CAN THE CURVATURE EFFECTS BE NEGLECTED IN THE EARLY UNIVERSE?

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In the discussion of GUTS in the early universe it is assumed that there exists a locally inertial space–time region large enough to contain a sufficient number of particles justifying the use of flat space statistical mechanics. We show that this assumption is false.

Early universe is being used extensively nowadays as the "poor man's high energy accelerator". All the calculations use flat-space quantum field theory and flat-space statistical mechanics to make predictions about the GUT epoch (temperatures of about 10^{15} GeV) [1]. It is necessary, therefore, to see whether the flat-space equilibrium statistical mechanics can be used in these calculations. Some aspects of using equilibrium statistical physics were discussed in ref. [2]. We consider here some complementary aspects.

Flat-space physics can always be used in the locally inertial frame surrounding an event. However, because of the curvature of space–time, such an inertial region is limited in size. Moreover, to be consistent one must have a sufficiently large number of particles (warranting the use of statistical mechanics) inside this inertial frame. As one approaches the singularity, the size of the region where inertial coordinates can be chosen decreases while the density of matter increases. It is of interest to see which effect predominates.

Consider a space–time with the metric $g_{ik}(\boldsymbol{x}, t)$ around an event x^i. Let us choose a locally inertial coordinate system around x^i, such that

$$g_{ik}\,(\text{at } x^i) = \eta_{ik}. \tag{1}$$

The metric in a neighbouring point y^i is ($\mathrm{d}x^i = y^i - x^i$)

$$g_{lm}(y^i) = \eta_{lm} + \frac{\partial g_{lm}}{\partial x^p}\,\mathrm{d}x^p + \frac{\partial^2 g_{lm}}{\partial x^p \partial x^q}\,\mathrm{d}x^p\,\mathrm{d}x^q, \tag{2}$$

$$\approx \eta_{lm} + RL^2. \tag{3}$$

Here R is a typical component of the curvature tensor, while L is $\approx |(y^i - x^i)|$, the coordinate length interval. Thus the inertial frames can be maintained only for length scales that satisfy

$$RL^2 \ll 1, \quad L^2 \ll 1/R. \tag{4}$$

In the radiation filled early universe,

$$R \approx 1/(ct)^2, \tag{5}$$

so that eq. (4) reads,

$$L/ct \equiv \epsilon \ll 1. \tag{6}$$

Physically ϵ represents the deviation from flatness along a linear scale.

Let us now consider the number of particles inside this coordinate volume L^3. (Notice that it is the coordinate volume L^3 (and *not* the proper volume) which is relevant for this discussion.)

$$N_{L^3} = \rho L^3 \approx \frac{g}{\pi^2}\left(\frac{kT}{c\hbar}\right)^3 L^3 = \frac{g}{\pi^2}\left(\frac{kTt}{\hbar}\right)^3 \epsilon^3. \tag{7}$$

In the radiation filled universe, (t_P is the Planck time, T_P the Planck temperature)

$$(t/t_\mathrm{P}) = (45/16\pi^3 g)^{1/2}(T_\mathrm{P}/T)^2 \tag{8}$$

so that,

$$N_{L^3} \approx \frac{g\epsilon^3}{\pi^2}\left(\frac{45}{16\pi^3 g}\right)^{3/2}\left(\frac{T_\mathrm{P}}{T}\right)^3 \approx \frac{1}{300}\left(\frac{1}{g^{1/2}}\right)\left(\frac{\epsilon T_\mathrm{P}}{T}\right)^3. \tag{9}$$

This leads to the surprising (and disturbing!) conclusion that, in the early epochs the number of particles inside a volume L^3 will not be statistically significant if ϵ has to be small. For example suppose we put $\epsilon \approx 10^{-2}$ (curvature neglected to one part in hundred; curvature effects in, e.g., standard solar system experiments of 1 part in 10^6 are measurable) and $T \approx 10^{16}$ GeV (GUTS era) and $g \approx 100$ (see ref. [3]) we get,

$$N_{L^3} \approx \tfrac{1}{3}, \qquad (10)$$

which is ridiculous if we want to do any statistical mechanics. If one requires $N \approx 10^6$ (validity for statistical mechanics) and $\epsilon \approx 10^{-2}$ (curvature neglected to one part in hundred) we get

$$T \approx 10^{-5}\, T_{\mathrm{P}} \approx 10^{14}\ \mathrm{GeV}. \qquad (11)$$

However, a large fraction of GUTS era is already supposed to have elapsed before $T \sim 10^{14}$ GeV (including the proton decay epoch).

Thus it seems that one cannot neglect effects of curvature in the GUTS epoch and still consistently use statistical mechanics. Further investigation is required to see how such effects can be taken into account.

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References

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