

A Minimal Model of Neutrino Flavor

Christoph Luhn*

*Institute for Particle Physics Phenomenology
University of Durham, Durham DH1 3LE, UK*

Krishna Mohan Parattu†

*Inter-University Centre for Astronomy and Astrophysics
Ganeshkhind, Pune 411007, India*

Akin Wingerter‡

*Laboratoire de Physique Subatomique et de Cosmologie
UJF Grenoble 1, CNRS/IN2P3, INPG
53 Avenue des Martyrs, F-38026 Grenoble, France*

Abstract

Models of neutrino mass which attempt to describe the observed lepton mixing pattern are typically based on discrete family symmetries with a non-Abelian and one or more Abelian factors. The latter so-called shaping symmetries are imposed in order to yield a realistic phenomenology by forbidding unwanted operators. Here we propose a supersymmetric model of neutrino flavor which is based on the group T_7 and does not require extra \mathbb{Z}_N or $U(1)$ factors, which makes it the smallest realistic family symmetry that has been considered so far. At leading order, the model predicts tribimaximal mixing which arises completely accidentally from a combination of the T_7 Clebsch-Gordan coefficients and suitable flavon alignments. Next-to-leading order (NLO) operators break the simple tribimaximal structure and render the model compatible with the recent results of the Daya Bay and Reno collaborations which have measured a reactor angle of around 9° . Problematic NLO deviations of the other two mixing angles can be controlled in an ultraviolet completion of the model.

*christoph.luhn@durham.ac.uk

†krishna@iucaa.ernet.in

‡akin@lpsc.in2p3.fr

1. Introduction

The triplication of chiral families remains one of the biggest mysteries in particle physics. A clue towards unraveling the principle behind the fact that quarks and leptons come in three copies is provided by the observation of a remarkable lepton mixing pattern: contrary to the quark sector, the mixing of the leptons is described by two large and one small angle. Until recently, the results of neutrino oscillation experiments were well compatible with a PMNS matrix of the intriguingly simple structure [1–3]

$$U_{\text{PMNS}} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1)$$

This so-called tribimaximal mixing suggests an intimate connection of the three generations of leptons, which can be realized in the framework of non-Abelian discrete family symmetries. Imposing such a horizontal symmetry G allows to unify different generations into a multiplet of the given non-Abelian group. With three families, the physically interesting groups should have a triplet representation, limiting possible choices to subgroups of $U(3)$. Many of these have been successfully applied to construct models of tribimaximal lepton mixing, see for instance [4–6] and references therein.

In general, family symmetry models can be classified according to the origin of the symmetry of the neutrino mass matrix. We will assume that neutrinos are Majorana particles; then their mass matrix is always symmetric under a Klein symmetry $\mathbb{Z}_2 \times \mathbb{Z}_2$. Working in a basis where the charged leptons are diagonal,¹ the explicit form of the Klein symmetry expressed in terms of 3×3 matrices is dictated by the PMNS mixing matrix and can be determined as

$$U_{\text{PMNS}}^* \begin{pmatrix} (-1)^p & 0 & 0 \\ 0 & (-1)^q & 0 \\ 0 & 0 & (-1)^{p+q} \end{pmatrix} U_{\text{PMNS}}^T. \quad (2)$$

Here $p = 0, 1$ and $q = 0, 1$, yielding a symmetry group of four elements. This neutrino flavor symmetry can arise as a residual symmetry of the underlying family symmetry G , in other words, the four elements of Eq. (2) can also be elements of the imposed family symmetry. Models of this type are called direct models [14]. In indirect models, on the other hand, the above Klein symmetry is not a subgroup of G . Models of this class are typically based on the type I seesaw mechanism [15–18] with the assumption of sequential dominance [19–21], i.e. a hierarchy among the three terms arising from three right-handed neutrinos. Here, the main role of the family symmetry consists in explaining special vacuum configurations of the flavon fields that break the family symmetry [14]. In addition to these pure classes of models there are semi-direct models

¹This classification can also be applied to GUT models where the charged lepton mass matrix is typically only approximately diagonal. In such a setup, the total PMNS mixing matrix will involve charged lepton corrections that have to be taken into account separately, see e.g. [7–11], leading to characteristic mixing sum rules [12, 13].

in which one of the \mathbb{Z}_2 factors of the Klein symmetry arises as a residual symmetry of G , while the other factor arises accidentally. In fact, the famous Altarelli-Feruglio model based on A_4 [22, 23] belongs to the semi-direct class as the tribimaximal μ - τ symmetry is not part of A_4 .

Hitherto, regardless of the type of model, the non-Abelian family symmetry has always been augmented by extra Abelian factors such as \mathbb{Z}_N and $U(1)$ in order to yield realistic phenomenology. These shaping symmetries were crucial for controlling the coupling of the Standard Model neutral flavon fields to the leptons. In [22, 23], for instance, the neutrino and charged lepton sectors are separated by means of a \mathbb{Z}_3 shaping symmetry. In that sense, one should speak more precisely of the Altarelli-Feruglio $A_4 \times \mathbb{Z}_3$ model, and the family symmetry G is thus a group of order 36, rather than 12.

Mindful of this subtlety of defining the full family symmetry of a model, a systematic scan over 76 different groups has been performed in [24] with the purpose of studying whether or not there is an inherent connection between A_4 and tribimaximal mixing. The results of the scan proved that there are indeed several, even more minimal, groups which are capable of describing tribimaximal mixing (assuming the imposed simple alignment of the flavon *vevs* can be justified). The smallest such group was identified to be T_7 , a group of order 21 which is sometimes also called the Frobenius group $\mathbb{Z}_7 \rtimes \mathbb{Z}_3$ [25–27]. Note that this non-Abelian symmetry does not require any extra shaping symmetry. It is furthermore interesting to point out that T_7 has no \mathbb{Z}_2 subgroups, and hence the Klein symmetry of the neutrino mass matrix cannot be part of this family symmetry. As such, a corresponding T_7 model would be neither direct nor semi-direct, but rather of indirect type. Not necessarily requiring the seesaw mechanism, it would – to the best of our knowledge – be the first indirect model which is not based on the assumption of sequential dominance and the quadratic appearance of flavon fields in the effective neutrino mass term.

It is the purpose of this article to present the details of a complete T_7 model of leptons which yields tribimaximal mixing at leading order, including a discussion of the vacuum alignment. We put particular emphasis on explaining how the Klein symmetry arises completely accidentally from a combination of the T_7 Clebsch-Gordan (CG) coefficients and suitable flavon alignments. As tribimaximal mixing has been ruled out by the Daya Bay [28] and Reno [29] measurements of a reactor mixing angle θ_{13} of around 9° , it is necessary to investigate if next-to-leading order (NLO) effects can generate large enough deviations from the tribimaximal leading order prediction. With the Klein symmetry arising purely accidentally, NLO terms are bound to perturb the tribimaximal mixing pattern. While switching on θ_{13} , NLO corrections will in general also give rise to perturbations of the atmospheric and, more critically, the solar mixing angle which may be phenomenologically unacceptable. However, these unwanted NLO corrections can be suppressed in an ultraviolet completed version of the T_7 model involving special messenger fields.

The remainder of this article is organized as follows. We define and discuss the minimal T_7 model in Section 2, both at leading as well as next-to-leading order. The ultraviolet completion of the model is presented in Section 3. Section 4 addresses the question of vacuum alignment, and the conclusions are drawn in Section 5. The relevant group

Field	L	e^c	μ^c	τ^c	h_u	h_d	$\tilde{\varphi}$	φ
T_7	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	$\mathbf{3}$
$U(1)_Y$	-1	2	2	2	1	-1	0	0
$U(1)_R$	1	1	1	1	0	0	0	0

Table 1: The transformation properties of the minimal T_7 model of leptons.

theoretic details of T_7 are laid out in Appendix A, including the generators of the representations in a basis with a diagonal order three element as well as the corresponding CG coefficients.

2. The minimal T_7 model

2.1. Tribimaximal mixing at leading order

In this section we describe a minimal model of leptons based on the family symmetry T_7 . This non-Abelian finite group comprises 21 elements and has three singlet as well as two triplet representations, which we denote by $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{3}$ and $\overline{\mathbf{3}}$, respectively. As the model does not feature any shaping symmetries, its structure is solely determined by the family symmetry T_7 as well as the gauge symmetry $SU(2)_L \times U(1)_Y$. We work in a supersymmetric framework with two Higgs doublets, h_u and h_d , transforming trivially under T_7 . The three generations of left-handed lepton doublets L are unified in a triplet representation of T_7 , while the right-handed charged leptons e^c , μ^c and τ^c live in the three distinct one-dimensional representations. In order to break the family symmetry, two flavon fields are introduced, namely $\tilde{\varphi}$ and φ transforming as a $\overline{\mathbf{3}}$ and $\mathbf{3}$, respectively. The particle content of this model is summarized in Tab. 1. For later purposes we have also listed the hypercharges as well as the charges under the $U(1)_R$ symmetry which is required for the F -term flavon alignment mechanism discussed in Section 4.

With these T_7 assignments, the leading order superpotentials of the charged lepton and the neutrino sector read

$$W_\ell = y_e \frac{\tilde{\varphi}}{\Lambda} L e^c h_d + y_\mu \frac{\tilde{\varphi}}{\Lambda} L \mu^c h_d + y_\tau \frac{\tilde{\varphi}}{\Lambda} L \tau^c h_d, \quad (3)$$

$$W_\nu = y_1 \frac{\tilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u, \quad (4)$$

respectively. Here, the y 's are dimensionless coupling constants, and Λ denotes a common cut-off above the scale of family symmetry breaking. The neutrino mass terms originate from the Weinberg operator [30] augmented by a flavon field.

It is important to note that only one flavon, $\tilde{\varphi}$, enters the charged lepton sector at leading order. The other flavon, φ , cannot couple to the charged leptons due to the absence of a singlet representation in the T_7 Kronecker product $\mathbf{3} \otimes \mathbf{3}$, see Appendix A. We will see below that the vev of $\tilde{\varphi}$ can be chosen such that it breaks T_7 down to \mathbb{Z}_3 , thus leading to

a diagonal charged lepton mass matrix. In the neutrino sector, both flavons are present, as the symmetric product of $\mathbf{3} \otimes \mathbf{3}$ contains both a $\mathbf{3}$ and a $\bar{\mathbf{3}}$.

In order to find the vacuum alignments that lead to tribimaximal mixing, we insert the flavon *vevs* in Eq. (3) and Eq. (4) in their most general form. Contracting the T_7 indices using the CG coefficients given in Appendix A and inserting the Higgs *vevs* v_u and v_d , we obtain the respective mass matrices. In the left-right convention for the charged leptons, where the left-handed particles are to the left of the Yukawa matrix, we find

$$M_\ell = \left[\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \frac{\langle \tilde{\varphi}_1 \rangle}{\Lambda} + \begin{pmatrix} 0 & 0 & y_\tau \\ y_e & 0 & 0 \\ 0 & y_\mu & 0 \end{pmatrix} \frac{\langle \tilde{\varphi}_2 \rangle}{\Lambda} + \begin{pmatrix} 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \\ y_e & 0 & 0 \end{pmatrix} \frac{\langle \tilde{\varphi}_3 \rangle}{\Lambda} \right] \frac{v_d}{\sqrt{3}}, \quad (5)$$

$$M_\nu = y_1 \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{\langle \tilde{\varphi}_1 \rangle}{\Lambda} + \begin{pmatrix} 0 & \omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & \omega \end{pmatrix} \frac{\langle \tilde{\varphi}_2 \rangle}{\Lambda} + \begin{pmatrix} 0 & 0 & \omega^2 \\ 0 & \omega^2 & 0 \\ \omega^2 & 0 & 0 \end{pmatrix} \frac{\langle \tilde{\varphi}_3 \rangle}{\Lambda} \right] \frac{v_u^2}{3\Lambda} \\ + y_2 \left[\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \frac{\langle \varphi_1 \rangle}{\Lambda} + \begin{pmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix} \frac{\langle \varphi_2 \rangle}{\Lambda} + \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \frac{\langle \varphi_3 \rangle}{\Lambda} \right] \frac{v_u^2}{3\sqrt{2}\Lambda}. \quad (6)$$

This simple pattern suggests the following flavon vacuum configuration to obtain tribimaximal mixing:

$$\langle \tilde{\varphi} \rangle = v_{\tilde{\varphi}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \varphi \rangle = v_\varphi \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \quad (7)$$

The alignment of $\tilde{\varphi}$ ensures a diagonal charged lepton mass matrix in Eq. (5), and brings the first line in Eq. (6) into tribimaximal form. The alignment of φ is of an extremely simple form and generates a contribution to M_ν which, again, has tribimaximal structure. Adopting the flavon alignments of Eq. (7), our minimal T_7 model therefore predicts the following charged lepton and neutrino mass matrices at leading order,

$$M_\ell = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \frac{v_{\tilde{\varphi}}}{\Lambda} \frac{v_d}{\sqrt{3}}, \quad (8)$$

$$M_\nu = \left[y_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{v_{\tilde{\varphi}}}{\Lambda} + y_2 \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \frac{v_\varphi}{\sqrt{6}\Lambda} \right] \frac{v_u^2}{3\Lambda}, \quad (9)$$

implying a tribimaximal PMNS mixing matrix and the complex valued neutrino masses

$$(m_1^\nu, m_2^\nu, m_3^\nu) = (2y_1 v_{\tilde{\varphi}} + \sqrt{6}y_2 v_\varphi, 2y_1 v_{\tilde{\varphi}}, -2y_1 v_{\tilde{\varphi}} + \sqrt{6}y_2 v_\varphi) \frac{v_u^2}{6\Lambda^2}. \quad (10)$$

Before continuing to the study of NLO effects, let us pause for a moment and appreciate the beauty of the leading order model. With regard to the charged lepton sector, we

observe that the alignment of $\tilde{\varphi}$ is left invariant by the diagonal order-three generator d of T_7 , see Tab. 5 in Appendix A. The corresponding \mathbb{Z}_3 symmetry which is thus preserved in the charged lepton sector after the family symmetry is broken enforces a diagonal M_ℓ . In contrast, the tribimaximal Klein symmetry $\mathbb{Z}_2 \times \mathbb{Z}_2$ of M_ν arises completely accidentally from a combination of the T_7 CG coefficients and suitable flavon alignments (which will be derived and thus justified in Section 4). This situation is reminiscent of the Altarelli-Feruglio model [22, 23] where one \mathbb{Z}_2 is contained in A_4 while the other \mathbb{Z}_2 , corresponding to a μ - τ symmetry, arises accidentally. In the case of T_7 , none of the two \mathbb{Z}_2 factors of the tribimaximal Klein symmetry is a part of the underlying family symmetry. As a consequence, one expects no protection of the tribimaximal structure when NLO terms are taken into account.

2.2. Next-to-leading order effects

The next-to-leading order superpotential for the charged and neutral leptons contains all terms up to mass dimension six and seven, respectively, that are invariant under the gauge and family symmetries. Here, we only indicate the terms that are not already part of the leading-order superpotential:

$$\begin{aligned} \Delta W_\ell = \frac{1}{\Lambda^2} & \left(C_1^e L e^c h_d \varphi \varphi + C_2^e L e^c h_d \varphi \tilde{\varphi} + C_3^e L e^c h_d \tilde{\varphi} \tilde{\varphi} + \right. \\ & C_1^\mu L \mu^c h_d \varphi \varphi + C_2^\mu L \mu^c h_d \varphi \tilde{\varphi} + C_3^\mu L \mu^c h_d \tilde{\varphi} \tilde{\varphi} + \\ & \left. C_1^\tau L \tau^c h_d \varphi \varphi + C_2^\tau L \tau^c h_d \varphi \tilde{\varphi} + C_3^\tau L \tau^c h_d \tilde{\varphi} \tilde{\varphi} \right), \end{aligned} \quad (11)$$

$$\begin{aligned} \Delta W_\nu = \frac{1}{\Lambda^3} & \left(C_1^\nu (L L)_{\mathbf{3}} h_u h_u \varphi \varphi + C_2^\nu (L L)_{\overline{\mathbf{3}}} h_u h_u \varphi \varphi + C_3^\nu (L L)_{\mathbf{3}} h_u h_u \tilde{\varphi} \tilde{\varphi} + \right. \\ & \left. C_4^\nu (L L)_{\overline{\mathbf{3}}} h_u h_u \tilde{\varphi} \tilde{\varphi} + C_5^\nu (L L)_{\mathbf{3}} h_u h_u \varphi \tilde{\varphi} + C_6^\nu (L L)_{\overline{\mathbf{3}}} h_u h_u \varphi \tilde{\varphi} \right). \end{aligned} \quad (12)$$

The subscripts in Eq. (12) indicate that we have contracted the family indices of the corresponding product to obtain a $\mathbf{3}$ and $\overline{\mathbf{3}}$, respectively. As before, contracting the family and gauge indices, and substituting the *vevs* for the Higgs and flavon fields gives:

$$\begin{aligned} \Delta M_\ell = \frac{v_d}{\Lambda^2} \frac{1}{3} & \left[C_1^e \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} v_\varphi^2 + C_2^e \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\tilde{\varphi}} + C_3^e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} v_{\tilde{\varphi}}^2 \right. \\ & + C_1^\mu \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} v_\varphi^2 + C_2^\mu \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\tilde{\varphi}} + C_3^\mu \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} v_{\tilde{\varphi}}^2 \\ & \left. + C_1^\tau \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} v_\varphi^2 + C_2^\tau \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \frac{v_\varphi}{\sqrt{3}} v_{\tilde{\varphi}} + C_3^\tau \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_{\tilde{\varphi}}^2 \right], \end{aligned} \quad (13)$$

$$\begin{aligned} \Delta M_\nu = \frac{v_u^2}{\Lambda^3} \frac{1}{9\sqrt{2}} & \left[C_1^\nu \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} v_\varphi^2 + \sqrt{3} C_2^\nu \begin{pmatrix} 2 & -\omega^2 & -\omega \\ -\omega^2 & 2\omega & -1 \\ -\omega & -1 & 2\omega^2 \end{pmatrix} v_\varphi^2 + \sqrt{6} C_3^\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_{\tilde{\varphi}}^2 \right. \\ & \left. + \sqrt{6} C_4^\nu \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} v_{\tilde{\varphi}}^2 + \sqrt{2} C_5^\nu \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} v_\varphi v_{\tilde{\varphi}} + C_6^\nu \begin{pmatrix} 2 & -\omega & -\omega^2 \\ -\omega & 2\omega^2 & -1 \\ -\omega^2 & -1 & 2\omega \end{pmatrix} v_\varphi v_{\tilde{\varphi}} \right]. \end{aligned} \quad (14)$$

A quick inspection of Eq. (13) shows that the contributions proportional to C_1^e, C_1^μ, C_1^τ are identically zero and have no effect whatsoever on the mass matrices. The contributions proportional to C_3^e, C_3^μ, C_3^τ do not disturb the diagonal structure of M_ℓ and hence do not affect the bi-unitary transformations that diagonalize it and that enter in the definition of U_{PMNS} . In contrast, C_2^e, C_2^μ, C_2^τ will lead to a departure from tribimaximal mixing, where it has to be noted that the magnitudes of the coefficients have to be weighted by the corresponding Yukawa couplings, and hence the effect of C_2^e will be negligible.

From Eq. (14) we see that the term proportional to C_1^ν is identically zero and will not have an impact on the mixing matrix. The term corresponding to C_3^ν is non-vanishing, but commutes with the two generators of the tribimaximal Klein symmetry (see e.g. [4] for the explicit form of the generators), and as a consequence does not disturb the tribimaximal form of the mixing matrix.

The NLO terms that do cause a departure from tribimaximal mixing are those corresponding to $C_2^\nu, C_4^\nu, C_5^\nu, C_6^\nu$. The contribution proportional to C_4^ν is invariant under a $\mu - \tau$ symmetry (which corresponds to one of the generators of the tribimaximal Klein symmetry) and thus leaves θ_{23} and θ_{13} unchanged. Of particular interest is the term proportional to C_5^ν , since its effect is such that θ_{12} is almost unchanged, θ_{23} stays within its 3σ interval, and θ_{13} receives a sizable contribution. Therefore it seems to be a good strategy to try and suppress all NLO contributions to the superpotential except the one proportional to C_5^ν . Thus, we will keep the deviation of θ_{12} and θ_{23} from the tribimaximal case small, and at the same time achieve a non-vanishing θ_{13} in agreement with experiment. In Section 3 we present a renormalizable, ultraviolet completion of our model that meets exactly these criteria outlined above. Before that, however, we will explore in Section 2.3 the phenomenology of our model at the effective level. We first consider the model in its most general form without any specific assumptions about the coefficients. At the end of Section 2.3 we also discuss the case where all coefficients which break the tribimaximal structure, except for C_5^ν , are set to zero.

2.3. Phenomenology

We now compare our results with experiment [28, 29, 31–33]. To that end, we first have to specify the coefficients in the leading and next-to-leading order superpotential (cf. Eq. (3), Eq. (4), Eq. (11), Eq. (12)). The mixing angles, CP phases and masses will then be uniquely determined.

The leading-order superpotential gives tribimaximal mixing which is form-diagonalizable [34]. As such, the mixing angles are in principle independent of the coefficients $y_e, y_\mu, y_\tau, y_1, y_2$. However, changing the coefficients affects the neutrino masses and can lead to a reordering of the mass eigenstates that in turn has an impact on the mixing angles. Since we work in a basis where the charged lepton mass matrix is diagonal, we can identify y_e, y_μ, y_τ with the corresponding Yukawa couplings for which we substitute their experimentally determined values [35]. For y_1 and y_2 , we choose values such that we can fit the mass ratio $\Delta m_{31}^2/\Delta m_{21}^2$.

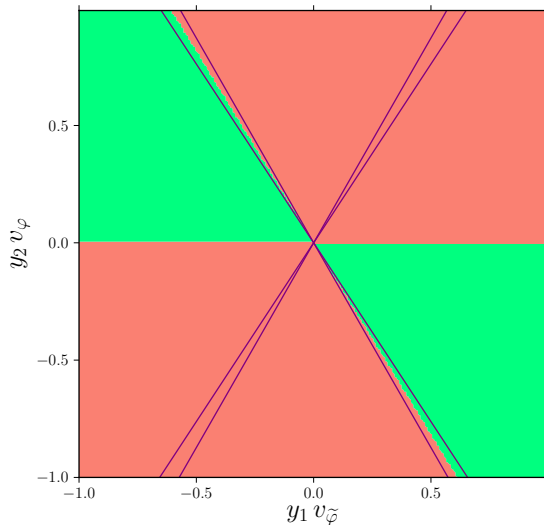
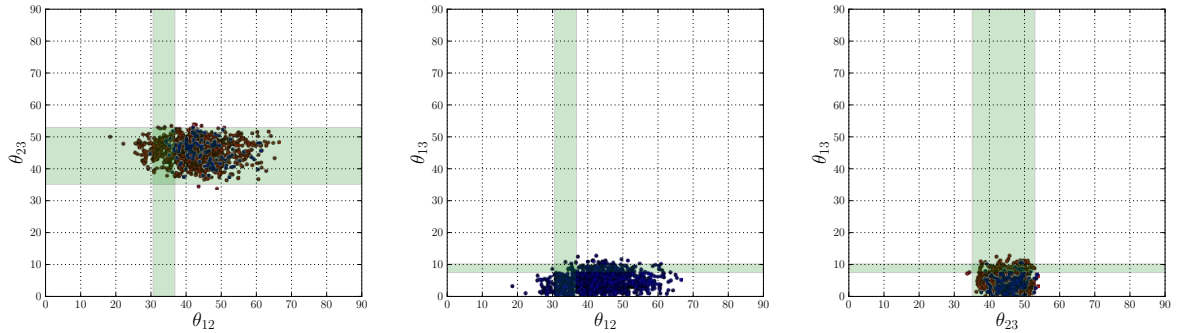


Figure 1: Contour lines of $\Delta m_{31}^2/\Delta m_{21}^2 \equiv 30$ as a function of $y_1 v_{\tilde{\varphi}}$ and $y_2 v_{\varphi}$ for normal neutrino mass ordering. The green region corresponds to values of the parameters where the mixing angles are tribimaximal.

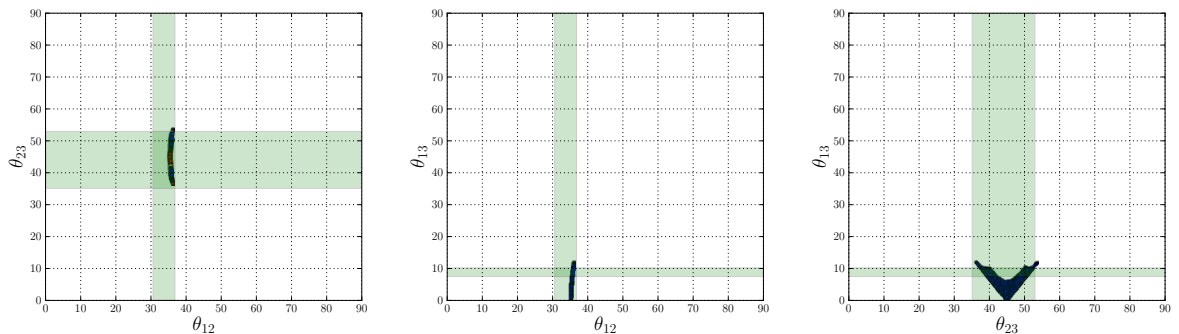
In Fig. 1 we present the contour lines of $\Delta m_{31}^2/\Delta m_{21}^2 \equiv 30$ as a function of $y_1 v_{\tilde{\varphi}}$ and $y_2 v_{\varphi}$ (cf. Eq. (9) on page 4) for normal neutrino mass orderings. Here we do not consider the case of inverted neutrino mass orderings as it would require $y_1 v_{\tilde{\varphi}} \ll y_2 v_{\varphi}$. The green region in Fig. 1 corresponds to values of the parameters for which we obtain tribimaximal mixing, whereas in the red region, the mixing matrix corresponds to a permutation of the columns of U_{PMNS} , i.e. tribimaximal mixing and the desired mass hierarchy cannot be simultaneously satisfied. Note that we have chosen y_1 and y_2 to be real for presentational purposes only. In the general case, y_1 and y_2 are complex, and to fit the mass ratio is equally easy.

In the next-to-leading order superpotential, we assume that the fundamental scale of the theory is about one order of magnitude larger than the flavon *vevs* and fix the ratio at $v_{\varphi}/\Lambda = v_{\tilde{\varphi}}/\Lambda = 0.10$ (see Eq. (13) and Eq. (14)). For the coefficients $C_1^{\nu}, \dots, C_3^{\tau}$, we choose 1000 sets of complex numbers whose real and imaginary parts are uniformly distributed between -2.5 and +2.5. For each such set, we calculate the corrections to the charged lepton and neutrino mass matrices and recalculate the mixing angles and masses. Note that now, M_{ℓ} is not necessarily diagonal, and we have to resort to the general definition of U_{PMNS} in terms of the bi-unitary transformations that diagonalize the charged lepton and neutrino mass matrices.

We present the results in Fig. 2(a) on the following page. The LO order prediction has given way to a “blob” centered around the tribimaximal values for the mixing angles. Remarkably, 99% of the points lie in the 3σ interval [31] of θ_{23} , whereas 13% also satisfy the 3σ bounds for θ_{12} (see first panel of Fig. 2(a)). As was to be expected, the strongest constraint comes from θ_{13} . From the third panel of Fig. 2(a) we see that low values



(a) $v_\varphi/\Lambda = v_{\tilde{\varphi}}/\Lambda = 0.10$ and $-2.5 \leq \Re C_i^\alpha, \Im C_i^\alpha \leq 2.5$.



(b) $v_\varphi/\Lambda = 0.25$, $v_{\tilde{\varphi}}/\Lambda = 0.05$, $-2 \leq \Re C_5^\nu, \Im C_5^\nu \leq 2$, all other coefficients set to zero.

Figure 2: The mixing angles for a given set of leading order coefficients $y_1, y_2, y_e, y_\mu, y_\tau$ and 1000 sets of random values for the next-to-leading order coefficients C_1^ν, \dots, C_3^τ . Our choice for the coefficients $y_1 = 0.50, y_2 = -1.52 y_1 v_{\tilde{\varphi}}/v_\varphi$ correspond to $\Delta m_{31}^2/\Delta m_{21}^2 \sim 30$ in the LO approximation. The green bands correspond to the 3σ intervals [31] for the mixing angles (normal hierarchy in the case of θ_{23} and θ_{13}). In each diagram, the color of the markers indicates whether the data point lies in the 3σ interval of the respective angle that is not plotted (blue) or not (red).

for θ_{13} are preferred, but 10% of the points can accommodate a large reactor angle in agreement with experiment. This number drops to 1% when we take the constraints from the other two angles into account.

In Fig. 2(b) we present the results for a model where all terms except C_5^ν that lead to departures from tribimaximal mixing are suppressed. This will correspond to the case of the renormalizable model that we will discuss in Section 3. As we can see from the first panel of Fig. 2(b), θ_{12} is practically unchanged from its tribimaximal value, whereas θ_{23} varies very little, and both angles are well within their 3σ error bands. At the same time, we can easily obtain a large θ_{13} in such a way that the experimental constraints on the other two angles are always satisfied (blue points in the third panel of Fig. 2(b)). At the same time we can observe an interesting correlation between θ_{23} and θ_{13} , namely the larger θ_{13} is, the more θ_{23} deviates from maximal mixing.² Unfortunately, this effect

²This numerical observation is a reflection of a well-known mixing sum rule relating the deviations of the atmospheric and reactor angles from their tribimaximal values via the CP phase δ [36, 37].

is symmetric around $\theta_{23} = 45^\circ$ and hence does not give a hint whether this deviation is positive or negative.

3. An ultraviolet completion of the model

We have shown in Section 2 how the minimal T_7 model, which predicts exact tribimaximal lepton mixing at leading order, receives in general significant corrections at next-to-leading order. Such deviations from the tribimaximal pattern are welcome in order to accommodate the observed large value of the reactor angle θ_{13} . On the other hand, the same set of corrections can potentially modify the other two mixing angles to values disfavored by current global fits. In particular, the solar angle should obtain only small corrections, as its tribimaximal value of $\theta_{12} \approx 35.3^\circ$ fits the experimental result already remarkably well. In order to obtain a deviation from tribimaximal mixing which stabilizes the solar angle in first approximation, one can make use of corrections of the so-called *trimaximal* type [38–46]. These are typically defined by corrections to the tribimaximal neutrino mass matrix whose eigenvectors are proportional to either $(1, 1, 1)^T$ [37] or $(2, -1, -1)^T$ [47]. Comparison with Eq. (14) reveals that there exists exactly one NLO correction which respects the trimaximal structure but breaks the tribimaximal one. This subleading contribution to the neutrino mass matrix is proportional to the coefficient C_5^ν and can be traced back to an NLO operator in Eq. (12) in which LL is contracted to a $\mathbf{3}$ of T_7 . Notice that the other two terms with LL contracted to a $\mathbf{3}$, i.e. those proportional to C_1^ν and C_3^ν , do not lead to deviations from the tribimaximal structure.

This begs the question of how to remove or suppress the unwanted NLO operators while keeping the one proportional to C_5^ν . To this end, it is useful to recall that we have so far only discussed the model at the effective level, i.e. writing down all (non-renormalizable) terms which are allowed by the imposed symmetries. However, any particular ultraviolet completion of such an effective model does not give rise to all effective NLO terms. The underlying renormalizable model will involve messenger fields which have to be introduced to mediate the effective leading order operators. These necessary messengers typically do not generate all NLO terms [48]. If one particular NLO term is desired, as is the case in our T_7 model, an extra messenger has to be introduced. In the following we adopt the strategy of considering an ultraviolet completion of the effective T_7 model such that the operator proportional to C_5^ν is allowed, while the bothersome NLO terms are forbidden or sufficiently suppressed. We begin the discussion of the ultraviolet completion of the model with the charged lepton sector, before turning to the more interesting neutrino sector which will be responsible for the breaking of the tribimaximal to the trimaximal pattern.

3.1. The charged lepton sector

The leading order operators in Eq. (3) can be obtained by imposing a T_7 triplet messenger pair Θ, Θ^c with the transformation properties as given in Tab. 2. The resulting

Field	Δ	Θ	Θ^c	Σ	Σ^c	Ω	Ω^c
T_7	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\bar{\mathbf{3}}$
$U(1)_Y$	2	-2	2	-2	2	1	-1
$U(1)_R$	0	1	1	2	0	1	1

Table 2: The charge assignments of the (effective) Higgs $SU(2)_L$ triplet Δ as well as of the messenger fields required for the ultraviolet completion of the minimal T_7 model.

renormalizable charged lepton superpotential takes the form

$$W_\ell^{\text{ren}} \sim Lh_d\Theta^c + \Theta\tilde{\varphi}e^c + \Theta\tilde{\varphi}\mu^c + \Theta\tilde{\varphi}\tau^c + \Theta\Theta^c(M_\Theta + \varphi + \tilde{\varphi}) , \quad (15)$$

where we have suppressed all dimensionless coupling coefficients. Note that Θ and Θ^c allow for a bilinear mass term. However, due to the absence of any shaping symmetry, we can also couple a flavon triplet to this product, thus leading to a trilinear term which, after family symmetry breaking, gives a correction to the messenger mass. This is a general feature for heavy messenger fields which transform as triplets under T_7 . Ignoring the correction to the messenger mass, we can integrate out the pair Θ , Θ^c and obtain the effective superpotential of Eq. (3) without any higher order corrections. Hence, in the present ultraviolet completion, the only source of deviations from a diagonal charged lepton mass matrix originates from the aforementioned trilinear terms correcting the messenger mass. Such a correction can be rendered sufficiently small by assuming $\langle\tilde{\varphi}\rangle, \langle\varphi\rangle \ll M_\Theta$. In the following, we will therefore ignore any NLO terms in the charged lepton sector.

3.2. The neutrino sector

We now turn to the neutrino sector where we want to formulate a renormalizable theory which, at leading order, gives rise to the effective superpotential in Eq. (4). The most popular possibility to derive the Weinberg operator from renormalizable terms is provided by the famous type I seesaw mechanism [15–18]. However, applied to the minimal T_7 model without any shaping symmetry, the type I seesaw mechanism does not yield the tribimaximal structure at leading order. This can be traced back to the fact that the triplet representations of T_7 are complex. Putting the L into the $\mathbf{3}$ would entail to introduce right-handed neutrinos N^c in the $\bar{\mathbf{3}}$ so that the Dirac neutrino Yukawa term $Lh_u N^c$ leads to a Dirac mass matrix that is proportional to the identity matrix. Then the tribimaximal structure of the effective light neutrino mass matrix would be inherited directly from a tribimaximal right-handed neutrino mass matrix M_R . With N^c being a $\bar{\mathbf{3}}$ rather than a $\mathbf{3}$, the alignments of the $\bar{\mathbf{3}}$ flavon $\tilde{\varphi}$ and the $\mathbf{3}$ flavon φ would have to be exchanged to get M_R of tribimaximal form. However, changing the alignment of the flavon $\tilde{\varphi}$ coupling to the charged leptons to $(1, 1, 1)^T$ destroys the diagonal charged lepton mass matrix in Eq. (5). As a result, under the fairly general set of assumptions

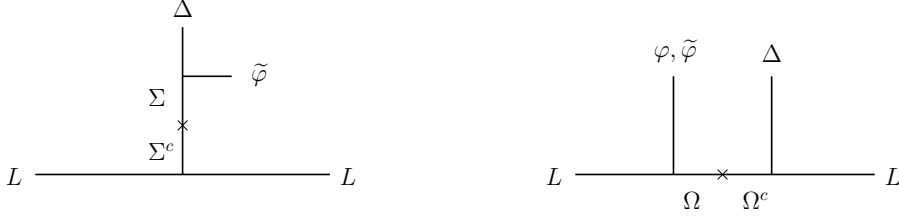


Figure 3: The diagrams contributing to the leading order neutrino superpotential of Eq. (4).

that we made above, the total leading order lepton mixing would no longer feature the tribimaximal pattern.

As an alternative to the type I seesaw mechanism, we can adopt the type II seesaw [49–53]. For the sake of clarity, we assume a Higgs $SU(2)_L$ triplet Δ in the remainder of this section. It is straightforward to replace such a Higgs field by the product of two h_u doublets which couple to the square of the lepton doublet L via an $SU(2)_L$ triplet messenger field. The type II seesaw mechanism seems particularly suited for our purposes as we wish to obtain the NLO correction to the neutrino mass matrix proportional to the coupling constant C_5^y , which arises from contracting LL to a $\mathbf{3}$ of T_7 . This can be naturally achieved by invoking a messenger field Σ^c in the $\bar{\mathbf{3}}$ representation which leads to the renormalizable term $LL\Sigma^c$. The $U(1)_Y$ and $U(1)_R$ charges required for the existence of this term are given in Tab. 2. Demanding a bilinear mass term fixes the charge assignments of Σ which, in turn, can couple to the Higgs field Δ and the flavon $\tilde{\varphi}$. A similar coupling to the other flavon φ is forbidden by the T_7 family symmetry. We are therefore forced to introduce another pair of messengers Ω, Ω^c , which allows to generate the second leading order contribution to M_ν proportional to y_2 , cf. Eq. (6). Their charge assignments are again given in Tab. 2.

With this particle content, the renormalizable neutrino superpotential reads

$$\begin{aligned}
 W_\nu^{\text{ren}} \sim & LL\Sigma^c + \Sigma\tilde{\varphi}\Delta + \Sigma\Sigma^c(M_\Sigma + \varphi + \tilde{\varphi}) \\
 & + L\varphi\Omega + L\tilde{\varphi}\Omega + \Omega^c\Delta L + \Omega\Omega^c(M_\Omega + \varphi + \tilde{\varphi}) \\
 & + L\Sigma^c\Omega^c + \Sigma^c\Omega^c\Omega^c,
 \end{aligned} \tag{16}$$

where we have suppressed all dimensionless coupling constants. As already discussed for the charged lepton sector, the masses of the T_7 triplet messengers receive corrections from the flavon *vevs*. These will break the tribimaximal structure at higher order. Ignoring the corrections to the messenger masses as well as the two operators in the third line of Eq. (16) results in the effective superpotential of Eq. (4). The corresponding diagrams are sketched in Fig. 3, where the right diagram contributes to the term proportional to y_2 , while the term proportional to y_1 arises from both diagrams.

We point out that the left diagram in Fig. 3 is needed only for generating the desired trimaximal NLO correction while suppressing other unwanted NLO terms. In other words, if we wanted to end up with tribimaximal neutrino mixing, the diagram with

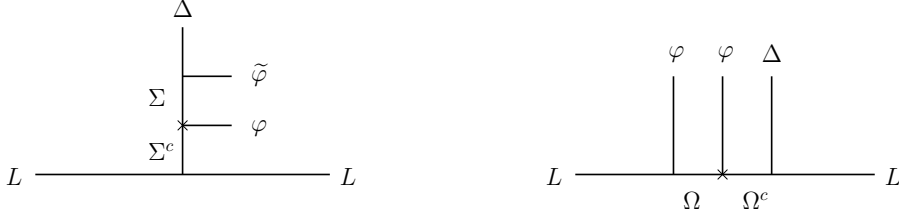


Figure 4: Two diagrams contributing to the NLO neutrino superpotential. The diagram on the left gives the desired trimaximal correction, while the diagram on the right should be suppressed in order to avoid large deviations of the solar angle from its tribimaximal value of $\theta_{12} \approx 35.3^\circ$.

the Ω messenger would have been perfectly sufficient. In that case we would have to impose the condition $\langle \tilde{\varphi} \rangle, \langle \varphi \rangle \ll M_\Omega$, similar to the situation in the charged lepton sector. However, since accurate tribimaximal mixing is ruled out by the Daya Bay and Reno measurements, we are forced to consider *sizable* NLO corrections. Due to the structure of the diagram with the Ω messenger, the NLO terms obtained from attaching a flavon to the cross representing the mass term $M_\Omega \Omega \Omega^c$ would give rise to corrections that would shift the solar angle away from its experimentally allowed region. Hence, the trimaximal NLO correction must originate from another diagram, namely the one with the Σ messenger, more precisely, the left diagram of Fig. 3 with the flavon φ attached to the cross. The resulting diagram is sketched in Fig. 4, where one NLO diagram involving the Ω messenger is also shown. The NLO diagram on the right can be suppressed compared to the NLO diagram on the left by assuming a hierarchy in the messenger masses. This can be parameterized by

$$M_\Sigma \sim \epsilon^k M_\Omega, \quad (17)$$

where k is a positive integer and ϵ an expansion parameter around 0.2. With this hierarchy of messenger masses imposed we also have to assume a hierarchy in the two flavons *vevs*,

$$\frac{v_{\tilde{\varphi}}}{v_\varphi} \sim \frac{M_\Sigma}{M_\Omega} \sim \epsilon^k, \quad (18)$$

so that the leading order mass contributions derived from the two diagrams in Fig. 3 are of similar size. Note that the diagram on the right involving the flavon $\tilde{\varphi}$ gives a subdominant contribution which is of the same tribimaximal structure as the one obtained from the left diagram. Finally, we need to guarantee that the trimaximal NLO correction of the left diagram in Fig. 4 is sizable enough to account for the observed value of reactor angle θ_{13} . This requires the correction to the mass of the Σ messenger originating from the flavon φ to be of order ϵ , i.e.

$$\frac{v_\varphi}{M_\Sigma} \sim \epsilon. \quad (19)$$

Defining $M \sim M_\Omega$ we can summarize the requirements on the hierarchies as

$$v_{\tilde{\varphi}} \sim \epsilon^{2k+1} M, \quad v_\varphi \sim \epsilon^{k+1} M, \quad M_\Sigma \sim \epsilon^k M, \quad M_\Omega \sim M, \quad (20)$$

yielding neutrino mass contributions proportional to $\frac{v_{\tilde{\varphi}}}{M_{\Sigma}} \sim \frac{v_{\varphi}}{M_{\Omega}} \sim \epsilon^{k+1}$ at leading order, with a trimaximal NLO correction proportional to ϵ^{k+2} . In comparison, the right diagram in Fig. 4 gives a contribution proportional to ϵ^{2k+2} which is suppressed by a factor of ϵ^k relative to the desired NLO correction.

With the above assumptions on the hierarchies it is possible to suppress all unwanted contractions in Eq. (12) while keeping the desired one proportional to C_5^{ν} . This holds true even if we take into consideration possible higher order diagrams obtained from the two terms of the third line of Eq. (16). These terms can give rise to diagrams which involve one Σ messenger as well as one or two Ω messengers. Multiplying the appropriate flavon fields, one can easily find that the maximal contribution derived from the renormalizable term $L\Sigma^c\Omega^c$ is of order ϵ^{2k+2} , while the one derived from $\Sigma^c\Omega^c\Omega^c$ is of order ϵ^{3k+3} . This shows that these higher order corrections are also suppressed with respect to the trimaximal NLO correction by at least a factor of ϵ^k .

We conclude this section by mentioning that we have also checked these results numerically for the case of $k = 2$. Integrating out the messengers we have first determined the effective neutrino mass matrix, from which we have calculated the mixing angles using the Mixing Parameter Tools provided with the REAP package [54]. This numerical check confirms the results of this section, showing how an ultraviolet completion of the minimal T_7 model can lead to deviations from tribimaximal mixing which are compatible with a sizable reactor angle as well as a solar angle that is close to its tribimaximal value.

4. Vacuum alignment

4.1. General discussion

Any model of flavor that tries to explain a particular mixing pattern by means of a family symmetry which gets broken by flavon *vevs* has to justify the imposed alignments. In this section we study the possibilities of achieving the vacuum configuration required by the minimal T_7 model. Following the idea of [23], we demand that supersymmetry be unbroken at the scale of family symmetry breaking. This so-called F -term alignment mechanism introduces driving fields whose F -terms are set to zero, thus entailing the important F -term conditions. Just like the regular flavons, driving fields are neutral under the $SU(2)_L \times U(1)_Y$ gauge symmetry. With the five irreducible representations of T_7 and no \mathbb{Z}_N shaping symmetry, one can introduce driving fields in at most five different representations. Hence, we obtain five distinct F -term conditions which we discuss in the following. Throughout this section we will drop all dimensionless coupling as well as overall CG coefficients.

A driving field D_1 transforming in the $\mathbf{1}$ of T_7 allows for two invariant terms in the renormalizable flavon superpotential

$$W_{\text{flav}} \supset D_1 (M^2 + \tilde{\varphi}\varphi) , \quad (21)$$

which lead to the F -term condition

$$\frac{\partial W_{\text{flav}}}{\partial D_1} = M^2 + \langle \tilde{\varphi}_1 \rangle \langle \varphi_1 \rangle + \langle \tilde{\varphi}_2 \rangle \langle \varphi_2 \rangle + \langle \tilde{\varphi}_3 \rangle \langle \varphi_3 \rangle = 0. \quad (22)$$

Similarly, the driving fields D_r in the representations $\mathbf{r} = \mathbf{1}', \mathbf{1}'', \mathbf{3}, \bar{\mathbf{3}}$ of T_7 lead to the conditions:

$$\frac{\partial W_{\text{flav}}}{\partial D_{\mathbf{1}'}} = \langle \tilde{\varphi}_1 \rangle \langle \varphi_2 \rangle + \langle \tilde{\varphi}_2 \rangle \langle \varphi_3 \rangle + \langle \tilde{\varphi}_3 \rangle \langle \varphi_1 \rangle = 0, \quad (23)$$

$$\frac{\partial W_{\text{flav}}}{\partial D_{\mathbf{1}''}} = \langle \tilde{\varphi}_1 \rangle \langle \varphi_3 \rangle + \langle \tilde{\varphi}_2 \rangle \langle \varphi_1 \rangle + \langle \tilde{\varphi}_3 \rangle \langle \varphi_2 \rangle = 0, \quad (24)$$

$$\begin{aligned} \frac{\partial W_{\text{flav}}}{\partial D_{\mathbf{3}}} &= M_1 \begin{pmatrix} \langle \tilde{\varphi}_1 \rangle \\ \langle \tilde{\varphi}_2 \rangle \\ \langle \tilde{\varphi}_3 \rangle \end{pmatrix} + \begin{pmatrix} \langle \tilde{\varphi}_1 \rangle^2 + 2\langle \tilde{\varphi}_2 \rangle \langle \tilde{\varphi}_3 \rangle \\ \omega^2(\langle \tilde{\varphi}_3 \rangle^2 + 2\langle \tilde{\varphi}_1 \rangle \langle \tilde{\varphi}_2 \rangle) \\ \omega(\langle \tilde{\varphi}_2 \rangle^2 + 2\langle \tilde{\varphi}_3 \rangle \langle \tilde{\varphi}_1 \rangle) \end{pmatrix} \\ &+ \begin{pmatrix} \langle \varphi_1 \rangle^2 - \langle \varphi_2 \rangle \langle \varphi_3 \rangle \\ \langle \varphi_2 \rangle^2 - \langle \varphi_3 \rangle \langle \varphi_1 \rangle \\ \langle \varphi_3 \rangle^2 - \langle \varphi_1 \rangle \langle \varphi_2 \rangle \end{pmatrix} + \begin{pmatrix} \langle \tilde{\varphi}_1 \rangle \langle \varphi_1 \rangle + \omega \langle \tilde{\varphi}_2 \rangle \langle \varphi_2 \rangle + \omega^2 \langle \tilde{\varphi}_3 \rangle \langle \varphi_3 \rangle \\ \langle \tilde{\varphi}_1 \rangle \langle \varphi_3 \rangle + \omega \langle \tilde{\varphi}_2 \rangle \langle \varphi_1 \rangle + \omega^2 \langle \tilde{\varphi}_3 \rangle \langle \varphi_2 \rangle \\ \langle \tilde{\varphi}_1 \rangle \langle \varphi_2 \rangle + \omega \langle \tilde{\varphi}_2 \rangle \langle \varphi_3 \rangle + \omega^2 \langle \tilde{\varphi}_3 \rangle \langle \varphi_1 \rangle \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (25) \end{aligned}$$

$$\begin{aligned} \frac{\partial W_{\text{flav}}}{\partial D_{\bar{\mathbf{3}}}} &= M_2 \begin{pmatrix} \langle \varphi_1 \rangle \\ \langle \varphi_2 \rangle \\ \langle \varphi_3 \rangle \end{pmatrix} + \begin{pmatrix} \langle \varphi_1 \rangle^2 + 2\langle \varphi_2 \rangle \langle \varphi_3 \rangle \\ \omega(\langle \varphi_3 \rangle^2 + 2\langle \varphi_1 \rangle \langle \varphi_2 \rangle) \\ \omega^2(\langle \varphi_2 \rangle^2 + 2\langle \varphi_3 \rangle \langle \varphi_1 \rangle) \end{pmatrix} \\ &+ \begin{pmatrix} \langle \tilde{\varphi}_1 \rangle^2 - \langle \tilde{\varphi}_2 \rangle \langle \tilde{\varphi}_3 \rangle \\ \langle \tilde{\varphi}_2 \rangle^2 - \langle \tilde{\varphi}_3 \rangle \langle \tilde{\varphi}_1 \rangle \\ \langle \tilde{\varphi}_3 \rangle^2 - \langle \tilde{\varphi}_1 \rangle \langle \tilde{\varphi}_2 \rangle \end{pmatrix} + \begin{pmatrix} \langle \tilde{\varphi}_1 \rangle \langle \varphi_1 \rangle + \omega^2 \langle \tilde{\varphi}_2 \rangle \langle \varphi_2 \rangle + \omega \langle \tilde{\varphi}_3 \rangle \langle \varphi_3 \rangle \\ \langle \tilde{\varphi}_3 \rangle \langle \varphi_1 \rangle + \omega^2 \langle \tilde{\varphi}_1 \rangle \langle \varphi_2 \rangle + \omega \langle \tilde{\varphi}_2 \rangle \langle \varphi_3 \rangle \\ \langle \tilde{\varphi}_2 \rangle \langle \varphi_1 \rangle + \omega^2 \langle \tilde{\varphi}_3 \rangle \langle \varphi_2 \rangle + \omega \langle \tilde{\varphi}_1 \rangle \langle \varphi_3 \rangle \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \quad (26) \end{aligned}$$

One can easily see that the desired alignments given in Eq. (7) are inconsistent with Eq. (23) and Eq. (24). Eq. (25) also does not admit Eq. (7) as a solution. Turning to Eq. (26), one can substitute $\langle \tilde{\varphi}_1 \rangle = v_{\tilde{\varphi}}$, $\langle \tilde{\varphi}_2 \rangle = \langle \tilde{\varphi}_3 \rangle = 0$ and $\langle \varphi_1 \rangle = \langle \varphi_2 \rangle = \langle \varphi_3 \rangle = \frac{v_{\varphi}}{\sqrt{3}}$ to obtain

$$\frac{1}{\sqrt{3}} M_2 v_{\varphi} + v_{\tilde{\varphi}}^2 + v_{\tilde{\varphi}}^2 + \frac{1}{\sqrt{3}} v_{\tilde{\varphi}} v_{\varphi} = 0, \quad (27)$$

$$\frac{1}{\sqrt{3}} M_2 v_{\varphi} + \omega v_{\varphi}^2 + \frac{1}{\sqrt{3}} \omega^2 v_{\tilde{\varphi}} v_{\varphi} = 0, \quad (28)$$

$$\frac{1}{\sqrt{3}} M_2 v_{\varphi} + \omega^2 v_{\varphi}^2 + \frac{1}{\sqrt{3}} \omega v_{\tilde{\varphi}} v_{\varphi} = 0. \quad (29)$$

Since $v_{\varphi} = 0$ is not of physical interest, we can cancel off v_{φ} as common factor in the second and third equation, which can then be solved easily to obtain $v_{\varphi} = \frac{M_2}{\sqrt{3}}$ and $v_{\tilde{\varphi}} = M_2$. Substituting this into the first equation, yields $2M_2^2 = 0$, which in turn would require $M_2 = 0$. Hence, the desired non-trivial alignment is also incompatible with Eq. (26).

So far we have assumed that *all* terms in Eq. (25) and Eq. (26) are present. However, in extra dimensional models it is possible to envisage a setup in which the two flavons $\tilde{\varphi}$ and φ are localized on different branes [22, 55]. Then, a given driving field could, in principle, couple exclusively to the $\tilde{\varphi}$ flavon, while another driving field could couple exclusively to φ . If both driving fields transform in the $\mathbf{3}$ of T_7 , it is straightforward

Field	χ	ξ'	ψ	$\tilde{\zeta}$	\tilde{D}_χ	D_ψ	$O_{\chi\tilde{\zeta}}$	$O_{\psi\tilde{\zeta}}$
T_7	3	1'	3	$\bar{3}$	$\bar{3}$	3	1	1
$\mathbb{Z}_N^{\text{hid}}$	x	x	y	z	$-2x$	$-2y$	$-x - z$	$-y - z$
$U(1)_R$	0	0	0	0	2	2	2	2

Table 3: The charge assignments of the hidden sector. x, y, z are positive integers smaller than N . The smallest possible value of N is 6 with $(x, y, z) = (2, 1, 5)$ or equivalently $(4, 5, 1)$, see discussion at the end of this section.

(though tedious for the alignment of $\tilde{\varphi}$) to verify that the two flavons feature the desired alignments up to T_7 transformed solutions.

We conclude that the F -term conditions of Eq. (23)-Eq. (26) cannot give rise to the alignments of Eq. (7) without a mechanism that effectively sets some of the coupling constants to zero.

4.2. Alignment through a hidden flavon sector

One of the attractive features of the T_7 model presented in Section 2 is that it does not require the introduction of an extra \mathbb{Z}_N shaping symmetry to arrive at tribimaximal mixing at leading order. Unfortunately, as we have found in Section 4.1, the flavons cannot be aligned without setting some coupling constants of the flavon superpotential to zero. If we do not want to resort to extra dimensions, we are forced to extend the minimal T_7 symmetry by introducing a new symmetry. However, we will show in this section how this can be achieved *without* assigning non-trivial charges under this new symmetry to the flavons $\tilde{\varphi}$ and φ , nor to the lepton and Higgs fields.

To this end, it is necessary to introduce new flavons which do not couple to the leptons. In that sense, one might call them “hidden flavons”, constituting a sequestered sector of the model. This separation is achieved by assuming these flavons (together with some new driving fields) to be the only fields which transform non-trivially under a hidden $\mathbb{Z}_N^{\text{hid}}$ symmetry. The flavons $\tilde{\varphi}$ and φ of the lepton (or visible) sector can then be aligned with respect to those hidden flavons via orthogonality conditions as we discuss now.

In the hidden sector, we introduce four flavons $\chi, \xi', \psi, \tilde{\zeta}$ which are aligned by virtue of four driving fields $\tilde{D}_\chi, D_\psi, O_{\chi\tilde{\zeta}}, O_{\psi\tilde{\zeta}}$. Their transformation properties are given in Tab. 3, where x, y, z are positive integers smaller than N . The minimal $\mathbb{Z}_N^{\text{hid}}$ symmetry and the corresponding values for x, y and z will be determined at the end of this section. The resulting terms of the renormalizable hidden flavon superpotential read

$$W_{\text{flav}}^{\text{hid}} = \tilde{D}_\chi (\chi\xi' + \chi\chi) + D_\psi \psi\psi + O_{\chi\tilde{\zeta}} \chi\tilde{\zeta} + O_{\psi\tilde{\zeta}} \psi\tilde{\zeta}. \quad (30)$$

Setting the F -terms of the driving fields to zero gives rise to the conditions

$$\frac{\partial W_{\text{flav}}^{\text{hid}}}{\partial \tilde{D}_\chi} = \begin{pmatrix} \langle \chi_2 \rangle \\ \langle \chi_3 \rangle \\ \langle \chi_1 \rangle \end{pmatrix} \langle \xi' \rangle + \begin{pmatrix} \langle \chi_1 \rangle^2 + 2\langle \chi_2 \rangle \langle \chi_3 \rangle \\ \omega(\langle \chi_3 \rangle^2 + 2\langle \chi_1 \rangle \langle \chi_2 \rangle) \\ \omega^2(\langle \chi_2 \rangle^2 + 2\langle \chi_3 \rangle \langle \chi_1 \rangle) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (31)$$

$$\frac{\partial W_{\text{flav}}^{\text{hid}}}{\partial D_\psi} = \begin{pmatrix} \langle \psi_1 \rangle^2 - \langle \psi_2 \rangle \langle \psi_3 \rangle \\ \langle \psi_2 \rangle^2 - \langle \psi_3 \rangle \langle \psi_1 \rangle \\ \langle \psi_3 \rangle^2 - \langle \psi_1 \rangle \langle \psi_2 \rangle \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (32)$$

$$\frac{\partial W_{\text{flav}}^{\text{hid}}}{\partial O_{\chi \tilde{\zeta}}} = \langle \chi_1 \rangle \langle \tilde{\zeta}_1 \rangle + \langle \chi_2 \rangle \langle \tilde{\zeta}_2 \rangle + \langle \chi_3 \rangle \langle \tilde{\zeta}_3 \rangle = 0, \quad (33)$$

$$\frac{\partial W_{\text{flav}}^{\text{hid}}}{\partial O_{\psi \tilde{\zeta}}} = \langle \psi_1 \rangle \langle \tilde{\zeta}_1 \rangle + \langle \psi_2 \rangle \langle \tilde{\zeta}_2 \rangle + \langle \psi_3 \rangle \langle \tilde{\zeta}_3 \rangle = 0. \quad (34)$$

These conditions can be solved exactly to give a total of 21 sets of solutions for the hidden flavon alignments. A brief description of how these can be derived is outlined below.

First, we consider Eq. (32). Without loss of generality we can distinguish two cases: $\langle \psi_1 \rangle = 0$ and $\langle \psi_1 \rangle \neq 0$. In the former case one quickly sees that it requires vanishing values for all the components of $\langle \psi \rangle$, thus giving only the trivial vacuum. For $\langle \psi_1 \rangle \neq 0$, we obtain three solutions, $\langle \psi \rangle \propto (1, 1, 1)^T$, $\langle \psi \rangle \propto (1, \omega^2, \omega)^T$ and $\langle \psi \rangle \propto (1, \omega, \omega^2)^T$, which are related to each other by the T_7 transformation d . We remark that a c transformation applied to these three solutions does not modify the alignments but only multiplies them with an irrelevant overall phase.

Next, we turn to Eq. (31). Again there are two cases to distinguish, $\langle \chi_1 \rangle = 0$ and $\langle \chi_1 \rangle \neq 0$. For the former case the solution is easily found to be $\langle \chi \rangle \propto (0, 0, 1)^T$. With $\langle \chi_1 \rangle \neq 0$ on the other hand, it is possible to show that there exist six different solutions, all of which are related to the previous solution by the T_7 symmetry transformation c : $\langle \chi \rangle \propto c^k(0, 0, 1)^T$, where $k = 0, 1, \dots, 6$. One can check that applying a d transformation to any of these seven solutions does not yield new alignments.

Combining our results so far, we have 21 different pairs of alignments $\langle \chi \rangle$ and $\langle \psi \rangle$. Being related by the 21 different T_7 symmetry transformations, they are physically equivalent and one can choose one pair without loss of generality. Having fixed the alignments of $\langle \chi \rangle$ and $\langle \psi \rangle$, Eq. (33) and Eq. (34) determine the alignment of $\langle \tilde{\zeta} \rangle$ uniquely to be perpendicular to both $\langle \chi \rangle$ and $\langle \psi \rangle$. This shows that in total there exist 21 sets of solutions for the hidden flavon alignments. For convenience we choose the following simple set

$$\langle \chi \rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \langle \psi \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \langle \tilde{\zeta} \rangle \propto \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}. \quad (35)$$

Let us now turn to the discussion of aligning the original flavon fields $\tilde{\varphi}$ and φ which enter in the lepton sector. Their alignment is dictated by new driving fields which couple the pre-aligned hidden flavons to $\tilde{\varphi}$ and φ . The transformation properties of the extra

Field	$O_{\chi\tilde{\varphi}}$	$O'_{\chi\tilde{\varphi}}$	$O_{\tilde{\zeta}\varphi}$	$O'_{\tilde{\zeta}\varphi}$
T_7	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}'$
$\mathbb{Z}_N^{\text{hid}}$	$-x$	$-x$	$-z$	$-z$
$U(1)_R$	2	2	2	2

Table 4: Charge assignments of the driving fields coupling the hidden flavons $\chi, \tilde{\zeta}$ to the flavons $\tilde{\varphi}, \varphi$.

driving fields $O_{\chi\tilde{\varphi}}, O'_{\chi\tilde{\varphi}}, O_{\tilde{\zeta}\varphi}$ and $O'_{\tilde{\zeta}\varphi}$ are listed in Tab. 4. With these assignments, the resulting renormalizable flavon superpotential consisting of all T_7 and $\mathbb{Z}_N^{\text{hid}}$ invariant terms reads

$$W'_{\text{flav}} = O_{\chi\tilde{\varphi}}\chi\tilde{\varphi} + O'_{\chi\tilde{\varphi}}\chi\tilde{\varphi} + O_{\tilde{\zeta}\varphi}\tilde{\zeta}\varphi + O'_{\tilde{\zeta}\varphi}\tilde{\zeta}\varphi. \quad (36)$$

The derived F -term conditions take the form

$$\frac{\partial W'_{\text{flav}}}{\partial O_{\chi\tilde{\varphi}}} = \langle \chi_1 \rangle \langle \tilde{\varphi}_1 \rangle + \langle \chi_2 \rangle \langle \tilde{\varphi}_2 \rangle + \langle \chi_3 \rangle \langle \tilde{\varphi}_3 \rangle = \langle \chi_3 \rangle \langle \tilde{\varphi}_3 \rangle = 0, \quad (37)$$

$$\frac{\partial W'_{\text{flav}}}{\partial O'_{\chi\tilde{\varphi}}} = \langle \chi_1 \rangle \langle \tilde{\varphi}_3 \rangle + \langle \chi_2 \rangle \langle \tilde{\varphi}_1 \rangle + \langle \chi_3 \rangle \langle \tilde{\varphi}_2 \rangle = \langle \chi_3 \rangle \langle \tilde{\varphi}_2 \rangle = 0, \quad (38)$$

$$\frac{\partial W'_{\text{flav}}}{\partial O_{\tilde{\zeta}\varphi}} = \langle \tilde{\zeta}_1 \rangle \langle \varphi_1 \rangle + \langle \tilde{\zeta}_2 \rangle \langle \varphi_2 \rangle + \langle \tilde{\zeta}_3 \rangle \langle \varphi_3 \rangle = \langle \tilde{\zeta}_1 \rangle (\langle \varphi_1 \rangle - \langle \varphi_2 \rangle) = 0, \quad (39)$$

$$\frac{\partial W'_{\text{flav}}}{\partial O'_{\tilde{\zeta}\varphi}} = \langle \tilde{\zeta}_3 \rangle \langle \varphi_1 \rangle + \langle \tilde{\zeta}_1 \rangle \langle \varphi_2 \rangle + \langle \tilde{\zeta}_2 \rangle \langle \varphi_3 \rangle = \langle \tilde{\zeta}_1 \rangle (\langle \varphi_2 \rangle - \langle \varphi_3 \rangle) = 0, \quad (40)$$

yielding the desired alignments, cf. Eq. (7),

$$\langle \tilde{\varphi} \rangle \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \varphi \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \quad (41)$$

Finally, we need to choose an appropriate $\mathbb{Z}_N^{\text{hid}}$, i.e. suitable values for N, x, y and z , so that no additional renormalizable terms, other than the ones discussed in this section, occur in the flavon superpotential of Eq. (30) and Eq. (36). Through a systematic search, we find that the smallest $\mathbb{Z}_N^{\text{hid}}$ that we can have is a $\mathbb{Z}_6^{\text{hid}}$ [although Eq. (30) alone can be obtained using a $\mathbb{Z}_5^{\text{hid}}$]. For a $\mathbb{Z}_6^{\text{hid}}$, we can have (x, y, z) as $(2, 1, 5)$ or $(4, 5, 1)$. Note that these two choices are equivalent solutions since the elements of the group corresponding to one set are the negatives of the elements corresponding to the other.

5. Conclusions

In the present work, we discussed a minimal model of neutrino flavor. In this context, the term “minimal” refers to the order of our flavor group T_7 , and to the number of flavon fields. It is interesting to note that the $\mathbb{Z}_2 \times \mathbb{Z}_2$ Klein symmetry is not a subgroup of T_7 , but arises completely accidentally. As such, it is – to the best of our knowledge – the first indirect model in which the flavon fields appear linearly (and not quadratically) in the leading order structure of the model. Another striking feature of our model is the absence of any Abelian shaping symmetries like \mathbb{Z}_N or $U(1)$. As a consequence, our flavor group T_7 with its 21 elements is smaller than any symmetry $A_4 \times \mathbb{Z}_N$ that has long been a paradigm for model building before Daya Bay’s and Reno’s measurements of a large reactor angle $\theta_{13} \simeq 9^\circ$. Our model predicts tribimaximal mixing at leading order, and we achieve a sizable θ_{13} by considering next-to-leading order corrections to the superpotential. It turns out that models in full agreement with experiment can be obtained for some generic values of the flavon *vevs* and coefficients in the superpotential. More elegantly, however, one can isolate the next-to-leading order contribution proportional to the coefficient C_5'' . Being of trimaximal type, this correction drives θ_{13} to sizable values while leaving the solar angle very close to its tribimaximal value. The atmospheric mixing angle is correlated to the size of θ_{13} and stays within the allowed 3σ region. In Section 3, we embedded our effective theory into a renormalizable model that naturally suppresses all next-to-leading order contributions except the trimaximal one which is responsible for the large reactor angle. In the last section, we discussed how the flavon fields can be dynamically aligned to yield the required symmetry breaking pattern. The alignment of the flavon fields necessitated the introduction of an Abelian shaping symmetry and so-called driving fields. However, one should note that the F -term alignment we considered in Section 4 is only one possible option and that the details of an elegant and simple mechanism of vacuum stabilization are still lurking in the dark.

Acknowledgments

We acknowledge useful discussions with Steve King and Stuart Raby. C.L. acknowledges partial support from the EU ITN grants UNILHC 237920 and INVISIBLES 289442 and would like to thank the LPSC Grenoble for their hospitality. K.P. is supported by the Shyama Prasad Mukherjee Fellowship from the Council of Scientific and Industrial Research (CSIR), India. C.L. and A.W. would like to thank the *Galileo Galilei Institute for Theoretical Physics* in Florence for their hospitality. We are greatly indebted to the *Centre de Calcul de l’Institut National de Physique Nucléaire et Physique des Particules* in Lyon for using their resources.

A. Generators and Clebsch-Gordan coefficients of T_7

In this appendix we list the relevant group theory of T_7 , which is sometimes also called the Frobenius group $\mathbb{Z}_7 \rtimes \mathbb{Z}_3$. The group is obtained from two generators c, d obeying

	d	c
$\mathbf{1}$	1	1
$\mathbf{1}'$	ω	1
$\mathbf{1}''$	ω^2	1
$\mathbf{3}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$\frac{\eta}{3} \begin{pmatrix} 1 + \eta + \eta^3 & \omega^2 + \omega\eta + \eta^3 & \omega + \omega^2\eta + \eta^3 \\ \omega + \omega^2\eta + \eta^3 & 1 + \eta + \eta^3 & \omega^2 + \omega\eta + \eta^3 \\ \omega^2 + \omega\eta + \eta^3 & \omega + \omega^2\eta + \eta^3 & 1 + \eta + \eta^3 \end{pmatrix}$
$\bar{\mathbf{3}}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$	$\frac{\eta^6}{3} \begin{pmatrix} 1 + \eta^6 + \eta^4 & \omega + \omega^2\eta^6 + \eta^4 & \omega^2 + \omega\eta^6 + \eta^4 \\ \omega^2 + \omega\eta^6 + \eta^4 & 1 + \eta^6 + \eta^4 & \omega + \omega^2\eta^6 + \eta^4 \\ \omega + \omega^2\eta^6 + \eta^4 & \omega^2 + \omega\eta^6 + \eta^4 & 1 + \eta^6 + \eta^4 \end{pmatrix}$

Table 5: The T_7 generators of the five irreducible representations in the basis where d is diagonal. Here we have defined $\omega = e^{\frac{2\pi i}{3}}$ and $\eta = e^{\frac{2\pi i}{7}}$.

the presentation, see e.g. [26],

$$\langle c, d | c^7 = d^3 = 1, d^{-1}cd = c^4 \rangle . \quad (42)$$

It has 21 elements and five irreducible representations, namely $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{3}$, and $\bar{\mathbf{3}}$. A pair of triplet generators satisfying the presentation in Eq. (42) can be found e.g. in [14, 26, 27], where the order-seven generator c is diagonal. For the purpose of the model in the present paper, it is however more convenient to work in a basis with a diagonal order-three generator d . Our choice of T_7 generators for all five irreducible representations is listed in Tab. 5, where we have defined

$$\omega = e^{\frac{2\pi i}{3}}, \quad \eta = e^{\frac{2\pi i}{7}} . \quad (43)$$

Although the order-seven generators of the triplet representations look rather involved, the Clebsch-Gordan coefficients take a relatively simple form. Omitting the trivial products, i.e. those involving the singlet $\mathbf{1}$ as well as products of only one-dimensional irreducible representations, the product rules are reported below. We use the convention that the components of the first representation of any given product $\mathbf{a} \otimes \mathbf{b}$ are denoted by a_i while we use b_i for the components of the second representation. The subscripts s and a stand for ‘‘symmetric’’ and ‘‘anti-symmetric’’, respectively. Note that we have also included the normalization factors of the Clebsch-Gordan coefficients.

$$\mathbf{1}' \otimes \mathbf{3} : \quad \mathbf{3} = a_1 \begin{pmatrix} b_2 \\ b_3 \\ b_1 \end{pmatrix}, \quad \mathbf{1}'' \otimes \mathbf{3} : \quad \mathbf{3} = a_1 \begin{pmatrix} b_3 \\ b_1 \\ b_2 \end{pmatrix}, \quad (44)$$

$$\mathbf{1}' \otimes \bar{\mathbf{3}}: \quad \bar{\mathbf{3}} = a_1 \begin{pmatrix} b_3 \\ b_1 \\ b_2 \end{pmatrix}, \quad \mathbf{1}'' \otimes \bar{\mathbf{3}}: \quad \bar{\mathbf{3}} = a_1 \begin{pmatrix} b_2 \\ b_3 \\ b_1 \end{pmatrix}, \quad (45)$$

$$\mathbf{3} \otimes \bar{\mathbf{3}}: \quad \mathbf{1} = \frac{1}{\sqrt{3}} (a_1 b_1 + a_2 b_2 + a_3 b_3), \quad (46)$$

$$\mathbf{1}' = \frac{1}{\sqrt{3}} (a_1 b_2 + a_2 b_3 + a_3 b_1), \quad (47)$$

$$\mathbf{1}'' = \frac{1}{\sqrt{3}} (a_1 b_3 + a_2 b_1 + a_3 b_2), \quad (48)$$

$$\mathbf{3} = \frac{1}{\sqrt{3}} \begin{pmatrix} a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3 \\ a_1 b_3 + \omega^2 a_2 b_1 + \omega a_3 b_2 \\ a_1 b_2 + \omega^2 a_2 b_3 + \omega a_3 b_1 \end{pmatrix}, \quad (49)$$

$$\bar{\mathbf{3}} = \frac{1}{\sqrt{3}} \begin{pmatrix} a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 \\ a_3 b_1 + \omega a_1 b_2 + \omega^2 a_2 b_3 \\ a_2 b_1 + \omega a_3 b_2 + \omega^2 a_1 b_3 \end{pmatrix}, \quad (50)$$

$$\mathbf{3} \otimes \mathbf{3}: \quad \mathbf{3}_s = \frac{1}{\sqrt{3}} \begin{pmatrix} a_1 b_1 + a_2 b_3 + a_3 b_2 \\ \omega(a_1 b_2 + a_2 b_1 + a_3 b_3) \\ \omega^2(a_1 b_3 + a_2 b_2 + a_3 b_1) \end{pmatrix}, \quad (51)$$

$$\bar{\mathbf{3}}_s = \frac{1}{\sqrt{6}} \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_2 b_2 - a_3 b_1 - a_1 b_3 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \end{pmatrix}, \quad (52)$$

$$\bar{\mathbf{3}}_a = \frac{1}{\sqrt{2}} \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}. \quad (53)$$

References

- [1] P. F. Harrison, D. H. Perkins, and W. G. Scott, “Tri-bimaximal mixing and the neutrino oscillation data,” *Phys. Lett.* **B530** (2002) 167, [hep-ph/0202074](#).
- [2] P. F. Harrison and W. G. Scott, “Symmetries and generalisations of tri-bimaximal neutrino mixing,” *Phys. Lett.* **B535** (2002) 163–169, [hep-ph/0203209](#).
- [3] Z.-Z. Xing, “Nearly tri-bimaximal neutrino mixing and CP violation,” *Phys. Lett.* **B533** (2002) 85–93, [hep-ph/0204049](#).
- [4] G. Altarelli and F. Feruglio, “Discrete Flavor Symmetries and Models of Neutrino Mixing,” *Rev. Mod. Phys.* **82** (2010) 2701–2729, [1002.0211](#).

- [5] H. Ishimori *et al.*, “Non-Abelian Discrete Symmetries in Particle Physics,” *Prog. Theor. Phys. Suppl.* **183** (2010) 1–163, [1003.3552](#).
- [6] W. Grimus and P. O. Ludl, “Finite flavour groups of fermions,” *J.Phys.A* **A45** (2012) 233001, [1110.6376](#).
- [7] C. Hagedorn, S. F. King, and C. Luhn, “A SUSY GUT of flavour with $S_4 \times SU(5)$ to NLO,” *JHEP* **1006** (2010) 048, [1003.4249](#).
- [8] I. K. Cooper, S. F. King, and C. Luhn, “SUSY $SU(5)$ with singlet plus adjoint matter and A_4 family symmetry,” *Phys.Lett.* **B690** (2010) 396–402, [1004.3243](#).
- [9] I. K. Cooper, S. F. King, and C. Luhn, “ $A_4 \times SU(5)$ SUSY GUT of flavour with trimaximal neutrino mixing,” *JHEP* **1206** (2012) 130, [1203.1324](#).
- [10] C. Hagedorn, S. F. King, and C. Luhn, “SUSY $S_4 \times SU(5)$ revisited,” [1205.3114](#).
- [11] S. F. King, C. Luhn, and A. J. Stuart, “A grand $\Delta(96) \times SU(5)$ flavour model,” [1207.5741](#).
- [12] S. King, “Predicting neutrino parameters from $SO(3)$ family symmetry and quark-lepton unification,” *JHEP* **0508** (2005) 105, [hep-ph/0506297](#).
- [13] S. F. King, “Parametrizing the lepton mixing matrix in terms of deviations from tri-bimaximal mixing,” *Phys. Lett.* **B659** (2008) 244–251, [0710.0530](#).
- [14] S. F. King and C. Luhn, “On the origin of neutrino flavour symmetry,” *JHEP* **0910** (2009) 093, [0908.1897](#).
- [15] P. Minkowski, “ $\mu \rightarrow e\gamma$ at a Rate of One Out of 1-Billion Muon Decays?,” *Phys. Lett.* **B67** (1977) 421.
- [16] M. Gell-Mann, P. Ramond, and R. Slansky, “The family group in grand unified theories,” *Supergravity* (1979) North–Holland, Amsterdam, [hep-ph/9809459](#).
- [17] T. Yanagida, “Horizontal symmetry and masses of neutrinos,” *Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe* (1979) KEK, Japan.
- [18] R. N. Mohapatra and G. Senjanovic, “Neutrino mass and spontaneous parity nonconservation,” *Phys. Rev. Lett.* **44** (1980) 912.
- [19] S. King, “Atmospheric and solar neutrinos with a heavy singlet,” *Phys.Lett.* **B439** (1998) 350–356, [hep-ph/9806440](#).
- [20] S. King, “Atmospheric and solar neutrinos from single right-handed neutrino dominance and $U(1)$ family symmetry,” *Nucl.Phys.* **B562** (1999) 57–77, [hep-ph/9904210](#).

- [21] S. King, “Constructing the large mixing angle MNS matrix in seesaw models with right-handed neutrino dominance,” *JHEP* **0209** (2002) 011, [hep-ph/0204360](#).
- [22] G. Altarelli and F. Feruglio, “Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions,” *Nucl.Phys.* **B720** (2005) 64–88, [hep-ph/0504165](#).
- [23] G. Altarelli and F. Feruglio, “Tri-Bimaximal Neutrino Mixing, A_4 and the Modular Symmetry,” *Nucl. Phys.* **B741** (2006) 215–235, [hep-ph/0512103](#).
- [24] K. M. Parattu and A. Wingerter, “Tribimaximal Mixing From Small Groups,” *Phys.Rev.* **D84** (2011) 013011, [1012.2842](#).
- [25] C. Luhn, S. Nasri, and P. Ramond, “Tri-Bimaximal Neutrino Mixing and the Family Symmetry $Z_7 \rtimes Z_3$,” *Phys. Lett.* **B652** (2007) 27–33, [0706.2341](#).
- [26] C. Luhn, S. Nasri, and P. Ramond, “Simple finite non-Abelian flavor groups,” *J.Math.Phys.* **48** (2007) 123519, [0709.1447](#).
- [27] C. Hagedorn, M. A. Schmidt, and A. Y. Smirnov, “Lepton Mixing and Cancellation of the Dirac Mass Hierarchy in SO(10) GUTs with Flavor Symmetries T_7 and $\Sigma(81)$,” *Phys. Rev.* **D79** (2009) 036002, [0811.2955](#).
- [28] **DAYA-BAY Collaboration** Collaboration, F. An *et al.*, “Observation of electron-antineutrino disappearance at Daya Bay,” *Phys.Rev.Lett.* **108** (2012) 171803, [1203.1669](#).
- [29] **RENO collaboration** Collaboration, J. Ahn *et al.*, “Observation of reactor electron antineutrino disappearance in the RENO experiment,” *Phys.Rev.Lett.* **108** (2012) 191802, [1204.0626](#).
- [30] S. Weinberg, “Baryon and Lepton Nonconserving Processes,” *Phys. Rev. Lett.* **43** (1979) 1566–1570.
- [31] G. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, *et al.*, “Global analysis of neutrino masses, mixings and phases: entering the era of leptonic CP violation searches,” *Phys.Rev.* **D86** (2012) 013012, [1205.5254](#).
- [32] D. Forero, M. Tortola, and J. Valle, “Global status of neutrino oscillation parameters after recent reactor measurements,” [1205.4018](#).
- [33] M. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, “Global fit to three neutrino mixing: critical look at present precision,” [1209.3023](#).
- [34] C. I. Low and R. R. Volkas, “Tri-bimaximal mixing, discrete family symmetries, and a conjecture connecting the quark and lepton mixing matrices,” *Phys. Rev.* **D68** (2003) 033007, [hep-ph/0305243](#).
- [35] **Particle Data Group** Collaboration, J. Beringer *et al.*, “Review of particle physics,” *Phys. Rev.* **D86** (2012) 010001.

- [36] S. King, “Tri-bimaximal Neutrino Mixing and θ_{13} ,” *Phys.Lett.* **B675** (2009) 347–351, [0903.3199](#).
- [37] S. F. King and C. Luhn, “Trimaximal neutrino mixing from vacuum alignment in A_4 and S_4 models,” *JHEP* **1109** (2011) 042, [1107.5332](#).
- [38] N. Haba, A. Watanabe, and K. Yoshioka, “Twisted flavors and tri/bi-maximal neutrino mixing,” *Phys.Rev.Lett.* **97** (2006) 041601, [hep-ph/0603116](#).
- [39] X.-G. He and A. Zee, “Minimal modification to the tri-bimaximal neutrino mixing,” *Phys.Lett.* **B645** (2007) 427–431, [hep-ph/0607163](#).
- [40] C. Lam, “Mass independent textures and symmetry,” *Phys.Rev.* **D74** (2006) 113004, [hep-ph/0611017](#).
- [41] W. Grimus and L. Lavoura, “A Model for trimaximal lepton mixing,” *JHEP* **0809** (2008) 106, [0809.0226](#).
- [42] C. H. Albright and W. Rodejohann, “Comparing trimaximal mixing and its variants with deviations from tri-bimaximal mixing,” *Eur.Phys.J.* **C62** (2009) 599–608, [0812.0436](#).
- [43] C. H. Albright, A. Dueck, and W. Rodejohann, “Possible alternatives to tri-bimaximal mixing,” *Eur.Phys.J.* **C70** (2010) 1099–1110, [1004.2798](#).
- [44] H. Ishimori, Y. Shimizu, M. Tanimoto, and A. Watanabe, “Neutrino masses and mixing from S_4 flavor twisting,” *Phys.Rev.* **D83** (2011) 033004, [1010.3805](#).
- [45] Y. Shimizu, M. Tanimoto, and A. Watanabe, “Breaking tri-bimaximal mixing and large θ_{13} ,” *Prog.Theor.Phys.* **126** (2011) 81–90, [1105.2929](#).
- [46] X.-G. He and A. Zee, “Minimal modification to tri-bimaximal mixing,” *Phys.Rev.* **D84** (2011) 053004, [1106.4359](#).
- [47] S. Antusch, S. F. King, C. Luhn, and M. Spinrath, “Trimaximal mixing with predicted θ_{13} from a new type of constrained sequential dominance,” *Nucl.Phys.* **B856** (2012) 328–341, [1108.4278](#).
- [48] I. de Medeiros Varzielas and L. Merlo, “Ultraviolet completion of flavour models,” *JHEP* **1102** (2011) 062, [1011.6662](#).
- [49] M. Magg and C. Wetterich, “Neutrino mass problem and gauge hierarchy,” *Phys.Lett.* **B94** (1980) 61.
- [50] J. Schechter and J. Valle, “Neutrino masses in $SU(2) \times U(1)$ theories,” *Phys.Rev.* **D22** (1980) 2227.
- [51] C. Wetterich, “Neutrino masses and the scale of $B - L$ violation,” *Nucl.Phys.* **B187** (1981) 343.

- [52] G. Lazarides, Q. Shafi, and C. Wetterich, “Proton lifetime and fermion masses in an $SO(10)$ model,” *Nucl.Phys.* **B181** (1981) 287.
- [53] R. N. Mohapatra and G. Senjanovic, “Neutrino masses and mixings in gauge models with spontaneous parity violation,” *Phys.Rev.* **D23** (1981) 165.
- [54] S. Antusch, J. Kersten, M. Lindner, M. Ratz, and M. A. Schmidt, “Running neutrino mass parameters in see-saw scenarios,” *JHEP* **0503** (2005) 024, [hep-ph/0501272](#).
- [55] B. D. Callen and R. R. Volkas, “Large lepton mixing angles from a 4+1-dimensional $SU(5) \times A_4$ domain-wall braneworld model,” *Phys.Rev.* **D86** (2012) 056007, [1205.3617](#).