

Post-COBE predictions for Inflationary Gravity Wave and Density Perturbation spectra *

VARUN SAHNI † and TARUN SOURADEEP *

Inter-University Centre for Astronomy and Astrophysics
Post Bag 4, Ganeshkhind, Pune 411 007
INDIA

ABSTRACT

We assess the relative contribution to the COBE - measured microwave anisotropy arising both from relic gravity waves as well as primordial density perturbations originating during inflation. We show that the gravity wave contribution to the CMBR anisotropy depends sensitively upon n - the primordial spectral index ($|\delta_k|^2 \propto k^n$), increasing as n deviates from a Harrison - Zeldovich spectrum ($n = 1$). As a result, for $n < 0.84$ the contribution from gravity waves towards $\delta T/T$ is greater than the corresponding contribution from density perturbations, whereas for $n > 0.84$ the reverse is true. ($n = 0.84$ corresponds to an expansion index $p = 13.5$ in models with power-law inflation $a \propto t^p$.) Our results show that for a scale-invariant Harrison-Zeldovich spectrum generated by chaotic inflation, gravity waves contribute approximately 24% to the CMBR anisotropy measured by COBE. Applying our results to the cold dark matter scenario for galaxy formation, we find that in general CDM models with tilted power spectra ($n < 1$), require the biasing parameter to be greater than unity, on scales of $16h_{50}^{-1} Mpc$. We also obtain an expression for the COBE - normalised amplitude and spectrum of the stochastic gravity wave background and compare it with the sensitivity of planned laser-interferometer gravity wave detectors.

email : † varun@iucaa.ernet.in
* tarun@iucaa.ernet.in

* To appear in the Proceedings of the First Iberian Meeting on Gravity, September 21 - 27, 1992, Evora, Portugal. Ed. M. Bento, O. Bertolami and J. Mourao (World Scientific Publishing Co.).

Both gravity waves and adiabatic density perturbations, characterised by a power spectrum $|\delta_k| \propto k^n$, are generically predicted by the Inflationary Universe scenario. On entering our horizon during the epoch of matter-radiation decoupling, density perturbations as well as gravity waves cause distortions in the cosmic microwave background radiation (CMBR) due to the well known Sachs-Wolfe effect. The anisotropy in the CMBR recently observed by COBE measures the amplitude of gravity waves as well as density perturbations providing thereby a direct probe of the inflationary epoch of our universe.

The close similarity between gravity waves and massless scalars, in a Friedman - Robertson - Walker (FRW) background originally pointed out by Grishchuk¹, makes it possible to express each polarisation state of the graviton in terms of solutions to the massless, minimally coupled Klein-Gordon equation: $h_{\times(+)} = \sqrt{8\pi G} \phi_k e^{-i\mathbf{k}\cdot\mathbf{x}}$

$$\ddot{\chi}_k + \left[k^2 - \frac{\ddot{a}}{a} \right] \chi_k = 0 \quad (1a)$$

where $\chi_k = \phi_k \times a(\eta)$, $a(\eta)$ is the scale factor of the Universe: $ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$, and k is the comoving wavenumber $k = 2\pi a/\lambda$. Eq.(1a) closely resembles the Schrodinger equation in quantum mechanics, with \ddot{a}/a , playing the role of the potential barrier 'V'. (The form of $V \equiv (\ddot{a}/a)$, is shown in Figure 1, for inflation followed by a matter dominated epoch.) Solutions of equation (1a) have some interesting properties, for instance, small wavelength solutions of (1a) are adiabatically damped:

$$\phi_k^+(\eta) \equiv \frac{\chi_k}{a(\eta)} \underset{k\eta \gg 2\pi}{\approx} \frac{1}{\sqrt{2ka(\eta)}} \exp(-ik\eta), \quad (1b)$$

whereas long wavelength modes asymptotically approach the form :

$$\phi_k(\eta) \equiv \frac{\chi_k}{a(\eta)} \underset{k\eta \ll 2\pi}{\approx} A(k) + B(k) \int \frac{d(\frac{\eta}{\eta_0})}{a^2}, \quad (1c)$$

where A and B are constants. The form of (1c) implies that the amplitude of a fluctuation (equivalently - gravity wave), freezes to a constant value when its wavelength (during inflation), becomes larger than the corresponding Hubble radius. Since modes are continuously being pushed outside the Hubble radius during inflation, a mode with a fixed wavenumber "k", which left the Hubble radius during inflation ($k = 2\pi \eta_1^{-1}$), will re-enter it later during the epoch of matter domination ($k = 2\pi \eta_0^{-1}$). The amplitude of the mode between these two epochs is super-adiabatically amplified, which in field-theoretic language corresponds to the quantum creation of gravitons¹ (see fig. 1).

The possibility that relic gravity waves may be generated during inflation, was first investigated by Starobinsky², who showed that the spectrum of gravitons is scale invariant in a fairly broad wavelength interval, if inflation proceeds exponentially and is followed by a radiation dominated epoch. The quantum creation of relic gravity waves was subsequently extended to other inflationary models by Abbott & Wise⁴ and Sahni⁶. (The effect of a post inflationary matter dominated epoch on the graviton spectrum was studied by Allen⁵, and also by Sahni⁶.) In particular it was demonstrated that, for power law expansion $a = (t/t_0)^p \equiv (\frac{\eta}{\eta_0})^{\frac{1-2\nu}{2}}$, ($\nu - \frac{3}{2} = \frac{1}{p-1} = -3\frac{1+w}{1+3w}$, where $w = \frac{p}{\epsilon}$ is the

equation of state of matter) Eq.(1a) can be solved exactly, with the following solution describing positive frequency waves in an FRW Universe in the adiabatic limit:

$$\phi_k^+(\eta) = \sqrt{\frac{\pi\eta_0}{4}} \left(\frac{\eta}{\eta_0}\right)^\nu H_\nu^{(2)}(k\eta) \quad (1d)$$

($\nu = \frac{3}{2}$ corresponds to exponential inflation, $w = -1$; $\nu > \frac{3}{2}$ to power law inflation, $-1 < w < -\frac{1}{3}$; $\frac{1}{2} < \nu < \frac{3}{2}$ to super-exponential inflation (pole - driven inflation), $w < -1$; and $-\frac{3}{2} \leq \nu \leq 0$ to matter dominated expansion, $0 \leq w \leq 1$.) From (1c) and (1d) it can be shown that, the amplitude of gravity waves on scales greater than the Hubble radius, has the time independent form⁶

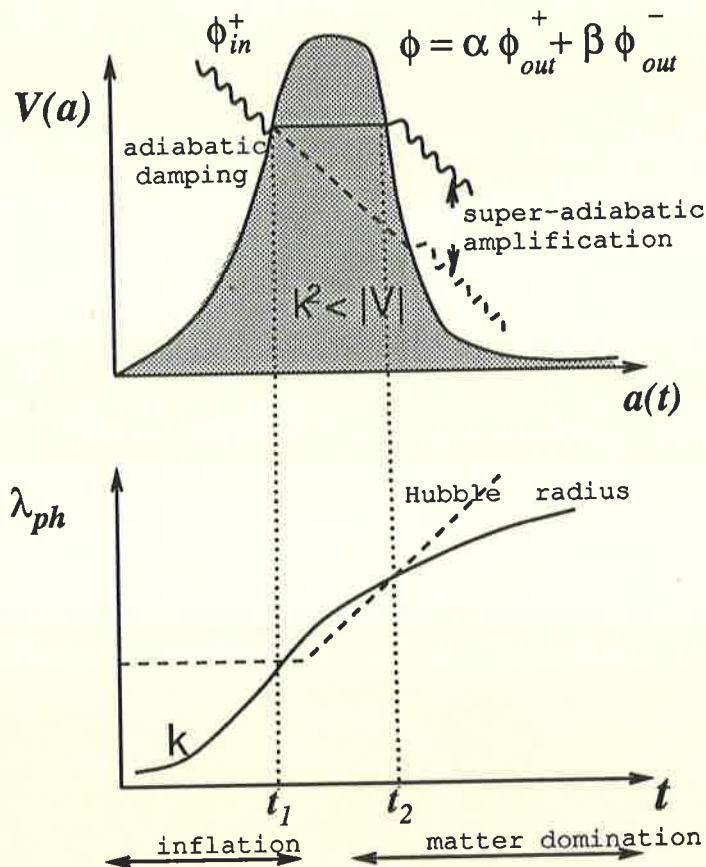


Fig. 1 The superadiabatic amplification of gravity waves is shown for a "potential barrier" $V[a(\eta)] = (\ddot{a}/a)$, for inflation ($a = (\eta_0/\eta)$) followed by radiation and matter domination³ ($a = a_0\eta(\eta + \tilde{\eta}_0)$). At early times $t < t_1$, the scalar field is in its vacuum state $\tilde{\phi}_k^{(+)}$. At late times $t \approx t_0$, the scalar field will in general be described by a linear superposition of positive and negative frequency solutions of eq. (1): $\tilde{\phi}_k(\eta) = \alpha\tilde{\phi}_k^{(+)} + \beta\tilde{\phi}_k^{(-)}$. A mode with comoving wavenumber $k = (2\pi a/\lambda)$ is shown to leave the Hubble radius during inflation (t_1), and re-enter it during matter domination (t_0). (Figure not drawn to scale.)

$$h_{+,x} = \sqrt{16\pi G} \frac{k^{\frac{3}{2}}}{\sqrt{2\pi}} |\phi_k| \Big|_{k|\eta_1|=2\pi} = \sqrt{16\pi} A(\nu) \left(\frac{H_1}{m_p} \right), \quad (2)$$

$$A(\nu) = \Gamma(\nu) \pi^{-\nu} \left(\nu - \frac{1}{2} \right)^{-1},$$

where we have summed over both polarisation states of the graviton. The value of ν is related to the spectral index n of density fluctuations as $\nu = (4-n)/2$ and H_1 is the value of the Hubble parameter at a time $k|\eta_1| = 2\pi$, when the given mode left the Hubble radius. For exponential inflation $\nu = \frac{3}{2}$ and $h_{+,x} = (2/\sqrt{\pi})(H_1/m_p)$. (m_p is the planck mass, $m_p = 1.2 \times 10^{19} GeV$.)

Both long wavelength gravity waves as well as adiabatic density perturbations generated during inflation, create potential fluctuations at the surface of last scattering, when their wavelength becomes smaller than the horizon size during the matter dominated epoch. This leads to distortions in the cosmic microwave background, described by the well known Sachs - Wolfe effect^{4,7} ($\delta T/T \sim \frac{1}{3} (\delta\varphi/c^2)$). Writing ($\delta T/T$) in terms of a multipole expansion:

$$\frac{\delta T}{T} = \sum_{l,m} a_{lm} Y_l^m(\alpha, \delta), \quad (3)$$

it can be shown that, for gravity waves^{4,8}

$$a_2^2 \equiv \langle |a_{2m}|^2 \rangle = 0.145 h_{+,x}^2 g(n), \quad g(n) = 5770.3 \times \int_0^\infty db \left(\frac{b}{2} \right)^{n-2} [I_2(b)]^2, \quad (4a)$$

where⁸

$$I_l(b) = \int_{k\eta_{rec}}^b \frac{dy}{(b-y)} \frac{J_{l+\frac{1}{2}}(b-y) J_{\frac{l}{2}}(y)}{(b-y)^{\frac{3}{2}} y^{\frac{3}{2}}}, \quad b = k\eta_0. \quad (4b)$$

We have numerically integrated (4b) to obtain $g(n)$ ($k\eta_{rec} \approx 0$), which is well approximated by the fitting formula $g(n) = \exp[0.6(n-1)]$ over the range $0.5 \leq n \leq 1$. The corresponding value of the *rms* quadrupole amplitude is

$$Q_T^2 = \frac{5}{4\pi} a_2^2 F_2 = 2.9 A^2(\nu) \left(\frac{H_1}{m_p} \right)^2 g(n) F_2. \quad (4c)$$

(For exponential inflation $\nu = \frac{3}{2}$, and $Q_T^2 = (2.9/4\pi^2)(H_1/m_p)^2 F_2$.) (The subscript "T", denotes the fact that the quadrupole anisotropy is induced by *tensor* waves, gravity waves being quadrupolar, do not contribute to a dipole component a_1 .) F_2 incorporates the finite beam width of the COBE-DMR instrument ($F_2 \approx 0.99$).

Scalar density perturbations generated during inflation, are characterised by a spectrum $|\delta_k|^2 = A k^n$, the scale invariant Harrison - Zeldovich spectrum $n = 1$, is predicted by inflationary models with exponential expansion. Models with power law inflation ($a(t) \propto t^p, p > 1$), which arise in a number of theories including extended inflation⁹, predict a more general value for the spectral index¹⁰, $n = (p-3)/(p-1) < 1$ ($n > 1$, is predicted for super-exponential inflation²⁸.)

The amplitude of density perturbations at horizon crossing ($k = H_0/c$) is given by¹¹

$$\left(\frac{\delta\rho}{\rho}\right)_H^2 \equiv \frac{k^3|\delta_k|^2}{2\pi^2} = \frac{\mathcal{A}}{2\pi^2} H_0^{n+3}. \quad (5)$$

(we assume $c = 1$, for simplicity.) The associated fluctuations in the microwave background are given by (for $l \geq 2$)

$$a_l^2 \equiv \langle |a_{lm}|^2 \rangle = \frac{H_0^4}{2\pi} \int_0^\infty \frac{dk}{k^2} |\delta_k|^2 j_l^2(kx) \quad (6a)$$

where $x = (2/H_0)$, is the present day horizon size ($j_l^2(kx)$ are spherical Bessel functions). For power law spectra $|\delta_k|^2 = \mathcal{A} k^n$, we obtain¹²

$$a_2^2 = \frac{\mathcal{A} H_0^{n+3}}{16} f(n) = \frac{\pi^2}{8} f(n) \left(\frac{\delta\rho}{\rho}\right)_H^2, \quad f(n) = \frac{\Gamma(3-n)\Gamma(\frac{3+n}{2})}{\Gamma^2(\frac{4-n}{2})\Gamma(\frac{9-n}{2})}. \quad (6b)$$

Values of n in the range $0.5 \leq n \leq 1.7$, are consistent with the recent COBE results¹³. For $n = 1$, (6b) reduces to $a_2^2 = (\pi/12) (\delta\rho/\rho)_H^2$.

The related value of the *rms* quadrupole amplitude is given by

$$Q_S^2 = \frac{5}{4\pi} a_2^2 F_2 = \frac{5\pi}{32} f(n) \left(\frac{\delta\rho}{\rho}\right)_H^2 F_2 \quad (7)$$

(the subscript "S" refers to *scalar* perturbations.)

As shown by a number of authors, scalar field fluctuations which left the Hubble radius during inflation, upon re-entering the horizon during radiation (matter) domination, give rise to a density fluctuation, whose *rms* value is given by¹⁴

$$\left(\frac{\delta\rho}{\rho}\right)_H = b \frac{H_1}{\dot{\phi}} \delta\phi \quad (8)$$

where H_1 is the Hubble parameter at a time t_1 , when a scale which reenters the horizon at t_0 , left the Hubble radius during the inflationary epoch (see Figure 1). For modes entering the horizon during radiation (matter) domination, $b = 4(\frac{2}{5})$. (The main contribution to the observed quadrupole anisotropy however, comes from modes entering the horizon during matter domination, we shall therefore assume $b = \frac{2}{5}$ in the ensuing discussion.) $\delta\phi$ arises because of quantum fluctuations in the scalar field during inflation. For modes entering the horizon today $\delta\phi$ is given by $\delta\phi = A(\nu)H_1$ (see (2)), from which we recover the standard result $\delta\phi = \frac{H_1}{2\pi}$, for exponential inflation^{14,15}.

For power law inflation, the inflaton potential has the form $V(\phi) = V_0 \exp(-\sqrt{16\pi/p} (\phi/m_p))$ which results in the following exact solution¹⁶ to the field equations: $H = (p/t)$, $(H/\dot{\phi}) = (\sqrt{4\pi p}/m_p)$, as a result

$$\left(\frac{\delta\rho}{\rho}\right)_H = A(\nu) \frac{\sqrt{16\pi p}}{5} \frac{H_1}{m_p}. \quad (9)$$

Combining Eqs. (7) and (9), the *rms* quadrupole amplitude of the temperature anisotropy

$$Q_S^2 = \frac{\pi^2}{10} p A^2(\nu) \left(\frac{H_1}{m_p} \right)^2 f(n) F_2 \quad (10)$$

($\nu - \frac{3}{2} = (p-1)^{-1} = (1-n)/2$), for power law inflation. (See also Fabbri, Lucchin and Matarrese; Lyth & Stewart.)¹⁷ The above result for power law inflation can be extended to cover models of slow-roll (quasi-exponential) inflation. As shown in Souradeep and Sahni¹⁸, Q_S^2 for a chaotic inflation model with a $m^2\phi^2$ potential corresponds to a power law model with $p = 2N$, where $N = H\Delta t$ is the number of e-foldings (Δt is the duration of the inflationary epoch and $H \approx \text{constant}$ is the Hubble parameter during inflation). In terms of the number of e-foldings in a model of slow-roll inflation, the following relation

$$2N = p = \frac{n-3}{n-1}, \quad (11)$$

fixes the spectral index of density perturbations n . As a result slow-roll inflation (chaotic $m^2\phi^2$ model) with minimal e-foldings $N = 67$ is equivalent to a power law model with $p = 134$ which corresponds to a spectral index $n = 0.985$.

Since the coefficients a_{lm} in (3) are Gaussian random variables, the variances Q_T^2 and Q_S^2 given by (4c) and (10) must be added, when considering the combined effect of both gravity waves and adiabatic density perturbations on the anisotropy of the CMBR. As a result, the net quadrupole anisotropy will be given by

$$Q_{\text{COBE-DMR}}^2 = Q_S^2 + Q_T^2 \quad (12)$$

where $Q_{\text{COBE-DMR}}$ is the *rms*-quadrupole normalised amplitude $Q_{rms-PS} = 5.86 \times 10^{-6}$ detected by COBE¹³. From (4c) and (10), we find that the ratio $(Q_T^2/Q_S^2) = (2.94(n-1)g(n)/(n-3)f(n))$ depends sensitively on n - the spectral index of density perturbations. We find that for $n \geq 0.84$, the contribution from gravity waves to the CMBR anisotropy dominates the contribution from scalar density perturbations, whereas for $n < 0.84$ the reverse is true. ($n = 0.84$ corresponds to $p = 13.5$.) The ratio (Q_T^2/Q_S^2) has been plotted against n in Figure 2. For slow-roll (chaotic) inflation we find that the microwave distortion due to gravity waves amounts to $\simeq 31\%$ of the distortion caused by density fluctuations, in agreement with earlier work by Starobinsky⁸ and contrary to recent claims by Krauss and White¹⁹.

Substituting the values of Q_T^2 and Q_S^2 in Eq.(12), we obtain

$$\left(\frac{H_1}{m_p} \right)^2 = \left[\frac{\pi^2}{10} \frac{n-3}{n-1} f(n) + 2.9 g(n) \right]^{-1} [A(\nu)]^{-2} \frac{Q_{\text{COBE-DMR}}^2}{F_2} \quad (13)$$

for power law inflation. Setting $p = 2N$ in Eq. (13) we can recover the result for slow-roll inflation¹⁸. As a result Eq.(13) allows us to determine the Hubble parameter during inflation both for power law, as well as for quasi-exponential inflation.

The value of (H_1/m_p) allows us to determine the value of Q_S^2 - the fraction of the quadrupole anisotropy contributed by density perturbations, as well as $(\delta\rho/\rho)_H^2$ - the amplitude of density fluctuations at horizon crossing (see (5), (9)). The value of $(\delta\rho/\rho)_H$ fixes the normalisation \mathcal{A} of the power spectrum of density fluctuations to be

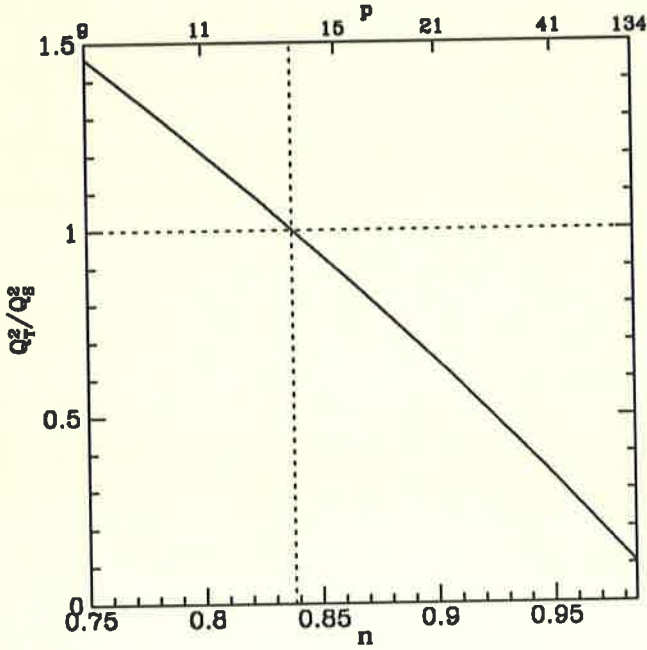


Fig. 2 The ratio of the Quadrupole anisotropy from tensor and scalar waves is plotted against both spectral index n and p (in $a \propto t^p$) for power law inflation. The maximum value $p = 134$ corresponds to $n = 0.985$ and is equivalent to ~ 67 e-foldings of slow-roll inflation.

$$\frac{\mathcal{A}}{2\pi^2} = \left[\frac{72.5}{16\pi} \frac{n-1}{n-3} g(n) + \frac{5\pi}{32} f(n) \right]^{-1} \left(\frac{c}{H_0} \right)^{n+3} \frac{Q_{\text{COBE-DMR}}^2}{F_2}, \quad (14)$$

(we have reintroduced c in the above expression for completeness and as noted earlier (Eq. 4c), the finite beam width factor $F_2 \approx 0.99$ for COBE-DMR). We now use the COBE-DMR results to normalise the spectrum and predict the value of the *rms* mass fluctuation on a given scale

$$\left(\frac{\Delta M}{M} \right)^2 (R) = \frac{1}{2\pi^2} \int k^2 dk |\delta_k|^2 W^2(kR), \quad (15)$$

where $W(kR)$ is the *top hat* window function ²⁰.

Our results for cold dark matter models with power law primordial spectra : $|\delta_k|^2 = \mathcal{A} k^n T_{k,\text{cdm}}$, (where $T_{k,\text{cdm}}$ is the CDM transfer function given in Appendix G of Bardeen et al.²¹) are shown in

Figure 3, for different values of the parameters n and h_{50} ($\Omega_{\text{cdm}} \simeq 0.9, \Omega_{\text{baryonic}} \simeq 0.1$, is assumed). (Values of n in the range $0.5 \leq n \leq 1.7$, are consistent with the COBE - DMR results¹³.) For a scale-invariant spectrum $n = 1$, our results agree with those of Bond and Efstathiou²². Our results also show that $\frac{\Delta M}{M}(16h_{50}^{-1} \text{Mpc}) \leq 1$ for $n \leq 0.87$. Since mass need not trace light, in models with nonbaryonic dark matter, our results can be taken to mean that a biasing factor ($b_{16} = (\frac{\Delta M}{M})_{16}^{-1}$) mostly greater than unity, is required in order to reconcile theoretical models based on power law inflation, with observations. From Figure 3 it follows that the value of b_{16} is sensitive to both n as well as h_{50} , decreasing with increasing n and h_{50} . It may be noted that since gravity waves contribute predominantly to the CMBR anisotropy for $n \leq 0.84$, the biasing factor $b_{16h_{50}^{-1}}$ is much larger than it would have been had only the contribution from scalar density perturbations to the CMBR been considered. This makes CDM models with power law inflation less compatible with the excess galaxy clustering observed in the APM survey, than had previously been assumed²³.

Having normalised the amplitude of scalar density perturbations generated during inflation, we now proceed to evaluate the normalised spectrum of relic gravity waves. During inflation (*ie* for $\eta < \eta_0 < 0$), gravitons are described by Eq.(1d) which represents the state corresponding to the adiabatic vacuum. During the matter dominated epoch however, gravitons are described by a state which is a linear superposition of positive and negative frequency solutions of Eq.(1a):

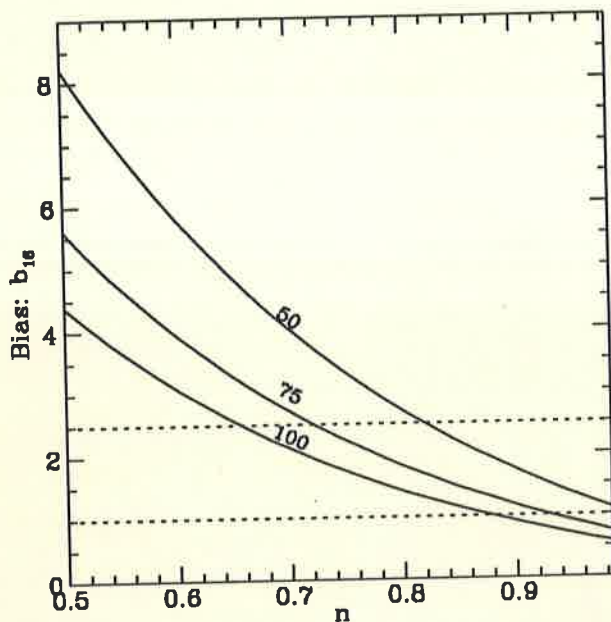


Fig. 3 The biasing factor $b_{16} = (\frac{\Delta M}{M})_{16}^{-1}$, is shown plotted against the spectral index n , for three values of the Hubble parameter: $H = 50, 75 \text{ \& } 100 \text{ kmsec}^{-1} \text{Mpc}^{-1}$. The dotted lines correspond to $b_{16} = 1$ and $b_{16} = 2.5$.

$$\tilde{\phi}_k(\eta) = \alpha \tilde{\phi}_k^{(+)} + \beta \tilde{\phi}_k^{(-)} \quad (16a)$$

where

$$\tilde{\phi}_k^{(+,-)}(\eta) = \sqrt{\frac{\pi\eta_0}{4}} \left(\frac{\eta}{\eta_0}\right)^\mu H_{|\mu|}^{(2,1)}(k\eta) \quad (16b)$$

$\eta > |\eta_0|$. μ can be related to the equation of state of matter $\mu = \frac{3}{2} \frac{w-1}{1+3w}$ where $w = P/\epsilon$, ($-\frac{3}{2} \leq \mu \leq 0$ for reasonable equations of state for matter: $0 \leq w \leq 1$).

The Bogoliubov coefficients α and β can be obtained by matching (1d) and (16) at wavelengths larger than the horizon size via (1c). As a result we obtain for $k\eta_0 < 2\pi$,

$$\alpha \pm \beta = \pm i\gamma^{\pm 1} \left(\frac{k\eta_0}{2}\right)^{\mp(\nu+|\mu|)} \quad (17)$$

where $\gamma = \pi^{-1}\Gamma(\nu)\Gamma(1+|\mu|)$, and $|\alpha|^2 - |\beta|^2 = 1$, (see Sahni⁶ (1990) for details). (For $k\eta_0 > 2\pi$ the adiabatic theorem gives $\alpha \simeq 1, \beta \simeq 0$.)

The energy density of created gravitons can now be determined exactly by substituting the expression for $\tilde{\phi}_k(\eta)$ in (16), with the values of the Bogoliubov coefficients given in Eq.(17) in the expression²⁹

$$\epsilon_g = \langle T_0^0 \rangle = \frac{1}{2\pi^2 a^2} \int dk k^2 \left(|\dot{\tilde{\phi}}_k|^2 + k^2 |\tilde{\phi}_k|^2 \right), \quad (18)$$

(both polarisation states of the graviton have been included in Eq.(18).)

Of greater relevance to us is the spectral energy density of gravity waves, $\epsilon(\omega) = \omega \frac{d\epsilon_g}{d\omega}$ which can be derived from Eq.(18) and has the simple form⁶

$$\begin{aligned} \epsilon(\tilde{\omega}) &= b_2^2 \tilde{\omega}^{1-2\bar{\nu}} \left(\frac{H_1}{m_p}\right)^2 \epsilon_m \quad \text{for } 1 < \tilde{\omega} < \frac{3}{4\pi} \Omega_r^{-\frac{1}{2}}, \\ \epsilon(\tilde{\omega}) &= b_1^2 \tilde{\omega}^{3-2\bar{\nu}} \left(\frac{H_1}{m_p}\right)^2 \epsilon_r \quad \text{for } \frac{3}{4\pi} \Omega_r^{-\frac{1}{2}} < \tilde{\omega} \end{aligned} \quad (19)$$

where $\tilde{\omega} = \frac{k\eta}{2\pi} = \frac{\lambda_h}{\lambda}$, is the dimensionless wavenumber expressed in units of the horizon scale (λ being the physical wavelength and λ_h being the present scale of the horizon, $\lambda_h \simeq 2 \cdot 10^{28} h_{50}^{-1} \text{ cm}$). $1 < \tilde{\omega} < \frac{3}{4\pi} \Omega_r^{-\frac{1}{2}}$ corresponds to wavelengths larger than the horizon size at matter radiation equality, ϵ_m and ϵ_r are the energy densities of matter and radiation respectively; $\epsilon_m \approx \epsilon_{cr} = 4.2 \cdot 10^{-9} h_{50}^2 \text{ ergs/cm}^3$, $\Omega_r = \frac{\epsilon_r}{\epsilon_m} \simeq 10^{-4} h_{50}^{-2}$, h_{50} is the present value of the Hubble parameter in units of 50 km/sec/Mpc.

From Eq.(19) we see that the spectrum of the gravity wave background is uniquely specified once $\left(\frac{H_1}{m_p}\right)$ - the dimensionless value of the Hubble parameter during inflation is established from Eq.(13). The ratio of the spectral energy density of gravity waves to the critical energy density $\Omega_g(\lambda) = \epsilon(\lambda)/\epsilon_{cr}$ and the dimensionless amplitude of gravity waves, $h_{\times+}(\lambda)$ have been plotted in Figure 4, for gravity waves created during exponential inflation, slow-roll (quasi-exponential) inflation and power law inflation. (For the corresponding analytical expressions describing $\Omega_g(\lambda)$, see Sahni⁶ and Grishchuk & Solokhin²⁴.) The quantities $h_{\times+}(\lambda)$ and $\Omega_g(\lambda)$ are directly related by $h_{\times+}^2(\lambda) = 24(\lambda/\lambda_h)\Omega_g(\lambda)$. We find that the amplitude of gravity waves is significantly smaller than the sensitivity of the current generation of terrestrial bar and beam detectors²⁵. The best

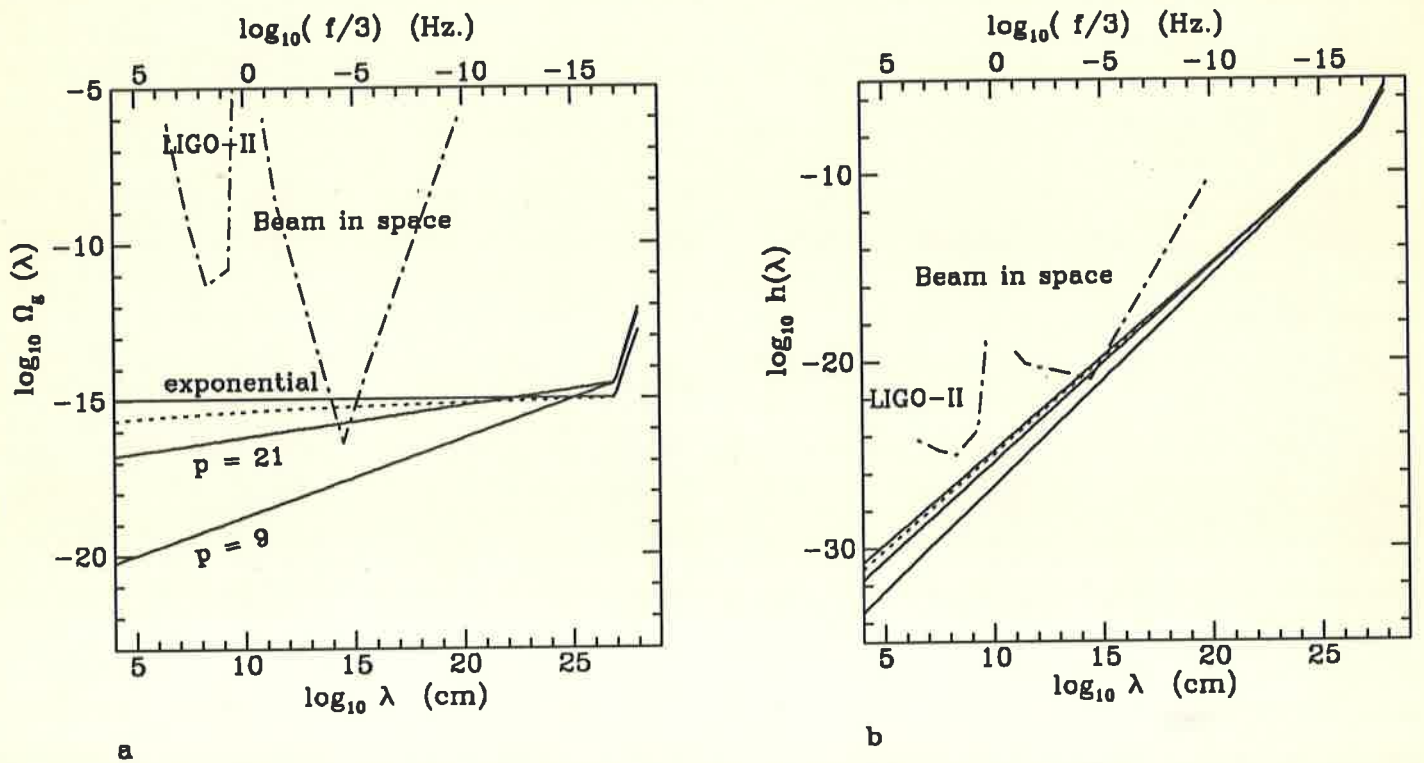


Fig. 4 The COBE-normalised spectral energy density (in units of the critical density) and the amplitude, $h_{\times+}$, of gravity waves is shown (in figures a and b respectively) as a function of the wavelength (and frequency f), for exponential inflation, quasi-exponential inflation (dotted line), and power law inflation $a \propto t^p$ with $p = 21$ and $p = 9$. For comparison, the expected sensitivity of the Laser Interferometer Gravitywave Observatory (LIGO), (Christensen 1992) and of the projected "Beam in space" (space - interferometer) has also been plotted (Thorne 1988).

hope for the detection of the stochastic gravity wave background appears to lie with the *space - interferometer*, whose development is still in the conceptual stage ²⁶.

After completing this work, we became aware of papers²⁷ by Davis et al.; Salopek; Liddle & Lyth; Lucchin, Matarrese & Mollerach ; and Lidsey and Coles, in which results overlapping with those of this paper and ref [18] were obtained.

References

1. Grishchuk, L.P., *Zh. Eksp. Teor. Fiz.* **67** (1974) 825 [*Sov. Phys. JETP* **40**,(1975) 409]
2. Starobinsky, A.A., *JETP Lett.* **30** (1979) 719
3. Sahni, V., *Class. Quantum Grav.* **5** (1988) L113
4. Abbott, L.F. and Wise, M.B., *Nucl. Phys.* **135B** (1984) 279
5. Allen, B., *Phys. Rev. D* **37** (1988) 2078
6. Sahni, V., *Phys. Rev. D* **42** (1990) 453
7. Sachs, R.K. and Wolfe, A.M., *Astrophys. J.* **147** (1967) 73
Rubakov, V.A., Sazhin, M.V. and Veryaskin, A.V., *Phys. Lett.* **115B** (1982) 189
Fabbri, R. and Pollock, M.D., *Phys. Lett.* **125B** (1983) 445
8. Starobinsky, A.A., *Pis'ma Astron. Zh.* **11** (1985) 323 [*Sov. Astron. Lett.* **11** (1985) 133]
9. La, D. and Steinhardt, P.J., *Phys. Rev. Lett.* **62** (1989) 376
10. Lucchin, L.F. and Matarrese, S., *Phys. Lett.* **164B** (1985) 282
Lucchin, L.F. and Matarrese, S., *Phys. Rev. D* **32** (1985) 1316
11. Kolb, E.W. and Turner, M.S., *The Early Universe*, (Addison - Wesley Publishing Company 1990)
12. Gradshteyn, I.S. and Ryzhik, M., *Table of Integrals, Series and Products*, (Acad. Press Inc., N.Y., 1980) p. 692
13. Smoot, C. F. et al., *Astrophys. J. Lett.* **396** (1992) L1
14. Starobinsky, A.A., *Phys. Lett.* **117B** (1982) 175
Guth, A. and Pi, S.Y., *Phys. Rev. Lett.* **49** (1982) 1110
Hawking, S.W., *Phys. Lett.* **115B** (1982) 295
Bardeen, J.M., Steinhardt, P., and Turner, M., *Phys. Rev. D* **28** (1983) 679
15. Bunch, T. S. and Davies, P. C. W., *Proc. R. Soc.* **A360** (1978) 117
Linde, A.D., *Phys. Lett.* **116B** (1982) 335

- Vilenkin, A. and Ford, L. H., *Phys. Rev.* **D25** (1982) 1231
16. Burd, A.B. and Barrow, J.D., *Nucl. Phys.* **B308** (1988) 929
17. Fabbri, R., Lucchin, L.F. and Matarrese, S., *Phys. Lett.* **166B** (1986) 49
- Lyth, D.H. and Stewart, E.D., *Phys. Lett.* **274B** (1992) 168
18. Souradeep, T. and Sahni, V., IUCAA preprint/July-07-92 (1992),
- Souradeep, T. and Sahni, V., *Mod. Phys. Lett.* **A7** (1992) 3541
19. Krauss, L. M. and White, M., *Phys. Rev. Lett.* **69** (1992) 869
20. Peebles, P.J.E., *The Large Scale Structure of the Universe* (Princeton University, Princeton 1980)
21. Bardeen, J.M., Bond, J.R., Kaiser, N, and Szalay, A.S., *Astrophys. J.* **30** (1986) 15
22. Bond, J.R. and Efstathiou, G., *Astrophys. J.* **285** L45, (1984) L45
- Bond, J.R. and Efstathiou, G., *MNRAS* **226** (1987) 655
23. Liddle, A. R., Lyth, D. H., and Sutherland W. J. , *Phys. Lett.* **B274** (1992) 244
24. Grishchuk, L.P. and Solokhin, M., *Phys. Rev. D* **43** (1991) 2566
25. Christensen, N., *Phys. Rev. D* **46** (1992) 5250
26. Thorne, K.S., in *300 Years of Gravitation*, eds. S.W. Hawking and W. Israel, (Cambridge Univ. Press, Cambridge, 1988) p.330
27. Davis R.L., Hodges, H.M., Smoot, G.F., Steinhardt, P.J. and Turner, M.S., *Phys. Rev. Lett.* **69** (1992) 1856
- Salopek, D. S., *Phys. Rev. Lett.* **69** (1992) 3602
- Liddle, A. R. and Lyth, D. H., *Phys. Lett.* **B291** (1992) 391
- Lucchin, F., Matarrese, S. and Mollerach, S., *Astrophys. J. Lett.* **401** (1992) L49
- Lidsey, J.E. and Coles, P., *MNRAS* **258** (1992) 57p
28. Gasperini, M. and Giovannini, M., *Phys. Lett.* **B282** (1992) 36
- Gasperini, M. and Veneziano, G., CERN preprint: CERN-TH.6572/92
29. Ford, L.H. and Parker, L., *Phys. Rev. D* **16** (1977) 1601