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Book Review

The Mathematical Theory of Black Holes. By S. Chandrasekhar. Clarendon Press, Oxford; Oxford University Press, New York, 1983, xxi + 646 pp, $110.00 (cloth).

No reviewer can do justice to this book without matching the enormous effort that has gone toward its preparation. For example, in the bibliographical notes at the end of Chapter 9, the reader who wishes to undertake a careful scrutiny of the developments described in that chapter is invited to study the author's derivations "in some 600 legal-size pages and in six additional notebooks" deposited in the Joseph Regenstein Library of the University of Chicago. The remarks below fall short of such an effort by a considerable margin.

Black holes began to gain popularity in the 1960s. Today they mean different things to different people. The astrophysicists look upon them as nature's ideal energy machines that may one day solve the riddles of a variety of cosmic objects. To theoretical physicists, black holes are elegant examples of thermodynamic systems that, through the Hawking effect, offer a link between general relativity on the one hand and particle physics and quantum theory on the other.

None of these topics are covered in this book. As the author states in the preface, "This book is an expression of the author's perspective with the limitations which that implies." More significantly, the preface carries a quote from William Shakespeare which includes the line "... what I have to say is of mine own making ...." Of this latter point the reader will find ample evidence throughout this book.

What is the author's own perspective on black holes? The key word in the title of the book is "mathematical": the mathematical elegance, the mathematical exactness, and the mathematical thoroughness that guide the author in his description of black holes. The exact solutions of Einstein's equations describing black hole space-time, null and timelike geodesics in these space-times and the propagation (reflection and transmission) of waves in them, perturbations and stability of black holes, and limits on energy extraction from them are the main topics covered here.

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To give a brief outline of the contents: The opening chapter gives a terse introduction to the relativity theory and its techniques that are needed later in the book. Topics like differential forms, covariant and Lie derivatives, and torsion and curvature precede Einstein's equation, while tetrads, the Newman–Penrose formalism, Petrov classification, optical scalars, and the Goldberg–Sachs theorem follow it. This stiff dose is certainly not for someone trying to learn relativity *ab initio*. Chapter 2 describes stationary axisymmetric space-times and introduces a metric of sufficient generality. The tetrad formalism is introduced and Maxwell's equations are written out in full. The next three chapters introduce the reader to the Schwarzschild and the Reissner–Nordström solutions. Familiar though these topics are to relativists, the contents of these chapters are hardly to be found in standard textbooks. In many places the treatment of results is the author's own even when describing original results of others; for example, we get here a comprehensive picture of the perturbations of the Schwarzschild black hole which includes results derived by several workers.

The author has reserved his *tour-de-force*, however, for the Kerr solution, which describes the massive spinning black hole. It is to the properties of this black hole that the rest of the book is devoted. This reviewer recalls attending the first Texas Symposium in Dallas in 1963 when the Kerr solution was reported to the relativists on the last day when the bulk of the excitement of the conference was over. It would have been hard to predict then that that short communication would one day blossom into the material of Chapters 6–11.

The mathematical elegance of the Kerr black hole lies, according to the author, in the separability of the Hamilton–Jacobi equation and of the wave equations when written in angular coordinates. It is this property that allows one to study in explicit terms the various properties of Kerr space-time and that leads the author to the conclusion that black hole is really a simple object.

While the large number of equations (there are 3620 in the book) make the text formidable to read, the authors clear style of presentation alleviates the difficulty. Topics like the dragging of inertial frames, the particle orbits and null geodesics, axial and polar perturbations, the reflection and transmission of waves, and the Penrose process are explained beautifully. The comments on the various references in the bibliographies at the ends of the chapters are very readable and should not be missed.

Dare one point to a few blemishes on such a monumental work? This reviewer would have liked to see a discussion of the Killing equations and Penrose diagrams in the opening chapter. Although the area theorem is referred to, a comprehensive discussion of the laws of black hole physics is lacking. Even if the bibliography is not claimed to be complete, it is sur-

prising that there is no reference to Szekeres in the discussion of the Kruskal (–Szekeres) diagram.

The account here is evidently limited to classical gravity. But some classical results can be understood better with a perspective from quantum physics. The concepts of superradiance and the Klein paradox (introduced only, by the way, on p. 561) would have gained greater significance for the reader if their relationship to quantum effects in curved space-time were described.

The exclusion of spacelike geodesics from the discussion is surprising in a work that is so thorough in describing the mathematical properties of space-time. This exclusion means that the peculiar behavior of tachyons (if they exist) in black hole space-times, which was discussed by several authors in the 1970s, remains untouched. Even if one does not believe in these esoteric particles, one cannot deny that the discussion of spacelike geodesics is still needed in other contexts such as for quantum propagators in curved space-time.

A mathematical point about the Schwarzschild solution that has bothered this reviewer remains unanswered here. In the Newtonian theory the potential of a point mass M is obtained by solving the equation $\nabla^2\phi = -4\pi GM\delta(\mathbf{r})$ to give $\phi = GM/r$. In the Schwarzschild solution the metric is obtained by solving the field equations in empty space-time ($T_{ik} = 0$). This gives an arbitrary constant which is identified with M by comparison with the Newtonian solution asymptotically. Is it or is it not possible to obtain the Schwarzschild black hole solution by directly relating it to the source singularity at $r = 0$?

Professor Chandrasekhar's books have all been looked upon as the last words in their respective fields, encompassing as they do complete mathematical treatments of the topics in question in a systematic manner. This book is no exception, and, in spite of the limitations implied by the author's perspective, it will prove to be a lasting landmark in the literature on general relativity. The reader needs to be warned, however, that this book is the "last word" and not an "introduction." Much though the author is impressed by the mathematical elegance of rotating black holes, he is not a missionary talking to cannibals. Rather he is like a guru from the Orient who is willing to impart treasures of knowledge to a student if he works hard for it.

To such hard working seekers of knowledge this book is strongly recommended.

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