

Cosmic inventory of energy densities: issues and concerns

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Abstract The dynamics of our universe is characterised by the density parameters for cosmological constant (Ω_V), nonbaryonic darkmatter (Ω_{wimp}), radiation (Ω_R) and baryons (Ω_B). To these parameters — which describe the smooth background universe — one needs to add at least another dimensionless number ($\sim 10^{-5}$) characterising the strength of primordial fluctuations in the gravitational potential, in order to ensure formation of structures by gravitational instability. I discuss several issues related to the description of the universe in terms of these numbers and argue that we do not yet have a fundamental understanding of these issues.

0.1 An inventory of energy densities

Based on dynamical considerations, we can divide the energy densities contributing to the expansion of the universe into those due to (i) radiation and relativistic particles (with an equation of state $p = w\rho$ and $w = 1/3$), (ii) baryons ($w = 0$), (iii) non relativistic, non baryonic dark matter ($w = 0$) and (iv) cosmological constant ($w \approx -1$). Current observations suggest that the top two positions are held by cosmological constant ($\Omega_V \approx 0.6$) and non baryonic dark matter ($\Omega_{\text{wimp}} \approx 0.35$) for which we have no laboratory evidence. [The third position will go to massive neutrinos if the recent observational results are confirmed.] The baryons in the universe contribute $\Omega_B \approx 0.02h^{-2}$ which is about an order of magnitude larger than the baryons seen in the form of luminous matter, indicating that at least part of the baryons are dark. The energy density in electromagnetic radiation is dominated by CMBR and contributes about $\Omega_R \approx 2.5 \times 10^{-5}h^{-2}$.

These energy densities drive the expansion of a homogeneous and isotropic universe which is completely structureless. The existence of small scale struc-

tures in the universe is believed to be due to gravitational instability which has amplified small initial fluctuations. Such a paradigm requires the existence of nonzero fluctuations in the gravitational potential energy due to primordial density fluctuations. Observations suggest that this can be characterised by a dimensionless number ($\sim 10^{-5}$) in our universe.

I shall now try to describe some of the issues related to these numbers which requires investigation, focussing on the questions: Do we understand any of these components at a fundamental level or can we relate them to one another in a meaningful way? Unfortunately, the answer today is ‘no’.

0.2 Cosmological constant: The theoretist’s nightmare

Current observations suggest that nearly 95 per cent of the matter in the universe is non baryonic. Cosmologists, over years, have learned to live with the existence of dark matter which is structurally very different from the normal baryonic matter one is familiar with in the laboratory. But recent observations of supernova and CMBR seem to indicate that there are at least *two* different components of dark matter, one made of weakly interacting massive particles contributing about $\Omega_{\text{wimp}} \approx 0.35$ and another made of some form of energy with negative pressure contributing about $\Omega_V \approx 0.6$. The simplest choice for the latter is a cosmological constant with an equation of state $p = w\rho$ where $w = -1$. Observations are, however, consistent with a somewhat broader range of negative values for w and it is possible to come up with more exotic equations of state (with negative pressure) for this component. If these observations do not go away (and it looks unlikely that they will!), we have a serious theoretical problem in our hands.

Conventionally, there are two ways of interpreting the cosmological constant. In the first approach, one introduces Λ as a parameter in Einstein’s theory of gravity just as the newtonian constant G is introduced as a parameter in this theory. In such a case, the action for matter coupled to gravity will have the form

$$A = \frac{c^3}{16\pi G} \int (R - 2\Lambda) \sqrt{-g} d^4x + \int \mathcal{L}_{\text{matter}} \sqrt{-g} d^4x \quad (1)$$

where $\mathcal{L}_{\text{matter}}$ is the matter lagrangian. In this approach, the numerical value of Λ needs to be determined from observations, just like the numerical value of G . In classical gravity, it is not possible to construct a dimensionless number using the fundamental constants Λ, G and c which appear in the action; hence the relative value of Λ and G does not make any sense and depends on the system of units. The situation, however, is different once we add the planck constant \hbar into the fray. (This does not require quantum gravity; even in the description of quantum fields in a classical curved background, one naturally ends up getting \hbar in the analysis as in the case of, for example, particle creation by an expanding universe.) It is then possible to form the dimensionless number $\Lambda(8\pi G\hbar/c^3) \equiv \Lambda L_P^2 \approx 10^{-120}$ where $L_P \approx 10^{-32}$ cm is the planck length. The fact that this dimensionless number — made out of fundamental constants

appearing in our theory — is so tiny, has been a source of mystery. If no other inputs are available, one would have expected such a small number to be actually equal to zero and one would look for a deep symmetry in the theory which requires vanishing of Λ ; that is, the $\Lambda\sqrt{-g}$ term cannot exist in (1) because of certain symmetry just as electromagnetic action cannot have a $m^2 A_i A^i$ term (making photon massive) because of gauge invariance. No such exact symmetry is known.

There is, however, an alternative — and more compelling — way of interpreting the cosmological constant which make matters worse. In equation (1) we can always add to the matter lagrangian any arbitrary constant. A change $\mathcal{L}_{\text{matter}} \rightarrow \mathcal{L}_{\text{matter}} - V_0$ is identical to the replacement $\Lambda \rightarrow \Lambda + 8\pi G V_0 / c^3$. Since observations can only determine the net Λ , it follows that we cannot just treat Λ as a parameter in the theory; it takes contributions from the matter lagrangian and only the total value has observational consequence. After adding all the matter sector contributions, the net result should finally satisfy the constraint $\Lambda_{\text{net}} L_P^2 \lesssim 10^{-120}$. As the universe evolves and cools some of the symmetries present at high energies will be spontaneously broken. This could lead to a $V_0 \approx E^4$ where E is the energy scale at which the symmetry is broken. For GUTs, $E \approx 10^{14}$ GeV and for electro-weak phase transition, $E \approx 100$ GeV. These contributions needs to be precisely cancelled by an original Λ term so that Λ_{net} is very tiny or zero. The only symmetry principle which we know that demands Λ_{net} to identically vanish is supersymmetry. But since supersymmetry is badly broken in nature, we would expect a contribution to Λ_{net} at the scale $V_0 \approx E_{\text{susy}}^4$ where E_{susy} is the energy scale at which supersymmetry is broken. This scale is still too high for comfort.

All these difficulties related to the cosmological constant will plague us even if observations suggest that $\Lambda_{\text{net}} = 0$. The cosmological observations suggesting that $\Lambda \approx H_0^{-2}$ adds a new layer of complications. Since the rate of expansion of the universe depends on the epoch while Λ_{net} is treated as a numerical constant in the action at low energies (below the electro weak scale, say), it is not clear why the relation $\Lambda \approx H_0^2$ should hold *around the present epoch*. We now need to explain: (i) why ΛL_P^2 is small but non zero and (ii) why is ρ_Λ comparable to other dominant energy densities in the universe around the present epoch.

Ever since recent cosmological observations suggested the existence of a nonzero cosmological constant, there has been a flurry of theoretical activity to “explain” it, none of which even gets to the first base. One class of models invokes some version of anthropic principle; but since anthropic principle never predicted anything, I do not consider it part of scientific methodology. The second class of models use a scalar field with an “appropriate” potential $V(\phi)$ to “explain” the observations. In this case, we will take the Λ term in (1) to be zero [for unknown reasons] and will have a dynamically evolving cosmological “constant” due to the potential $V(\phi)$ in $\mathcal{L}_{\text{matter}}$. These models are all, however, trivial and have no predictive power because it is always possible to choose a $V(\phi)$ to account for any sensible dynamical evolution of the universe. Since the triviality of these models (which are variously called “quintessence”, “dark energy”) does not seem to have been adequately emphasised in literature, let me briefly comment on this issue.

Consider any model for the universe with a *given* $a(t)$ and some known forms of energy density $\rho_{\text{known}}(t)$ (made of radiation, matter etc) both of which are observationally determined. It can happen that this pair does not satisfy the Friedmann equation for an $\Omega = 1$ model. To be specific, let us assume $\rho_{\text{known}} < \rho_c$ which is substantially the situation in cosmology today. If we now want to make a consistent model of cosmology with $\Omega = 1$, say, we can invoke a scalar field with the potential $V(\phi)$. It is trivial to choose $V(\phi)$ such that we can account for *any* sensible pair $[a(t), \rho_{\text{known}}(t)]$ along the following lines: Using the given $a(t)$, we define two quantities $H(t) = (\dot{a}/a)$ and $Q(t) \equiv 8\pi G\rho_{\text{known}}(t)/3H^2(t)$. The required $V(\phi)$ is given parametrically by the equations:

$$V(t) = (1/16\pi G)H(1 - Q) \left[6H + (2\dot{H}/H) - (\dot{Q}/1 - Q) \right] \quad (2)$$

$$\phi(t) = \int dt [H(1 - Q)/8\pi G]^{1/2} \left[\dot{Q}/(1 - Q) - (2\dot{H}/H) \right]^{1/2} \quad (3)$$

All the potentials invoked in the literature are special cases of this formula. This result shows that *irrespective of what the future observations reveal about $a(t)$ and $\rho_{\text{known}}(t)$* one can always find a scalar field which will “explain” the observations. Hence this approach has no predictive power. What is worse, most of the $V(\phi)$ suggested in the literature have no sound particle physics basis and — in fact — the quantum field theory for these potentials are very badly behaved on nonexistent.

It is worth realising that the existence of a non zero cosmological constant will be a statement of fundamental significance and constitutes a conceptual contribution of cosmology to quantum gravity. The tendency of some cosmologists to treat Ω_Λ as one among a set of, say, 17 parameters [like $\Omega_{\text{rad}}, \Omega_B, n, \dots$] which needs to be fixed by observations, completely misses the point. Cosmological constant is special and its importance transcends cosmology. Unfortunately, we do not have at present a fundamental understanding of cosmological constant from any of the approaches to quantum gravity. There are no nontrivial string theoretical models [1] incorporating $\rho_V > 0$; loop gravity can incorporate it but does not throw any light on its value. It should be stressed that the nonzero value for $\rho_V \neq 0$ does *not* imply deSitter (or even asymptotically deSitter) spacetime. Hence the formalism should be capable of handling ρ_V without deSitter geometry.

To give an example of a more fundamental way of thinking about cosmological constant, let me describe an idea in which cosmological constant is connected with the microstructure of spacetime. In this model we start with $\Lambda = 0$ but generate a small value for this parameter from two key ingredients: (i) discrete spacetime structure at Planck length and (ii) quantum gravitational uncertainty principle. To do this, we first note that cosmological constant can be thought of as a lagrange multiplier for proper volume of spacetime in the action functional for gravity by rewriting the first term of (1) as:

$$\hbar^{-1}A_{\text{grav}} = \frac{1}{2L_P^2} \int d^4x R \sqrt{-g} - \frac{\Lambda}{L_P^2} \int d^4x \sqrt{-g}; \quad (4)$$

In any quantum cosmological models which leads to large volumes for the universe, phase of the wave function will pick up a factor of the form $\Psi \propto \exp(-i(\Lambda/L_P^2)\mathcal{V})$, where \mathcal{V} is the four volume [2], from the second term in (4). Treating $(\Lambda/L_P^2, \mathcal{V})$ as conjugate variables (q, p) , we can invoke the standard uncertainty principle to predict $\Delta\Lambda \approx L_P^2/\Delta\mathcal{V}$. Now we make the crucial assumption regarding the microscopic structure of the spacetime: Assume that there is a zero point length [3] of the order of L_P so that the volume of the universe is made of several cells, each of volume L_P^4 . Then $\mathcal{V} = NL_P^4$, implying a Poisson fluctuation $\Delta\mathcal{V} \approx \sqrt{\mathcal{V}}L_P^2$. and leading to

$$\Delta\Lambda = \frac{L_P^2}{\Delta\mathcal{V}} = \frac{1}{\sqrt{\mathcal{V}}} \approx H_0^2 \quad (5)$$

which is exactly what cosmological observations imply! Planck length cutoff (UV limit) and volume of the universe (IR limit) combine to give the correct $\Delta\Lambda$.

After I gave this talk, I came to know that similar result was obtained earlier by Sorkin [4] based on a different model. The numerical result can of course arise in different contexts and it is probably worth discussing some of the conceptual components in my argument. The first key idea is that, in this approach, Λ is a stochastic variable with a zero mean and fluctuations. It is the rms fluctuation which is being observed in the cosmological context. This has two related implications: first, FRW equations now need to be solved with a stochastic term on the right hand side and one should check whether the observations can still be explained. Second, we may run into trouble if $\Lambda \approx H^{-2}$ at all epochs including the era of nucleosynthesis since it *might* violate the bounds. Another key feature is that stochastic properties of Λ need to be described by a quantum cosmological model. If the quantum state of the universe is expanded in terms of the eigenstates of some suitable operator (which does not commute the total four volume operator), then one should be able to characterise the fluctuations in each of these states. Generically, this will describe an ensemble of universes, with the fluctuations in cosmological constant inversely related to the size. Decoherence like arguments could then select out near classical, large volume universes. [5].

While I am not optimistic about the *details* of the above model, I find it attractive to think of the observed cosmological constant as arising from quantum *fluctuations* of some energy density rather than from bulk energy density. This is relevant in the context of standard discussions of the contribution of zero-point energies to cosmological constant. I would expect the correct theory to regularise the divergences and make the zero point energy finite and about L_P^{-4} (see eg., [6], [7]). This contribution is most likely to modify the microscopic structure of spacetime (e.g if the spacetime is naively thought of as due to stacking of Planck scale volumes, this will modify the stacking or shapes of the volume elements) and will not affect the bulk gravitational field when measured at scales coarse grained over sizes much bigger than the Planck scales. In other words, large amounts of energy is soaked up by the quantum microstructure of the spacetime itself like a sponge soaking up water. This process, however, will leave a small residual fluctuation which will depend on the volume of the space-

time region which is probed. I would conjecture that the cosmological constant we measure corresponds to this residue. It is small, in the sense that it has been reduced from L_P^{-4} to $L_P^{-4}(L_P H_0)^2$, which indicates the fact that fluctuations — when measured over a large volume — is small compared to the bulk value. It is the wetness of the sponge we notice, not the water content inside.

There are other ways of “explaining” $\Lambda \approx H^{-2}$ and I will mention just two possibilities. If one assumes that every patch of the universe with size L_P contained an energy E_P , then a universe with characteristic size H_0^{-1} will contain the energy $E = (E_P/L_P)H_0^{-1}$. The corresponding energy *density* will be $\rho_V = (E/H_0^{-3}) = (H_0/L_P)^2$ which is what one wants. The trouble, of course, is that we do not know why every length scale L_P should contain an energy E_P . Another possibility is to argue that the entropy of a patch of the universe containing vacuum energy density ρ_V is effectively the ratio between the Planck energy density $\rho_P = (E_P/L_P^3)$ and ρ_V ; that is, $S \approx (E_P/L_P^3 \rho_V)$. The idea being the most ordered state will have $\rho_V = \rho_P$ and lesser vacuum energy densities will allow for larger number of configurations. If we now equate this entropy to that of the deSitter horizon $S = (H_0^{-2}/L_P^2)$, we immediately get $\rho_V = (H_0/L_P)^2$. Though very suggestive, it is based on two unjustified, non-rigorous, conjectures. I mention these possibilities only because I believe the result $\rho_V = (H_0/L_P)^2$ will eventually arise from some deep geometrical feature rather than, say, from the slow roll over of a scalar field, introduced for this purpose.

0.3 Energy in primordial gravitational potential fluctuations

A strictly homogeneous and isotropic universe cannot develop significant amount of structure over the timescale $(G\rho)^{-1/2}$. The very fact that our universe has structures shows that there must have been small deviations from homogeneity in the energy density at earlier phases. The gravitational potential $\phi(t, \mathbf{x})$ due to fluctuations in the energy density $\delta\rho(t, \mathbf{x})$ satisfies an equation of the form $\nabla^2\phi \propto (\delta/a)$ in the relevant scales. Taking $\delta\rho$ to be a gaussian at sufficiently early epochs, it follows that the fluctuations in the gravitational potential can be characterised by the power per logarithmic band in the suitably defined \mathbf{k} space:

$$\Delta_\phi^2(t, k) = \frac{k^3 P_\phi(t, k)}{2\pi^2} \propto \frac{P_\delta(t, k)}{k} \quad (6)$$

where $P_\phi = |\phi_k|^2$ and $P_\delta = |\delta_k|^2$ are the power spectra of gravitational potential and density contrast. If this power is strongly scale dependent then we will either have copious production of small scale objects and blackholes (if the power at small scales dominate) or we will have stronger and stronger deviations from FRW universe at larger and larger scales (if the power at large scales dominate). To allow for the kind of universe we see, it is important that $\Delta_\phi^2(t, k)$ is (at least, approximately) scale invariant, requiring $P_\delta(t, k) = Ak$ with some constant A [8]. It follows that $\Delta_\phi^2(t, k) = \Delta_\phi^2(t)$; further at large scales, the evolution of the universe can be described by linear theory, and in a $\Omega = 1$ model,

$\Delta_\phi^2(t)$ does not evolve with time. We thus reach the remarkable conclusion that our universe is characterised by a dimensionless number $\Delta_\phi^2 \propto AH_0^4$. This number represents the strength of primordial gravitational fluctuations, or — equivalently — the strength of gravitational power at large scales. Observations of CMBR [9] suggest that this number is about 10^{-5} .

Considering the fact that all of structure formation is essentially a transfer of this power from large scales to small scales, it is surprising that this number has received so little attention in the literature. It is easy to argue that this number could not have been more than a factor 100 off in either direction and still lead to the kind of universe we live in. But we have no real clue as to why it is about 10^{-5} ! If the fluctuations were generated by inflation, then this number is related to the parameters in the lagrangian for the inflaton field. For natural values of these parameters, one gets totally wrong answers. (This is in spite of grandiose claims as to how CMBR verifies inflation; if we take an initial spectrum $P = Ak^n$ with two parameters A and n , any scale invariant mechanism will get $n = 1$ and the *real* test of the theory is to predict A - a test which inflation flunks). The model based on cosmic strings, in contrast, does get this number right as the dimensionless ratio $G\mu = (E_{\text{string}}/E_P)^2$; but, of course, this model has other problems.

0.4 Energy density of radiation

One of the most precisely measured energy densities is that due to cosmic microwave background radiation which dominates the radiation background in all wavebands. Today this energy density contributes $\Omega_R \approx 2.4 \times 10^{-5} h^{-2} (T/2.73 \text{ K})^4$. The numerical value is probably not of much significance since it changes with the epoch as $(1+z)^4$. It is, however, possible to construct a conserved dimensionless number $N = a_0^3 n_R$ which represents the total number of photons (or the entropy of the radiation field) in the universe. Taking $a_0 = H_0^{-1} |\Omega_0 - 1|^{-1/2}$, it follows that $N \gtrsim 3 \times 10^{86} h^{-3}$. [Note that, if Ω_0 is driven close to unity by inflation, then $\Omega_0 - 1$ could be contributed by the gravitational wave background generated in the same process.] This number, again, has no simple interpretation. In particular, if we assume that the universe had Planck temperature $T_P \approx 10^{19}$ GeV when its volume was about L_P^3 , then the current CMBR temperature should be about 10^{-29} K! Obviously, some physical process should have increased the entropy of the universe by a large factor (about 10^{90}) in order to keep the universe warm enough today. Until this process is identified and pinned down, we must accept that we do not understand the amount of energy density present in radiation today.

The above analysis is clearly related to the numerical value of the quantity $a(t)T_R(t)$ in our universe. If we assume that different particle species including the wimps were in equilibrium with radiation at sufficiently early epoch, then we can relate the number density of any non baryonic particle species to the number density of photons. Given the masses and interaction strength of the wimp, it is then straight forward to calculate Ω_{wimp} in terms of Ω_R . (In the case of massless neutrinos, for example, this is a fairly standard text book exercise but

the principle is the same for any other particle species.) Thus the computation of Ω_{wimp} is related to our understanding of the particle physics model *plus* our understanding of Ω_R .

0.5 Baryonic energy density

The most natural scenario for the universe would start with equal number of baryons and anti baryons leading to $\Omega_B = 0$ today. The fact that we have a tiny number density of net baryons with a photon to baryon ratio being $n_B/n_R \approx 2.7 \times 10^{-8} \Omega_B h^2 \simeq 5.4 \times 10^{-10}$ is yet another mystery in the current universe. (This is discussed in greater detail by Subir in his talk.)

Since bulk of the luminous matter is made of baryons, it is of course possible to estimate the value of Ω_B at different epochs. In particular, the estimate $\Omega_B h^2 \approx 0.02$ from the big bang nucleosynthesis seems to be consistent with the recent BOOMERANG data which — after some initial hiccups (see eg., [10]) — now gives $\Omega_B h^2 = 0.02 \pm 0.005$. The MAXIMA data, however, still gives $\Omega_B h^2 = 0.05 \pm 0.014$ which is on the higher side. It is, however, not clear whether these results are consistent with the measurement of baryonic density in the IGM. A careful study of Lyman- α absorbers in the IGM [11] suggest that $\Omega_B h^2 \gtrsim 0.022(\Gamma_{12}/1.2)^{1/2}$ where Γ is the photoionization rate due to the meta galactic ionizing flux in units of $10^{-12} s^{-1}$. Since one believes that $\Gamma \gtrsim 1.5$, this result suggests a baryonic density in IGM which is marginally inconsistent with the BBN. Only further work can show whether this result is due to inadequacies in modelling or whether it represents a genuine difficulty.

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