Evolution of primordial statistical isotropy violation

Indo-UK Meeting

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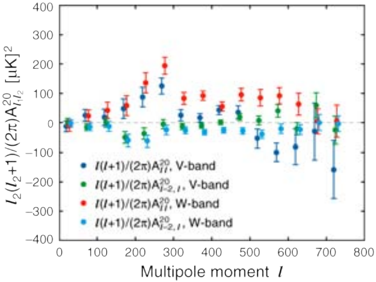
Motivation

Certain peculiarities of the recent CMB data, raises questions about modifying the standard model of cosmology

- Anomalies in the CMB maps (north-south asymmetry, cold spot) suggest that perturbations violate statistical isotropy.
- Statistically significant quadrupolar effect of SI violation observed at $\ell = 200$ (baryon acoustic peak of the CMB power spectrum)
- Motivates study of SI violations in the baryon photon fluid.



Bennett, C., et.al., 2011, ApJS, 192, 17

CMB anisotropy and statistical isotropy (SI)

CMB photon fluctuations and temperature anisotropy

$$\Delta(\vec{x}=0,\hat{n},\tau)=\Delta T(\hat{n})=\sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$$

Under SI condition, the 2-point correlation function on the sphere

$$\langle \Delta T(\hat{n})\Delta T(\hat{n'})\rangle = C(\hat{n},\hat{n'}) = C(\hat{n}.\hat{n'}) = \frac{1}{4\pi}\sum_{\ell}^{\infty}(2\ell+1)C_\ell P_\ell(\cos\theta)$$

C_ℓ - (diagonal covariance matrix in SH space)

$$\langle a_{\ell m} a^*_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}C_\ell$$

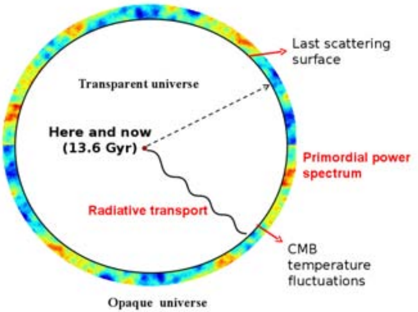
Fourier modes of the CMB photon fluctuations under SI

$$\begin{aligned}\Delta(\vec{x},\hat{n},\tau) &\rightarrow \tilde{\Delta}(\vec{k},\hat{n},\tau)\equiv\tilde{\Delta}(|k|,\hat{k}\cdot\hat{n},\tau)\rightarrow\tilde{\Delta}_\ell(k,\tau)\\ \tilde{\Delta}_\ell(k,\tau_0) &= \sum_{\ell',L}(...)j_L(k(\tau_0-\tau_s))\tilde{\Delta}_{\ell'}(k,\tau_s)\end{aligned}$$

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = 4\pi \int \frac{dk}{k} \frac{k^3 P(k)}{2\pi^2} |\tilde{\Delta}_\ell(k,\tau_0)|^2 \delta_{\ell\ell'} \delta_{mm'}$$

SI violation leads to $C(\hat{n}, \hat{n}') \neq C(\hat{n} \cdot \hat{n}')$ and off-diagonal terms in the above relation & can be seeded in

- SI violation in $P(\vec{k})$ - **Pullen & Kamionkowski (Phys.Rev.D76:103529,2007)**
- SI violation in $\tilde{\Delta}(\vec{k}, \hat{n}, \tau)$ - **Aich & Souradeep (Phys.Rev.D81:083008,2010)**
- SI violation due to compact spaces



Breakdown of Statistical isotropy and Bipolar formalism

A.Hajian & T. Souradeep, ApJ 597 L5 (2003)

Generalized Correlation function $C(\hat{n}, \hat{n}') \neq C(\hat{n} \cdot \hat{n}')$

$$C(\hat{n}, \hat{n}') = \sum_{\ell\ell' JN} A^{JN}_{\ell\ell'} \{Y_\ell(\hat{n}) \otimes Y_{\ell'}(\hat{n}')\}_{JN}$$

Bipolar Spherical Harmonics (BipoSH) basis

$$\{Y_\ell(\hat{n}) \otimes Y_{\ell'}(\hat{n}')\}_{JN} = \sum_{mm'} \mathcal{C}^{JN}_{\ell m \ell' m'} Y_{\ell m}(\hat{n}) \; Y_{\ell' m'}(\hat{n}')$$

BipoSH coefficients

$$A^{LM}_{\ell\ell'} = \int d\Omega_{\hat{n}} \int d\Omega_{\hat{n}'} C(\hat{n}, \hat{n}') \{Y_\ell(\hat{n}) \otimes Y_{\ell'}(\hat{n}')\}^*_{LM}$$

Off-diagonal elements of the covariance matrix

$$A^{LM}_{\ell\ell'} = \sum_{mm'} \langle a_{\ell m} a^*_{\ell' m'} \rangle (-1)^{m'} \mathcal{C}^{LM}_{\ell m \ell' -m'}$$

Deviation from SI

M. Aich & T. Souradeep, Phys.Rev.D81:083008,2010.

- Anisotropic CMB photon fluctuations $\tilde{\Delta}(\vec{k}, \hat{n}, \tau) \neq \tilde{\Delta}(k, \hat{k} \cdot \hat{n}, \tau)$
- Brightness fluctuations expanded in (BipoSH) series.

$$\tilde{\Delta}(\vec{k}, \hat{n}, \tau) = 4\pi \sum_{\ell_1 \ell_2 LM} \beta_{\ell_1 \ell_2} \tilde{\Delta}^{LM}_{\ell_1 \ell_2}(k, \tau) \boxed{\{Y_{\ell_1}(\hat{k}) \otimes Y_{\ell_2}(\hat{n})\}_{LM}}$$

BipoSH coefficients for an isotropic power spectrum $\langle \phi(\vec{k}) \phi^*(\vec{k'}) \rangle = P_0(k) \delta(\vec{k} - \vec{k'})$

$$A^{JN}_{\ell \ell'} = \int \frac{k^2 dk}{2\pi^2} \sum_{\ell_1 L L' M M'} (\ldots) \mathcal{C}^{JN}_{LML'-M'} \tilde{\Delta}^{LM}_{\ell_1 \ell}(k, \tau) \tilde{\Delta}^{L'M'}_{\ell_1 \ell'}(k, \tau)^*$$

Generalized evolution equation (in the asymptotic limit)

$$\tilde{\Delta}^{LM}_{\ell_1, \ell_2}(k, \tau) = \sum_{\ell} (\ldots) j_\ell(k \Delta \tau) \tilde{\Delta}^{LM}_{\ell_1 - \ell, \ell_2 - \ell}(k, \tau_s) \quad \ell, \ell_n \gg |\ell_n - \ell|, L$$

SI violating physical effects at last scattering

M. Aich & T. Souradeep, Phys.Rev.D81:083008,2010.

Fluctuations sourced by the bipolar dipole ($L = 1, M = \{-1, 0, 1\}$) match CMB anisotropy in presence of a homogeneous magnetic field at last scattering

$$\tilde{\Delta}(\vec{k}, \hat{n}, \tau_s) = 3i\sqrt{\frac{3}{2}} \sum_M \tilde{\Delta}^{1M}_{1,1}(k, \tau_s)(\hat{k} \times \hat{n})_M$$

Source term comprising magnetic field, velocity of plasma etc.

$$\langle \tilde{\Delta}(\vec{k}, \hat{n}, \tau_s)\tilde{\Delta}(\vec{k}, \hat{n}', \tau_s)\rangle \propto \tilde{\Delta}^{1M}_{1,1}(k, \tau_s)\tilde{\Delta}^{1M'}_{1,1}(k, \tau_s)[(\hat{n}.\hat{n}')(\hat{k}.\hat{k}) - (\hat{n}.\hat{k})(\hat{n}'.\hat{k})]$$

The angular correlations are

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = (\ldots)\delta_{\ell\ell'}\delta_{mm'} + (\ldots)\delta_{\ell\pm 2\ell'}\delta_{mm'}$$

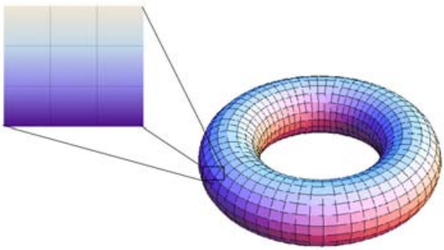
Results match - **Durrer et al., PRD 1998, Kahniashvili et al., PRD 2008.**

Compact Spaces

- Inflation → relatively smooth observable Hubble volume is a small patch of an extremely inhomogeneous and complicated spatial manifold.

Gott, MNRAS 193, 153, 1980; Cornish, Spergel & Starkman, PRL 77, 215, 1996.

- Non-trivial cosmic topology on surface with uniform curvature (Euclidean and hyperbolic)
- Planck has a working group on non-trivial cosmic topology.



SI violation on a flat torus T^3: maps points under $\hat{n} \to \hat{n} + \vec{a}L$

The correlation function

$$C(\hat{n}, \hat{n}') = \frac{1}{L^3} \sum_a P(k_a) \exp\left[-i \frac{2\pi \vec{a}.(\hat{n}-\hat{n}')\Delta\tau}{L}\right]$$

$\vec{a} \equiv (a_x, a_y, a_z)$, $k_a^2 = (2\pi/L)^2(\vec{a}.\vec{a})$, a vector with integer components & L is size of torus. **J.Richard Bond, Dmitry Pogosyan & Tarun Souradeep, Phys.Rev. D62 (2000) 043005**

Using the plane wave expansion

$$e^{i\frac{2\pi}{L}\vec{a}.\hat{n}\Delta\tau} = \sum_\ell (-i)^\ell (2\ell+1) j_\ell(2\pi a \Delta\tau/L) P_\ell(\hat{a}.\hat{n})$$

the Bipolar coefficients for a flat torus are

$$\begin{aligned} A^{JN}_{\ell\ell'} &= \frac{(4\pi)^2}{L^3} (i)^{\ell+\ell'} (-1)^{\ell'} \sum_a P\left(\frac{2\pi a}{L}\right) j_\ell\left(\frac{2\pi a \Delta\tau}{L}\right) j_{\ell'}\left(\frac{2\pi a \Delta\tau}{L}\right) \\ &\times \{Y_\ell(\hat{a}) \otimes Y_{\ell'}(\hat{a})\}_{JN} \end{aligned}$$

CMB photon fluctuations on compact spaces

$$\Delta(\vec{x},\hat{n},\tau)=\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot\vec{x}}\tilde{\Delta}(\vec{k},\hat{n},\tau)\phi(\vec{k})$$

On compact spaces, the k space gets discretized and Fourier sum is required $\int d^3k \rightarrow \left(\frac{2\pi}{L}\right)^3\sum_i$

$$\begin{aligned}\Delta(\vec{x},\hat{n},\tau) &= \frac{1}{L^3}\sum_i e^{i\vec{k_i}.\vec{x}}\tilde{\Delta}(\vec{k_i},\hat{n},\tau)\phi(\vec{k_i})\\ C(\hat{n},\hat{n'}) &= \frac{1}{L^3}\sum_i P(k_i)\tilde{\Delta}(\vec{k_i},\hat{n},\tau)\tilde{\Delta}(\vec{k_i},\hat{n}',\tau)\end{aligned}$$

$\langle\phi(k_i)\phi^*(k_j)\rangle = P(k_i)\delta_{ij}$ is the discrete primordial power spectrum.
The Bipolar coefficients in terms of the moments of the CMB photon fluctuations in Fourier space are

$$A^{JN}_{\ell\ell'}\sim\sum_a P\left(\frac{2\pi a}{L}\right)\tilde{\Delta}_\ell\left(\frac{2\pi a}{L},\tau\right)\tilde{\Delta}^*_{\ell'}\left(\frac{2\pi a}{L},\tau\right)\{Y_\ell(\hat{a})\otimes Y_{\ell'}(\hat{a})\}_{JN}$$

Summary & Future Work

- SI violation in CMB brightness fluctuations
  - Motivated by anomalies in CMB maps - SI deviation in the CMB brightness fluctuations, in addition to anisotropic $P(k)$
  - Due to SI violation, lower multipoles at the last scattering free stream to higher multipoles at the present epoch which are not dominated by the cosmic variance regime.
  - We have used our formalism to represent and match the well known case for SI violation in presence of a homogeneous magnetic field.
- SI violation in compact spaces
  - due to generation of fluctuations in a compact space.
  - We are interested in generalizing the approach for CMB polarization as well as on other non-trivial compact spaces.

Thanks for your attention!