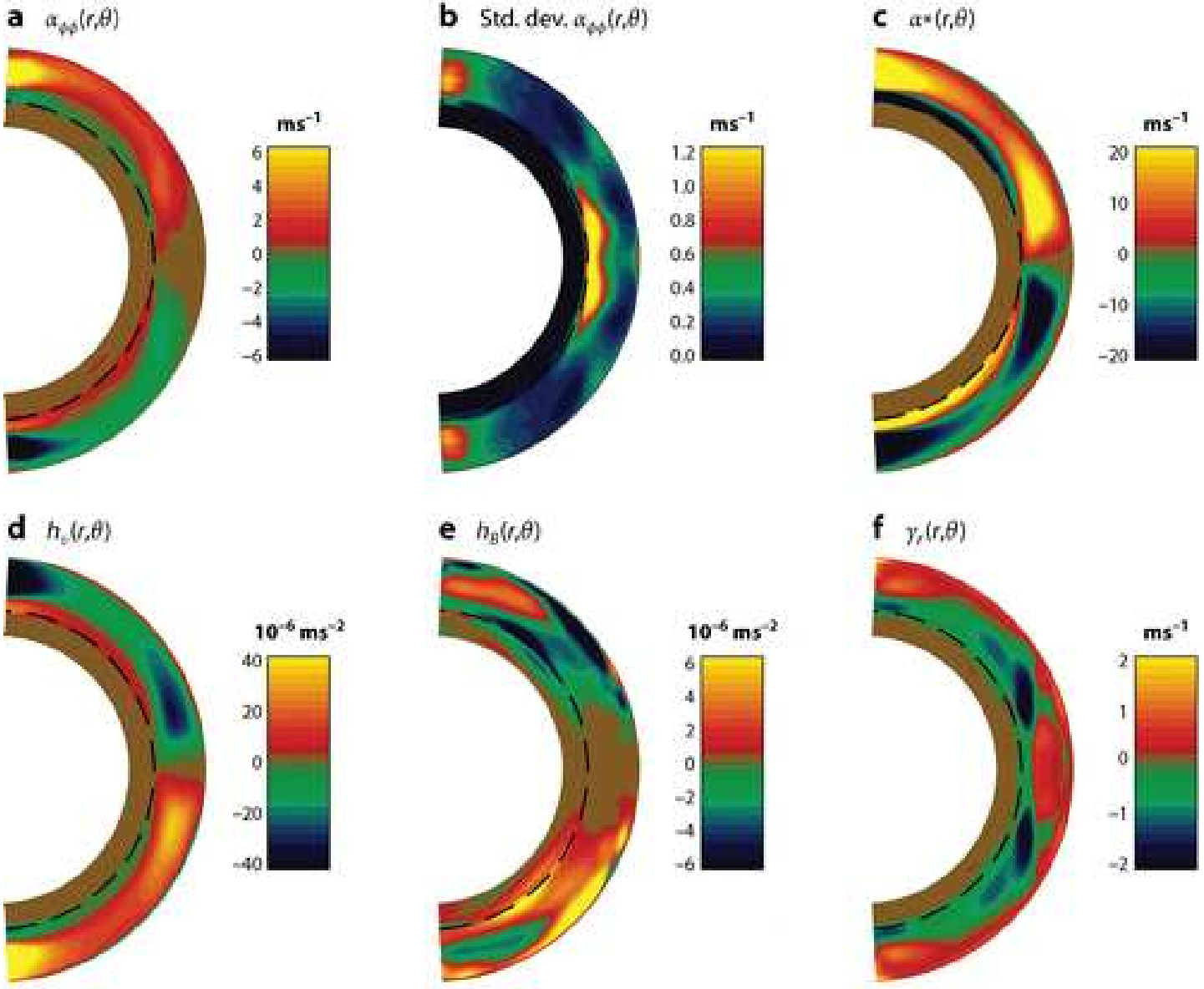
Convection in a spherical shell; modest $R_e = 36$, $R_m = 36 - 144$

Cycle period 33 years

Similar results: Brown et al., ApJ, 2011, 731:69; Racine et al. ApJ, 2011, 735, 46

$\bar{\mathcal{E}}$: simulations vs theory

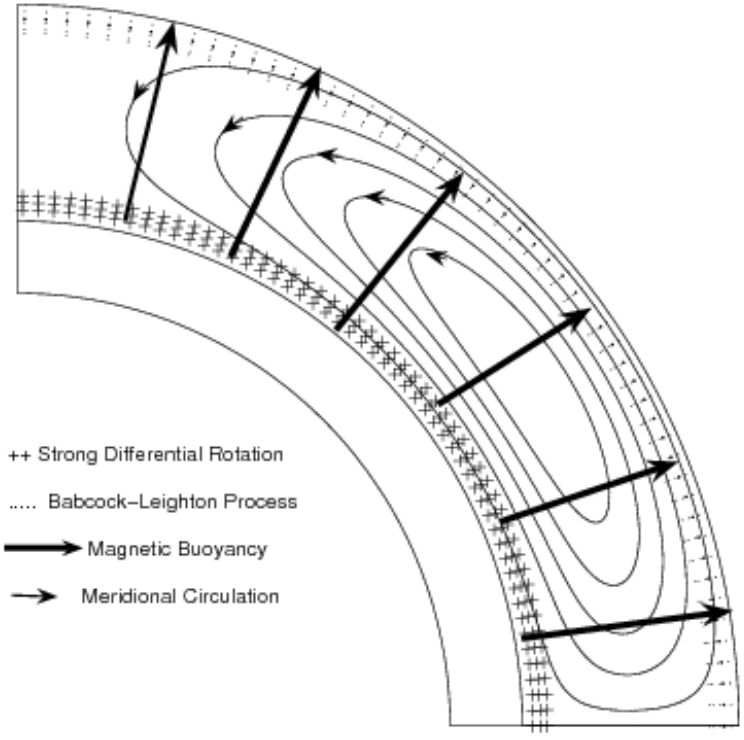
Convection in a spherical shell **(Racine et al., Ap.J. 735, 46, 2011)**

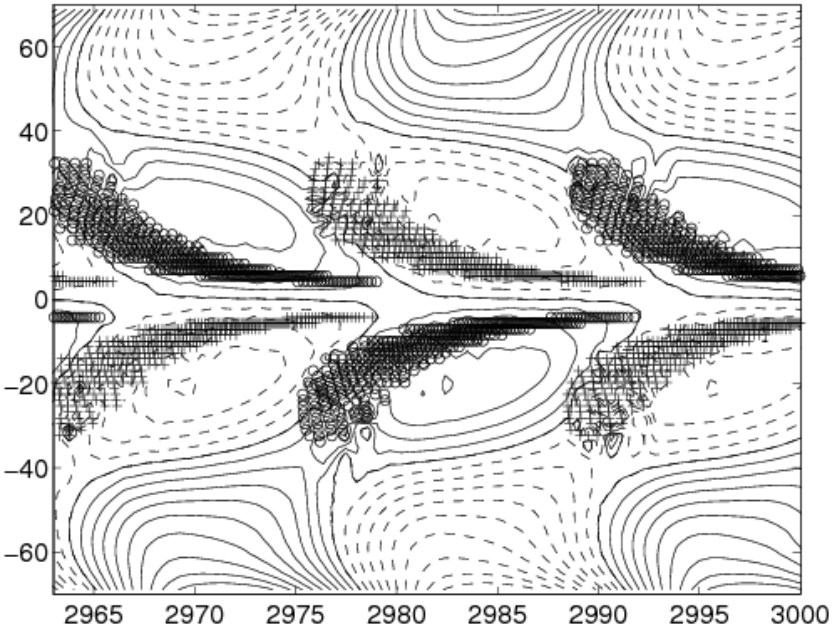


Charbonneau P. 2014.

Flux transport dynamos

Arnab Rai Choudhuri, and many more friends





- $B_{poloidal}$ to B_ϕ by differential rotation near tachocline
- Buoyant Rise of of toroidal field to form tilted sunspots
- Decay of tilted sunspots and advection to form $B_{poloidal}$
- Meridional circulation takes it down to tachocline

Can MFD work in presence of FD?

Subramanian and Brandenburg, MNRAS, 2014, 445, 2930

- **Helically forced turbulence with $k_f = 4$**
- **Consider various helicity, P_m and R_m.**
- **Expect α^2 LS dynamo + Fluctuation dynamo**
- **Check k dependent growth rate for magnetic energy and normalized magnetic helicity spectra.**

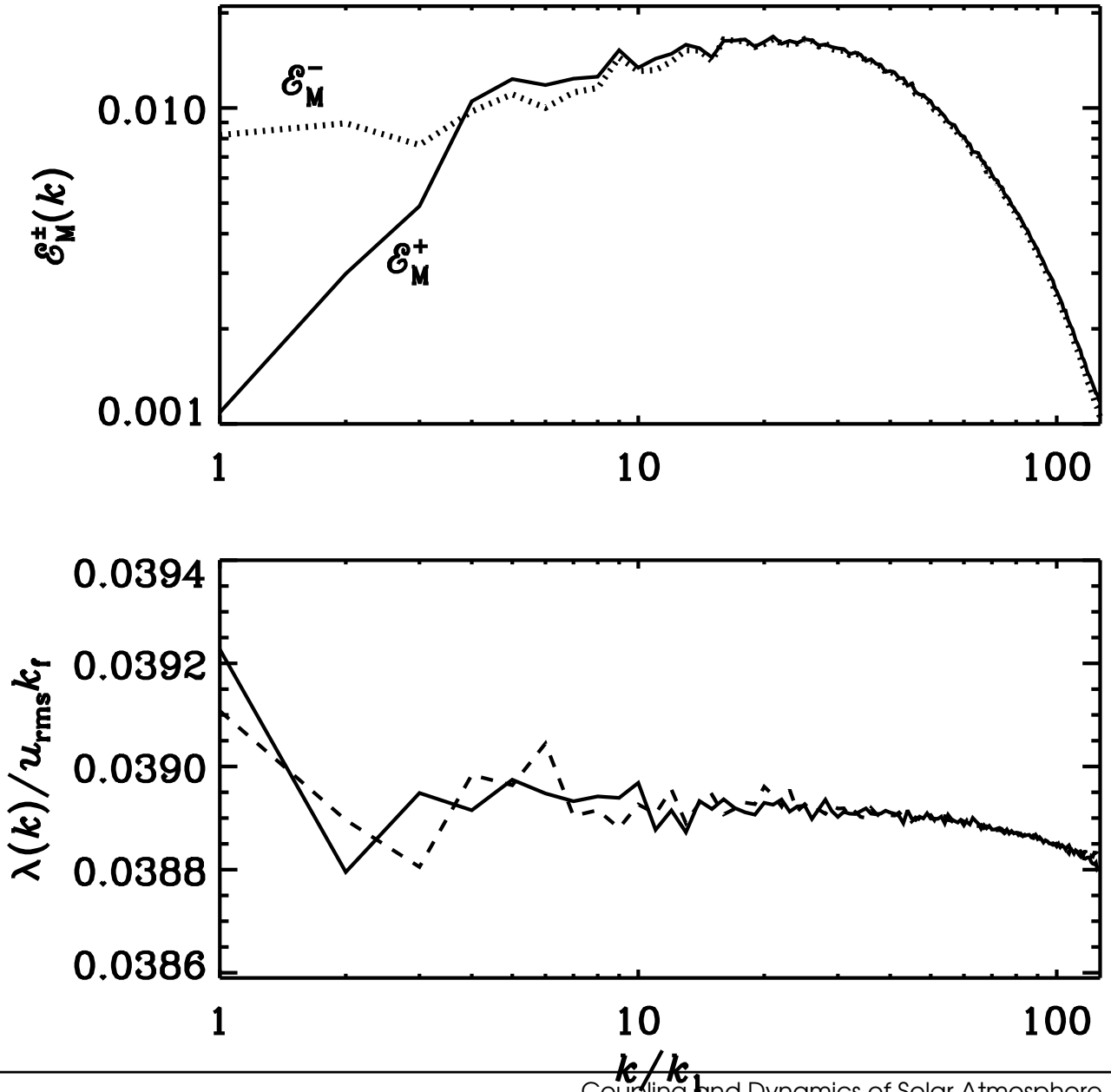
$$E_M(k,t) = E_{M0}(k)e^{\lambda(k)t}, \quad H_M(k,t) = H_{M0}(k)e^{\lambda(k)t}$$

- **Spectrum of positively and negatively polarized contributions**

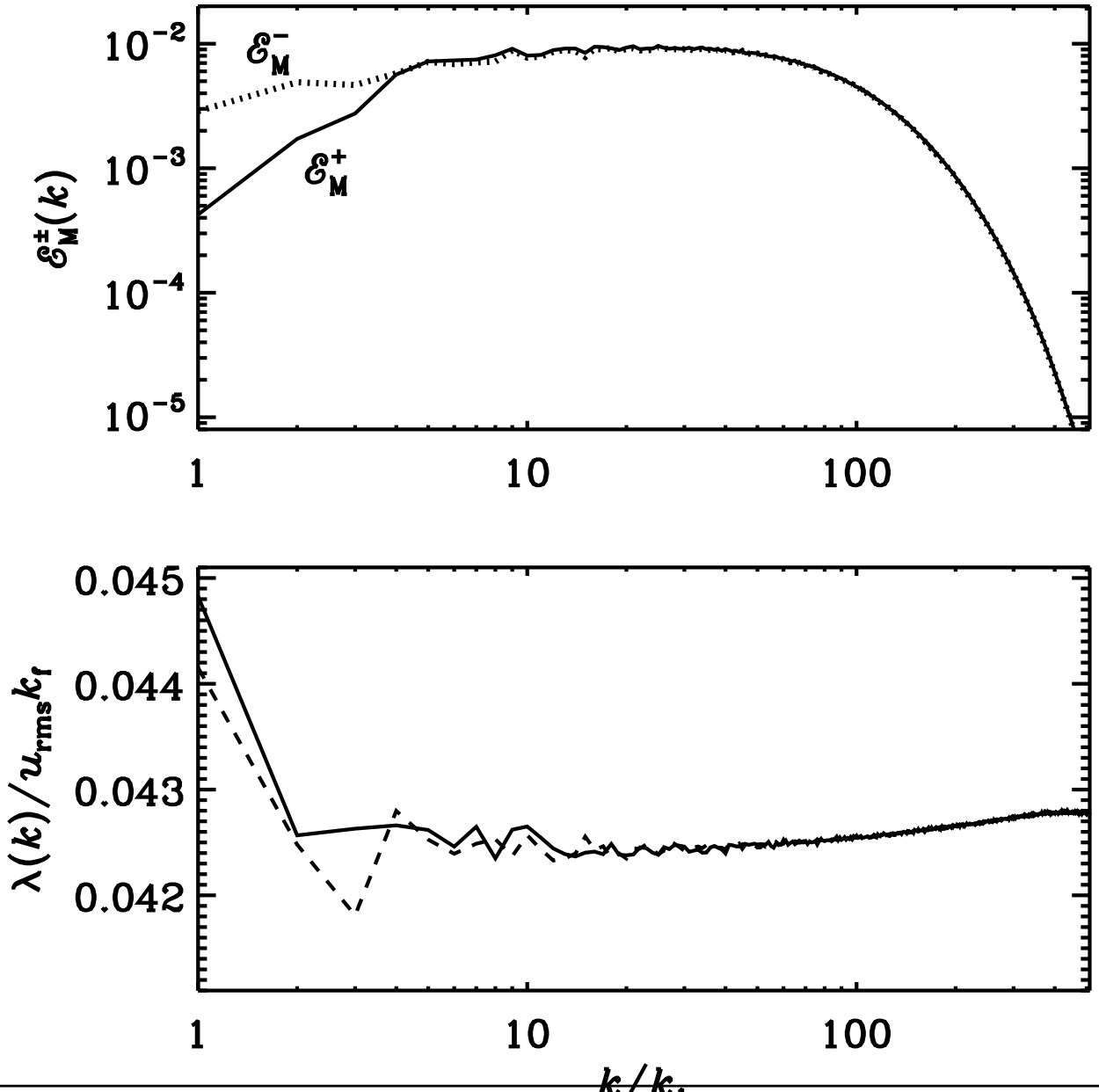
$$E^{\pm}_{M0} = \frac{1}{2}\left[E_{M0}(k) \pm \frac{1}{2}kH_{M0}(k)\right].$$

- **Mean/large scale field will show up as excess of E^{-}_{M0}**
- **Is there an evidence for a mean field in Kinematic stage?**

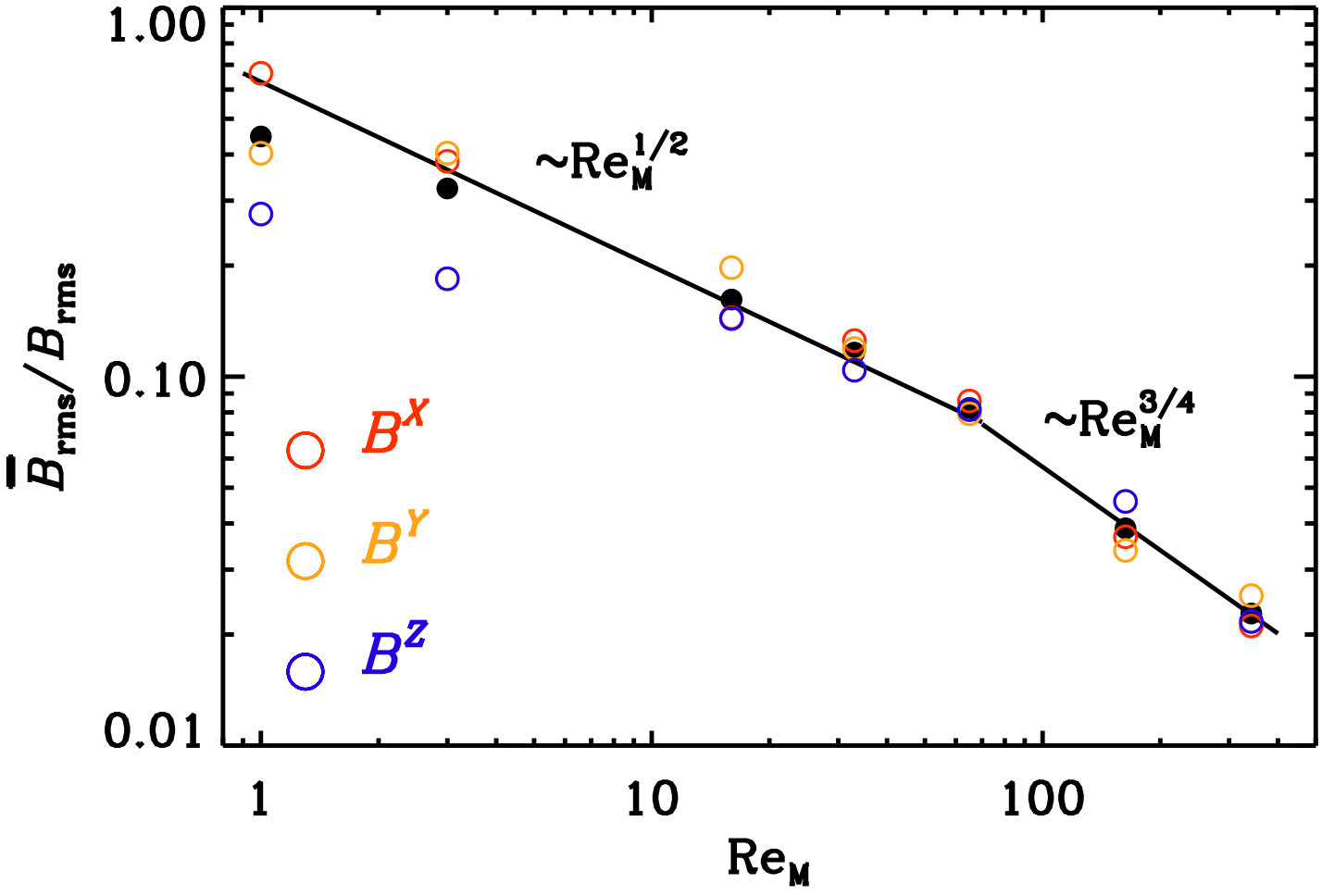
$P_m = 0.1$, $R_m = 160$, *fully helical*



$P_m = 0.1$, $R_m = 330$, *fully helical*



Mean B/fluctuations decreases with R_m

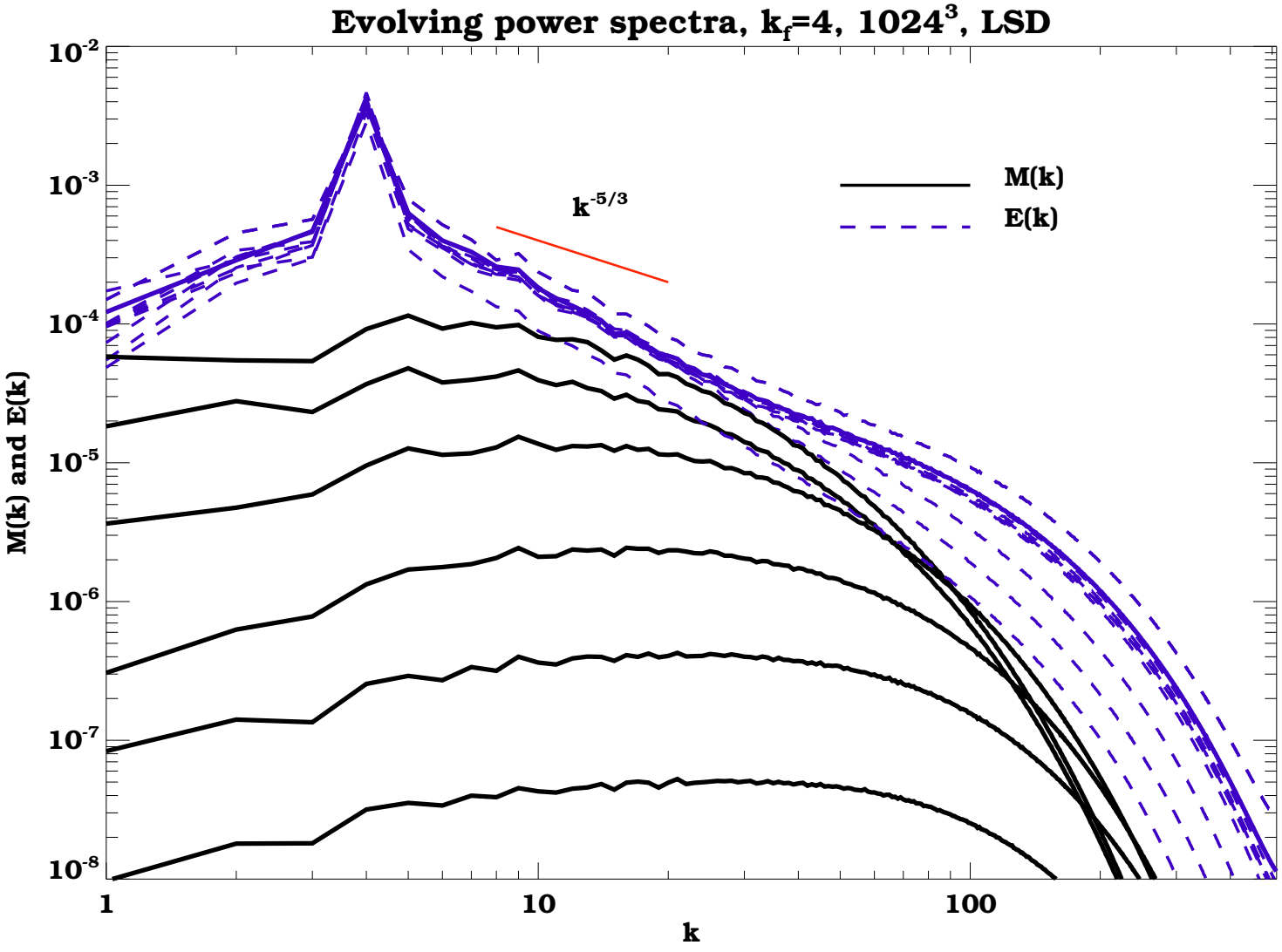


- Magnetic energy spectrum peaks at small resistive scale even when turbulence helical. Understood Via Kazantsev model.
- $\overline{B}_{rms}/B_{rms}$ decreases strongly with R_m at kinematic stage

Does Saturation Save LSD?

Pallavi Bhat, Axel Brandenburg, Kandaswamy Subramanian (2015?)

Power shifts to larger scales on saturation:-)



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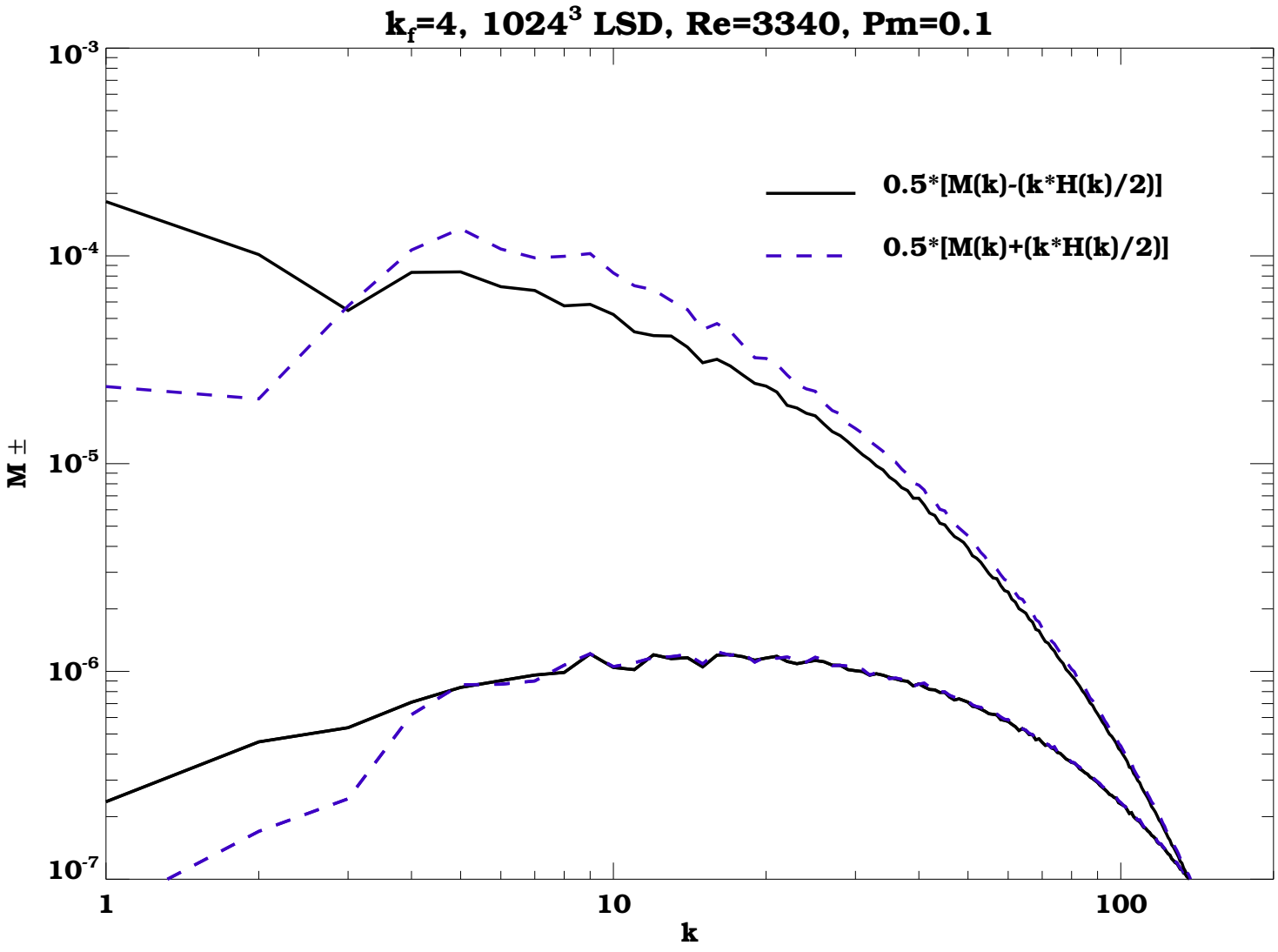
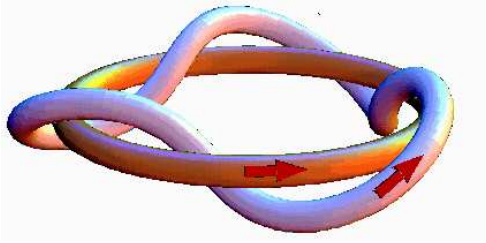


Magnetic Helicity conservation and LSDs

- **Magnetic Helicity** $H = \int_V \mathbf{A} \cdot \mathbf{B}\, dV$, $\quad \nabla \times \boldsymbol{A} = \boldsymbol{B}$

  $\mathbf{A}$ is vector potential, V is **closed** volume

- **Measures links and twists in** B



- **Total energy and magnetic helicity conserved in ideal systems**

- **Helicity is nearly conserved even when energy dissipated!**

How does the Sun's mean field helicity arise?

Helicity and dynamo quenching



Anvar and Natasha Shukurov 2009

- Helical motions transfer helicity between WRITHE AND TWIST Helicities
- Lorentz force of small-scale twist Helicity grows to kill the dynamo
- Unless one has helicity fluxes. But what is helicity density/flux?

Nonlinear saturation of helical dynamos

- $\overline{\mathcal{E}}$ transfers helicity between small-large scales
- Small scale current helicity grows to cancel kinetic α
- Nonlinear $\alpha = -(\tau/3)\langle \mathbf{v} \cdot \omega \rangle + (\tau/3\rho)(4\pi/c)\langle \mathbf{j} \cdot \mathbf{b} \rangle \rightarrow 0?$
- CATASTROPHIC QUENCHING OF DYNAMO?
- Need to get rid of small scale helicity, by Helicity fluxes? (Blackman & Field; Kleeorin et al).
  But what is gauge invariant helicity density and flux?
- Small scale helicity density h is density of correlated $\mathbf{b}$ field links (Subramanian & Brandenburg, ApJ Lett., 2006)

$$\partial h/\partial t + \nabla \cdot \mathbf{F} = -2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}} - 2\eta(4\pi/c)\overline{\mathbf{j} \cdot \mathbf{b}}$$

Large scale dynamos need helicity fluxes

Gauge-invariant helicity density/Flux?

- **Gauss's linking formula for magnetic helicity**

$$h_{\rm G} = \frac{1}{4\pi}\int\int \boldsymbol{b}(\boldsymbol{x})\cdot\left[\boldsymbol{b}(\boldsymbol{y})\times\frac{\boldsymbol{x}-\boldsymbol{y}}{|\boldsymbol{x}-\boldsymbol{y}|^3}\right]{\rm d}^3x\,{\rm d}^3y = \int \boldsymbol{a}\cdot\boldsymbol{b}\,{\rm d}^3x$$

- **Let** $\overline{b_i(\boldsymbol{x},t)b_j(\boldsymbol{y},t)} = M_{ij}(\boldsymbol{r},\boldsymbol{R})$**, with** $\boldsymbol{r}=\boldsymbol{x}-\boldsymbol{y}$**,** $\boldsymbol{R}=(\boldsymbol{x}+\boldsymbol{y})/2$
- **Correlation scale** $l \ll L \ll R_s$ **the system scale (Formally let** $L\to\infty$**)**
- **The ensemble average helicity is:** $\overline{h}_{\rm G} = \int {\rm d}^3R\, h(\boldsymbol{R})$

$$\overline{h}_{\rm G} = \int {\rm d}^3R\left[\frac{1}{4\pi}\int_{L^3} {\rm d}^3r\,\epsilon_{ijk}M_{ij}(\boldsymbol{r},\boldsymbol{R})\,\frac{r_k}{r^3}\right] = \int {\rm d}^3R\, h(\boldsymbol{R})$$

$h(\boldsymbol{R})$: **Gauge invariant helicity density of the random small scale field** b**.**
Density of correlated b field links
Subramanian & Brandenburg, ApJ Lett, 2006

A local helicity conservation equation

- **Local helicity conservation equation** :

$$\partial h/\partial t + \nabla \cdot \mathbf{F} = -2\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\mathbf{B}} - 2\eta(4\pi/c)\overline{\mathbf{j} \cdot \mathbf{b}}$$

¡

- **Simplest helicity flux due to advection**: $\mathbf{F}^{\rm bulk} = h\overline{\boldsymbol{U}}$
- **Next simplest diffusion**: $\mathbf{F} = -a\eta_t \nabla h$
- **Latest Vishniac flux and diffusive flux comes from triple correlations?**
- **Now in stationary limit;** $\overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} = -\frac{1}{2}\boldsymbol{\nabla} \cdot \mathbf{F} - \eta\overline{\boldsymbol{j} \cdot \boldsymbol{b}}$
- **Helicity fluxes can also generate** α_M **and then drive the dynamo!**
- **Combine helicity conservation and dynamo equations for dynamical quenching models**

Where do we stand?

- Sun has both small-scale and large-scale fields
- Dynamos required to maintain fields.
- Fluctuation dynamo can maintain the magnetic carpet
- How do MFD's operate and saturate in presence of fluctuations?
- For mean field dynamos: Helicity fluxes? α, η_t at large R_m?
- Kinematic models (like FTDs) have lots of success...but does sun do what they want?
- Are there other turbulent transport effect which are important?
- Direct simulations have become exciting tools to explore
- Understanding large R_m dynamo is work in progress!