down of the universal expansion. By determining it, one can decide whether our universe is open or closed.

The deceleration parameter is defined as

$$q \equiv \frac{-S\ddot{S}}{\dot{S}^2} = -\frac{1}{H^2(t)}\left(\frac{\ddot{S}}{S}\right). \tag{24}$$

From the field equations it is a straightforward procedure to show that

$$q = \left(\frac{4\pi}{3}G\tau^2\right)\rho = \rho/2\rho_c, \tag{25}$$

where $\rho_c = 3H^2/8\pi G$ is known as the 'closure density'.

For the Einstein–De Sitter model, $q = \frac{1}{2}$ and $\rho = \rho_c$. For $k = +1, q > \frac{1}{2}, \rho > \rho_c$ and $k = -1, q < \frac{1}{2}, \rho < \rho_c$. Thus, the value of q tells us whether the density in the universe is enough to close it or not. For $\tau \sim 10^{10}$ years, the closure density $\rho_c \sim 2 \times 10^{-29}$ gms/cc. The observed density of the luminous matter is of the order of 10^{-31} gms/cc. The deficit between this and ρ_c is known as the 'missing mass' in the universe, a term which seems to indicate an *a-priori* preference for a closed universe. The accurate measurement of q is important for a knowledge of the type of universe we live in.

We arrived at the linear Hubble law by the Taylor expansion of $S(t)$ to first order. If we retain the second-order term, we can show

$$z \approx HD_e + \tfrac{1}{2}(1 + q)(HD_e)^2. \tag{26}$$

Thus, q can, in principle, be deduced from the measurement of the redshift as a function of distance. But distance measurements are not accurate enough to yield an accurate value for q. The deceleration parameter can be deduced observationally from the magnitude-redshift relation. Again, the results are not conclusive. Other observational tests of cosmological models include radio-source counts and angular diameters of galaxies. The final verdict as to the nature of our universe remains in the future. Whether we live in an open or closed universe remains an open question.

8. Conclusion

In the foregoing, we have discussed very briefly some aspects of the general relativistic cosmological models and their implications to observations. The purpose of this is to serve as a short introduction to other chapters to follow that will treat in greater detail some of the basic features of our universe.

References

1. J. V. Narlikar, *Introduction to Cosmology*, Jones and Bartlett (1983).
2. A. K. Raychaudhuri, *Theoretical Cosmology*, Clarendon Press (1979).
3. S. Weinberg, *Gravitation and Cosmology*, John Wiley (1972).

5. Relics of the Big Bang

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1. The Early Universe

All Friedmann models have an epoch in the past when the scale factor S was zero. We refer to this epoch as the big bang epoch. To mathematicians, the big bang implies a breakdown of the concept of spacetime geometry, and they have come to recognize it as an inevitable feature of Einstein's general relativity. It is a feature that prevents the physicist from investigating what happened at $S = 0$ or prior to it. To some physicists, this abrupt termination of the past signifies an incompleteness of the theory of relativity. To them, a more complete theory of the future may show a way of avoiding the catastrophic nature of the $S = 0$ epoch. A universe that has been expanding forever or that has been oscillating between maximum and minimum (but finite) values of S, might result from such a theory.

Here we will continue to put our faith in the validity of general relativity and push our investigations into the past of the universe as close as possible to the $S = 0$ epoch. The purpose of such investigations will be to find out whether we can point to any present-day evidence that the universe indeed had a past epoch when S was close to zero. In short, we will be looking for relics of the big bang.

Pioneering work in this field was done by George Gamow in the mid-1940s. Gamow was concerned with the problem of the origin of elements. Starting from the (then available) basic building blocks of neutrons and protons, Gamow attempted to describe the formation of nuclei of deuterium, helium, and so on. The process envisaged by him involved nuclear fusion, that is, a process in which nuclei are formed by bringing together neutrons and protons. Astrophysicists were already sure by the 1940s that such processes operate inside stars, where the necessary conditions of high temperature and density were known to exist. Gamow pointed out that similar conditions must have existed in a typical Friedmann universe soon after the big bang.

We know from cosmological equations that the density of the universe was very high at small values of S. What about temperature? A simple calculation shows how the temperature also might have been high. This calculation requires the assumption that, at present, we have a radiation density u_0 that is a relic of an early hot era. With this assumption, the radiation energy density at a past epoch S is given by

$$u = u_0 \frac{S_0^4}{S^4}, \tag{1}$$

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where $S = S_0$ at the present epoch. Cosmological equations also tell us that at a critical value of the scale factor the contribution of radiation energy density equals that of matter energy density, and that prior to this epoch the former was more dominant. Gamow therefore assumed that, in the early epochs, the dynamics of expansion were determined by radiant energy rather than by matter in the form of dust.

If we wish to make a simplified calculation, we can assume that the radiation was in blackbody form with temperature T, so that

$$u = aT^4, \tag{2}$$

where a is the radiation constant. This means that in the early stages of the big bang universe

$$T^0_0 = aT^4, \qquad T^1_1 = T^2_2 = T^3_3 = -\tfrac{1}{3}aT^4. \tag{3}$$

We also anticipate that the space-curvature parameter k will not affect the dynamics of the early universe significantly, and set it equal to zero. Thus, from Einstein's equations for the $\binom{0}{0}$ component, we get

$$\frac{\dot{S}^2}{S^2} = \frac{8\pi Ga}{3c^2}T^4. \tag{4}$$

Further, from (1) and (2) we get

$$T = \frac{A}{S}, \quad A = \text{constant}. \tag{5}$$

Substituting (5) into (4) gives a differential equation for S that can be easily solved. Setting $t = 0$ at $S = 0$, we get

$$S = A\left(\frac{3c^2}{32\pi Ga}\right)^{-1/4} t^{1/2} \tag{6}$$

and, more importantly,

$$T = \left(\frac{3c^2}{32\pi Ga}\right)^{1/4} t^{-1/2} \tag{7}$$

Notice that all the quantities inside the parentheses on the right-hand side of the above equation are known physical quantities. Thus, we can express the above result in the following form

$$T_{\text{Kelvin}} = 1.52 \times 10^{10}\, t_{\text{seconds}}^{-1/2}. \tag{8}$$

In other words, about one second after the big bang the radiation temperature of the universe was 1.52×10^{10} K. The universe at this stage was certainly hot enough to facilitate nucleosynthesis, as Gamow supposed.

The idea of a hot big bang, as the above picture is called, depends therefore on the assumption that there is relic radiation present today. It is commonly believed

that the microwave background radiation first discovered in 1965 by Arno Penzias and Robert Wilson is this relic radiation. We will return to the details of this evidence later. For the present, we will accept this evidence as confirming Gamow's notion of the hot big bang and proceed further.

2. Thermodynamics of the Early Universe

Considerable progress has been made in our understanding of the properties of particles and their basic interactions, since the days when Gamow and his colleagues R. A. Alpher and R. Hermann did their calculations of primordial nucleosynthesis. In the following pages we will briefly outline the basic principles on which the modern calculations are usually based.

First, it is necessary to specify the building blocks from which nuclei were constructed in the early epochs. The physicist would naturally like to imagine that the universe started with the simplest possible material composition (whatever that may be!) and that more complex structures were built out of simpler ones by physical interactions. Thus, the cosmologist is forced to take stock of the knowledge of particle physics. While Gamow and his colleagues took the existence of particles like protons, neutrons, electrons, and so on for granted, modern particle physicists believe that a more basic framework accounts for the creation or existence of these particles.

Here we take up the story from the stage when baryons (neutrons and protons), leptons (electrons, muons, neutrinos, and their antiparticles) and photons (the particles of light) are already in existence and are in thermodynamic equilibrium as particles of an ideal gas. Later, we will consider the more speculative and earlier epochs and discuss how these particles came into existence.

Before proceeding with calculations, we must clarify what is meant by 'thermodynamic equilibrium' and 'ideal gas'. We have already mentioned that in these early epochs the dominant form of energy was in particles moving relativistically. The question arises, therefore, whether these particles were interacting with one another or whether they were moving freely. The ideal gas approximation implies that the particles were mostly moving freely. Such particles would interact and collide, of course, but these instances are assumed to have occupied very brief time spans, and their effects on motions may be otherwise neglected. We will shortly express this idea in a quantitative manner.

The collisions and scatterings of the particles would, however, have helped to redistribute their energies and momenta. If these redistributions occurred frequently enough, the system of particles as a whole would have reached a state of thermodynamic equilibrium. In this case for each species of particles there is a definite rule governing the number of particles in a given range of momentum. For thermodynamic equilibrium to be reached, the timescales for successive scatterings should be small compared to the expansion time scale for the universe. Again, we will express this idea quantitatively in a short while.

2.1. *Distribution Functions*

Assuming ideal gas approximation and thermodynamic equilibrium, it is then possible to write down the distribution functions for any given species of particles. Let us use the symbol A to denote typical species ($A = 1, 2, \ldots$). Thus, $n_A(P)\,\mathrm{d}P$ denotes the number density of species A in the momentum range $(P, P + \mathrm{d}P)$, where

$$n_A(P) = \frac{g_A}{2\pi^2 h^3} P^2 \left[\exp\left(\frac{E_A(P) - \mu_A}{kT} \right) \pm 1 \right]^{-1}. \tag{9}$$

In the above formula, T = temperature of the distribution, g_A = number of spin states of the species, k = Boltzmann constant, and

$$E_A^2 = c^2 P^2 + m_A^2 c^4 \tag{10}$$

is the energy corresponding to rest mass m_A of a typical particle. Thus, for electron $g_A = 2$; for the neutrino $g_A = 1$, $m_A = 0$, and so on. The + sign in Equation (9) applies to particles obeying the Fermi–Dirac statistics (these particles are called *fermions*), while the − sign applies to particles obeying the Bose–Einstein statistics (particles known as *bosons*). For example, electrons and neutrinos are fermions, and photons are bosons.

The quantity μ_A is the chemical potential of the species A. For a detailed discussion of chemical potentials, see any standard text on thermodynamics and statistical mechanics. We note here that in any reaction involving these particles, the μ_A are conserved (just as electric charge, energy, spin, and so on are conserved). Because photons can be absorbed or emitted in any number in a typical reaction, we set $\mu_A = 0$ for photons. Since particles and anti-particles (such as electrons and positrons) annihilate in pairs and produce photons, their chemical potentials are equal and opposite.

Apart from the dynamic quantities and the electric charge, several other quantities are found to be conserved in the interactions of particles. These are the baryon number, the muon lepton number, and the electron lepton number. In computing these numbers, a value of +1 is assigned to a particle and −1 to its antiparticle. The electron lepton number counts electrons (e^-) and their neutrinos (ν_e), while the muon lepton number counts muons (μ^-) and their neutrinos (ν_μ). Under these conservation rules, reactions like these

$$\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e, \qquad \mathrm{p} + \bar{\nu}_\mu \to \mu^+ + \mathrm{n}$$

are permitted, while a reaction like the following is not:

$$\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \nu_e$$

(Later we will consider the situation in which the baryon number is not conserved. At the epochs that we are concerned with here, however, we may safely assume the conservation of baryon number to apply.)

Hence, if we assume that in any reaction electric charge, the baryon number, the electron lepton number, and the muon lepton number are conserved, then we have only four independent chemical potentials – those corresponding to protons, electrons, electron neutrinos, and muon neutrinos. From (9) we see that the total number of particles per unit volume in each of these species is needed to determine the corresponding μ_A and that the number densities will be large for large $\mu_A > 0$. These number densities are not known with any degree of accuracy, except that (as we shall shortly see) the ratio

$$\frac{N_B}{N_\gamma} = \frac{\text{Number density of baryons}}{\text{Number density of photons}} \sim 10^{-8} - 10^{-10}$$

is small compared to 1.

The smallness of the baryon number density suggests that the number densities of leptons may also be small compared to N_γ, and it is usually assumed that this hypothesis provides a good justification for taking $\mu_A = 0$ for all species. We will assume that $\mu_A = 0$ for all species in our calculations to follow.

We then get the following integrals for the particle number density (N_A), the energy density (ε_A), pressure (p_A), and entropy density (S_A):

$$N_A = \frac{g_A}{2\pi^2 h^3} \int_0^\infty \frac{P^2\,\mathrm{d}P}{\exp[E_A(P)/kT] \pm 1}, \tag{11}$$

$$\varepsilon_A = \frac{g_A}{2\pi^2 h^3} \int_0^\infty \frac{P^2 E_A(P)\,\mathrm{d}P}{\exp[E_A(P)/kT] \pm 1}, \tag{12}$$

$$p_A = \frac{g_A}{6\pi^2 h^3} \int_0^\infty \frac{c^2 P^4 [E_A(P)]^{-1}\,dP}{\exp[E_A(P)/kT] \pm 1}, \tag{13}$$

$$s_A = (p_A + \varepsilon_A)/T. \tag{14}$$

2.2. *High- and Low-Temperature Approximations*

The above expressions become simplified for particles moving relativistically. In this case

$$T \gg \frac{m_A c^2}{k} \equiv T_A. \tag{15}$$

The details are given in Table I for the different species of interest. The numbers are expressed in units of the quantities for the photon ($g_A = 2$, symbol γ):

$$N_\gamma = \frac{2.404}{\pi^2} \left(\frac{kT}{ch} \right)^3, \qquad \varepsilon_\gamma = \frac{\pi^2 (kT)^4}{15 h^3 c^3} = 3p_\gamma,$$

$$s_\gamma = \frac{4\pi^2 k}{45} \left(\frac{kT}{ch} \right)^3. \tag{16}$$

Table I. Thermodynamic quantities for various particle species at $T \gg T_A$

Particle species A	Symbol	T_A (K)	g_A	N_A/N_γ	$\varepsilon_A/\varepsilon_\gamma$	s_A/s_γ
Electron	e^-	5.93×10^9	2	3/4	7/8	7/8
Positron	e^+		2	3/4	7/8	7/8
Muon	μ^-	1.22×10^{12}	2	3/4	7/8	7/8
Antimuon	μ^+		2	3/4	7/8	7/8
Muon, electron neutrinos	ν_μ, ν_e	0	1	3/8	7/16	7/16
and their antineutrinos	$\bar{\nu}_\mu, \bar{\nu}_e$		1	3/8	7/16	7/16
Pions	π^+	1.6×10^{12}	1	1/2	1/2	1/2
	π^-		1	1/2	1/2	1/2
	π^0		1	1/2	1/2	1/2
Proton	p	10^{13}	2	3/4	7/8	7/8
Neutron	n	$T_n - T_p \sim 1.5 \times 10^{10}$	2	3/4	7/8	7/8

In this approximation, consider the electrical potential energy of any two electrons separated by distance r. This is given by

$$U = \frac{e^2}{r}.$$

Now the average interelectron distance is given by $N_e^{-1/3} \sim ch/kT$. Thus, average interaction energy is

$$\langle U \rangle \sim \frac{e^2}{hc} kT.$$

However, kT measures the energy of motion of electrons. Thus, the interaction energy is $e^2/hc \sim 1/137$ of the energy of motion. Since this fraction is small, we are justified in treating the electrons as free gas.

By contrast, at low temperatures $T \ll T_A$ we have for all species with $m_A \neq 0$

$$N_A = \frac{g_A}{h^3}\left(\frac{m_A kT}{2\pi}\right)^{3/2} \exp\left(-\frac{T_A}{T}\right),$$

$$\varepsilon_A = m_A N_A, \qquad p_A = N_A kT, \qquad s_A = \frac{m_A N_A}{T} c^2. \tag{17}$$

We will often refer to this limit as the nonrelativisitic approximation. (For the photon and zero rest mass neutrino $T_A = 0$ and this approximation never applies.)

2.3. *The Behaviour of Entropy*

We now recall the conservation law satisfied by ε and p in the early stages of the expanding universe, the law given by

$$\frac{d}{dS}(\varepsilon S^3) + 3pS^2 = 0, \tag{18}$$

and use it in conjunction with the second law of thermodynamics. This law tells us that the entropy in a given volume S^3 stays constant as the volume expands adiabatically. From (14) we therefore get

$$\frac{d}{dt}(S^3 s) = \frac{d}{dt}\left[\frac{S^3}{T}(p + \varepsilon)\right] = 0, \tag{19}$$

where $s = \Sigma_A s_A$ is the total entropy of all the particles in the expanding volume.

Rewriting (19) with the help of (18) we get

$$0 = \frac{d}{dt}\left(\frac{S^3 p}{T}\right) + \frac{1}{T}\frac{d}{dt}(S^3\varepsilon) + (S^3\varepsilon)\frac{d}{dt}\left(\frac{1}{T}\right)$$

$$= \frac{d}{dt}\left(\frac{S^3 p}{T}\right) - \frac{3pS^2}{T}\dot{S} + S^3\varepsilon\frac{d}{dt}\left(\frac{1}{T}\right),$$

that is,

$$\frac{dp}{dT} = \frac{1}{T}(p + \varepsilon). \tag{20}$$

This relation can be directly derived from (12) and (13) by a simple manipulation of the integrals. Then, starting from (20), we can derive (19). We will use the constancy of

$$\sigma = \frac{S^3}{T}(p + \varepsilon) \tag{21}$$

in our later calculations.

In the high-temperature approximation we get $p = \varepsilon/3 \propto S^{-4}$ from (18). Hence, from the constancy of σ we recover the relation (5) $T \propto S^{-1}$. A simple relation like this does not hold if the high-temperature approximation is not valid.

3. Primordial Neutrinos

From Table I we see that for $T < 1.5 \times 10^{12}$ K, the only particles that can be present with appreciable number densities in thermal equilibrium are $\mu^\pm$, $e^\pm$, ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$, and γ. The baryons (p and n) and pions ($\pi^\pm$, π^0) will be cooled below their critical temperatures T_A, so that for them the low-temperature approximation holds. The photons, $e^{\pm}$ and $\mu^{\pm}$ follow their respective distributions of the type (9). The neutrinos, however, require some attention, since this phase happens to be crucial in determining the extent of their survival.

The neutrinos are absorbed, emitted, or scattered in reactions such as the following:

$$e^- + \mu^+ \longleftrightarrow \nu_e + \bar{\nu}_\mu, \qquad e^+ + \mu^- \longleftrightarrow \bar{\nu}_e + \nu_\mu,$$
$$\nu_e + \mu^- \longleftrightarrow \nu_\mu + e^-, \qquad \nu_e + \mu^+ \longleftrightarrow \nu_\mu + e^+,$$
$$\nu_e + e^- \longleftrightarrow \nu_e + e^-, \qquad \nu_e + e^+ \longleftrightarrow \nu_e + e^+,$$
$$\nu_\mu + \mu^+ \longleftrightarrow \nu_e + e^+, \qquad \nu_\mu + \mu^- \longleftrightarrow \nu_e + e^-.$$

These are all examples of weak interactions. For $T \lesssim T_\mu$, the cross-section of a typical reaction is of the order

$$\Sigma = \mathcal{G}^2 \hbar^{-4} (kT)^2 c^{-4} \tag{22}$$

where $\mathcal{G} = 1.4 \times 10^{-49}$ erg cm^{-3} is the weak interaction coupling constant. From (6) and Table I, we see that the number densities of participating particles $e^{\pm}$ is of the order

$$(kT/c\hbar)^3,$$

while for muons we should take account of (17) and introduce an exponential damping factor of

$$\exp\left(-\frac{T_\mu}{T}\right)$$

Thus, typical neutrino reaction rate is

$$\eta = c\Sigma \left(\frac{kT}{c\hbar}\right)^3 \exp\left(-\frac{T_\mu}{T}\right) = \mathcal{G}^2 \hbar^{-7} c^{-6} (kT)^5 \exp\left(-\frac{T_\mu}{T}\right). \tag{23}$$

We must now take note of the other rate that is relevant to the maintenance of the equilibrium of neutrinos – the rate at which a typical volume enclosing them expands. From Einstein's equations we get

$$H^2 = \frac{\dot{S}^2}{S^2} = \frac{8\pi G}{3c^2} \varepsilon \approx \frac{16\pi^3 G}{90\hbar^3 c^5} (kT)^4 \tag{24}$$

H, the Hubble constant at the particular epoch, measures the rate of expansion of the volume in question. Thus, the ratio of the reaction rate to the expansion rate is given by

$$\frac{\eta}{H} \sim G^{-1/2} \hbar^{-11/2} \mathcal{G}^2 c^{-7/2} (kT)^3 \exp\left(-\frac{T_\mu}{T}\right)$$

$$\sim \left(\frac{T}{10^{10}\,\mathrm{K}}\right)^3 \exp\left(-\frac{10^{12}\,\mathrm{K}}{T}\right)$$

$$= T_{10}^3 \exp\left(-\frac{1}{T_{12}}\right). \tag{25}$$

Here we have substituted the values of G, $\hbar$, $\mathcal{G}$, c, k, and T_μ and arrived at the above numerical expression. Further, we have written the temperature using the notation T_{10}, T_{12}, and so on. In general, T_n indicates temperatures expressed in units of 10^n K.

What does (25) tell us? As the temperature drops below 10^{12} K, the exponential decreases rapidly. This means that the reactions involving neutrinos run at slower rate compared to the expansion rate of the universe. The neutrinos then cease to interact with the rest of the matter and therefore drop out of thermal equilibrium as temperatures fall appreciably below $T_{12} = 1$. How far below?

The original theory of weak interactions suggested that this temperature may be around $T_{11} = 1.3$. In the late 1960s and early 1970s, successful attempts to unify the weak interaction with the electromagnetic interaction led to additional (neutral current) reactions that keep neutrinos interacting with other matter at even lower temperatures. We state here the outcome of these investigations: that the neutrinos can remain in thermal equilibrium down to temperatures of the order $T_{10} \simeq 1$.

However, even though neutrinos decouple themselves from the rest of the matter, their distribution function still retains its original form with the temperature dropping as $T \propto S^{-1}$. This is because as the universe expands the momentum and energy of each neutrino falls as S^{-1} and the number density of neutrinos falls as S^{-3}. Since the temperature of the rest of the mixture also drops as S^{-1}, and since the two temperatures were equal when the neutrinos were coupled with the rest of the matter, they continue to remain equal, even though neutrinos and the rest of the matter are no longer in interaction with one another.*

There is, however, another epoch when the neutrino temperature begins to differ from the temperature of the rest of the matter. We end this section with a discussion of this important phase in the early universe.

First consider the universe in the temperature range $T_{12} = 1$ to $T_{10} = 1$. In this phase we have the neutrinos, the electron positron pairs, and the photons, each with distribution functions of the type (9) in the high-temperature approximation (see Table I). Thus

$$\varepsilon = \varepsilon_{\nu_e} + \varepsilon_{\bar{\nu}_e} + \varepsilon_{\nu_\mu} + \varepsilon_{\bar{\nu}_\mu} + \varepsilon_{e^-} + \varepsilon_{e^+} + \varepsilon_\gamma$$

* Our remarks about neutrinos are meant to apply to all four species $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$.

Counting the various g-factors from Table I, we get

$$\varepsilon = \frac{9}{2} aT^4. \qquad (26)$$

Thus, in this period the expansion equation is modified from our simplified formula (4) to

$$\frac{\dot{S}^2}{S^2} = \frac{12\pi G a}{c^2} T^4 \qquad (27)$$

and the relation (7) is changed to

$$T = \left(\frac{c^2}{48\pi G a}\right)^{1/4} t^{-1/2} \qquad (28)$$

which we may rewrite as

$$T_{10} = 1.04\, t_{\text{seconds}}^{-1/2}. \qquad (29)$$

However, in the next phase the situations become complicated, as the $e^{\pm}$ pairs are no longer relativistic. Thus, the high-temperature approximation is no longer valid and we have to use the full formulae (12) and (13) to determine the ε and p and the expansion rate of the universe. We will not go into details of this phase but, instead, jump across to its end, when the pairs have annihilated, leaving only photons

$$e^- + e^+ \rightarrow \gamma + \gamma. \qquad (30)$$

Thus, the energy, originally in $e^{\pm}$ and photons, has now vested only in photons, raising their number and temperature. How can we evaluate this change? It is here that (21), telling us of the constancy of σ, comes to our help.

In the relativistic phase ($T_9 > 5$) of $e^{\pm}$ we have

$$\sigma = \frac{4S^3}{3T}\{\varepsilon_{e^-} + \varepsilon_{e^+} + \varepsilon_\gamma\} = \frac{11}{3} a(ST)^3. \qquad (31)$$

When the $e^{\pm}$ have annihilated and left only photons, we have the photon temperature T_γ given by

$$\sigma = \frac{4}{3}\frac{S^3}{T_\gamma}\varepsilon_\gamma = \frac{4}{3} a(ST_\gamma)^3. \qquad (32)$$

We now use the result that the neutrino temperature always changes as S^{-1}. Let us write it as

$$T_\nu = \frac{B}{S}, \quad B = \text{constant}. \qquad (33)$$

Then (31) gives

$$\sigma = \frac{11}{3} aB^3 \left\{\frac{T}{T_\nu}\right\}^3. \qquad (34)$$

Similarly (32) gives

$$\sigma = \frac{4}{3} aB^3 \left\{\frac{T_\gamma}{T_\nu}\right\}^3. \qquad (35)$$

Now in the preannihilation era $T = T_\nu$, so that (34) tells us $\sigma = (11/3)aB^3$. After annihilation σ must have the same value, so we may equate it to the value given by (35). Thus, we arrive at the conclusion that the photon temperature at the end of $e^{\pm}$ annihilation has risen above the neutrino temperature by the factor

$$\frac{T_\gamma}{T_\nu} = \left\{\frac{11}{4}\right\}^{1/3} \cong 1.4. \qquad (36)$$

So the present-day neutrino temperature is lower than the photon temperature by the factor $(1.4)^{-1}$. If we take the latter as ~ 3 K, the former is ~ 2.1 K.

In the above calculation, we have not taken account of neutrinos having a small but nonzero rest mass. Nor have we considered the question of the existence of more than two types of neutrinos in the primordial epochs. For example, particle physicists talk about the so-called τ-neutrino associated with the τ-lepton. We will take another look at neutrinos later when we will discuss these questions anew.

4. The Neutron/Proton Ratio

We have so far developed a picture of the early universe that is best expressed in the form of a time-temperature table of events, as shown in Table II.

In our discussion so far we have not paid much attention to baryons – the protons and neutrons that are also present in the mixture. In our approximation of setting the chemical potentials to zero we took the baryon number to be zero. The validity of the approximation depended on the baryon number density being several orders (8 to 10) of magnitude smaller than the photon density. Nevertheless, we must now take note of the existence of baryons, however small their number density; for we need them in order to consider Gamow's idea of nucleosynthesis in the hot universe.

First notice that the temperatures T_n and T_p of Table I are very high, so that the neutron and proton distribution functions follow the nonrelativistic approximations of (17).

Table II. A time-temperature table of events preceding nucleosynthesis in the early universe

Time since big bang (s)	Temperature (K)	Events
$\lesssim 10^{-4}$	$> 10^{12}$	Baryons, mesons, leptons, and photons in thermal equilibrium.
10^{-4}–10^{-2}	10^{12}–10^{11}	$\mu^{\pm}$ begin to annihilate and disappear from the mixture. Neutrinos begin to decouple from rest of matter.
10^{-2}–1	10^{11}–10^{10}	Neutrinos decouple completely. $e^{\pm}$ pairs still relativistic.
1–180	10^{10}–10^{9}	The pairs of $e^{\pm}$ annihilate and disappear, raising the photon gas temperature to ~1.4 times the temperature of neutrinos.

Thus we get

$$N_p = \frac{2}{h^3}\left(\frac{m_p kT}{2\pi}\right)^{3/2} \exp\left(-\frac{T_p}{T}\right),$$

$$N_n = \frac{2}{h^3}\left(\frac{m_n kT}{2\pi}\right)^{3/2} \exp\left(-\frac{T_n}{T}\right). \tag{37}$$

In this approximation, the neutron to proton number ratio is given by

$$\frac{N_n}{N_p} \simeq \exp\left(\frac{T_p - T_n}{T}\right) = \exp\left(-\frac{1.5}{T_{10}}\right). \tag{38}$$

The ratio therefore drops with temperature, from near 1:1 at $T \gtrsim 10^{12}$ K to about 5:6 at $T = 10^{11}$ K, and to 3:5 at 3×10^{10} K.

For thermodynamic equilibrium to be maintained, the reactions that convert neutrons to protons and vice-versa have to be rapid enough compared to the rate at which the universe expands. These interactions are none other than the weak interactions considered earlier when we discussed the decoupling of neutrinos from the rest of the primordial brew (see Section 3). There is one difference, however. In discussing the decoupling of neutrinos we were concerned mainly with the reaction of a neutrino with leptons like $e^{\pm}$, $\mu^{\pm}$, and the cross-section Σ given by (22) was determined for such interactions. Similarly, the reaction rate η given by (23) was obtained by multiplying by the number densities of participating leptons.

In the present case, the cross-section for a typical reaction like

$$\nu_e + n \leftrightarrow e^- + p$$

is larger than that for the pure leptonic reaction like

$$\nu_\mu + e \leftrightarrow \nu_e + \mu.$$

Also, the lepton densities used in (23) were considerably higher than the nucleon densities we are considering now. So the probability of a given nucleon interacting with any neutrino is higher than the probability of a given neutrino interacting with any nucleon. The result is that the effective temperature at which n and p cease to be in thermodynamic equilibrium is lower than the effective temperature for neutrino decoupling determined earlier.

Quantitatively, instead of $\Sigma \propto T^2$ as in (22), the cross-section in the present case goes as $\propto T$, and the effective decoupling temperature T_* at which the reaction rate is just about equal to H is $< 10^{10}$ K. Note that if the universe were expanding faster, T would be higher and the ratio N_n/N_p at decoupling as given by (38) would be higher.

Once the thermodynamic equilibrium ceases to be maintained, the N_n/N_p ratio is not given by (38) but by detailed consideration of specific reactions involving the nucleons.

As the universe cooled further, this ratio was therefore determined by the reactions that change protons to neutrons and vice-versa. These are essentially weak interactions of the type

$$n + \nu_e \leftrightarrow p + e^-, \qquad n \leftrightarrow p + e^- + \bar{\nu}_e.$$

The reaction rates are therefore determined by the cross-sections computed according to the weak interaction theory. Until the electro-weak gauge theory became established in the late 1970s, the *V-A* theory of weak interaction was used for these computations. We will not go into details of the calculation here, the purpose of which is to come up with a differential equation for the ratio

$$X_n = \frac{N_n}{N_p + N_n}. \tag{39}$$

If λ (n → p) denotes the rate at which neutrons are converted to protons and λ(p → n) the corresponding rate for protons changing to neutrons, then clearly X_n satisfies the equation

$$\frac{dX_n}{dt} = (1 - X_n)\cdot\lambda(p \to n) - X_n\cdot\lambda(n \to p). \tag{40}$$

The rates λ depend on distribution functions of leptons, which in turn depend on the temperature, which is related to the scale factor of the expanding universe. The integration of (40) has to be done numerically, and it is continued until all $e^{\pm}$ pairs have dropped out of the mixture – which happens at $T \gtrsim 10^9$ K.

When all $e^{\pm}$ have disappeared, it is still possible for the neutron to decay via the reaction

$$n \to p + e^- + \bar{\nu}_e$$

with a characteristic time $\tau = 1013$ s. So from the time the pairs disappear to the onset of nucleosynthesis, the neutron ratio X_n will decrease by the exponential factor $\exp(-t/\tau)$.

Thus, the ratio of neutrons to protons is uniquely determined at the time nucleosynthesis begins, once we know all the parameters of the weak interaction process. This is one good aspect of primordial nucleosynthesis theory, which we will now proceed to discuss.

5. The Synthesis of Helium and Other Nuclei

A typical nucleus Q is described by two quantities A = atomic mass and Z = atomic number, and is written*

$${}^{A}_{Z}Q.$$

This nucleus has Z protons and $(A-Z)$ neutrons. If m_Q is the mass of the nucleus, its binding energy is given by

$$B_Q = [Zm_p + (A - Z)m_n - m_Q]c^2. \tag{41}$$

Let us now consider a unit volume of cosmological medium containing N_N nucleons, bound or free. Since the masses of protons and neutrons are nearly equal, we may denote the typical nucleon mass by m. Thus $m_n \approx m_p = m$. If there are N_n free neutrons and N_p free protons in the mixture

$$X_n = \frac{N_n}{N_N}, \quad X_p = \frac{N_p}{N_N} \tag{42}$$

will denote the fractions by weight of free neutrons and free protons. If a typical bound nucleus Q has atomic mass A and there are N_Q of them in our unit volume, we may denote the weight fraction of Q by

$$X_Q = \frac{N_Q A}{N_N} \tag{43}$$

Now at very high temperatures ($T \gg 10^{10}$ K), the nuclei are expected to be in thermal equilibrium. However, even at these temperatures, $T \ll T_Q$ and (17) holds. Further, since we are now concerned with relative number densities, we can no longer ignore the chemical potentials. Thus,

$$N_Q = g_Q \left(\frac{m_Q kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{\mu_Q - m_Q c^2}{kT}\right), \tag{44}$$

where we reinstated the chemical potentials μ_Q. Since chemical potentials are conserved in nuclear reactions,

$$\mu_Q = Z\mu_p + (A - Z)\mu_n, \tag{45}$$

assuming that the nuclei were built out of neutrons and protons by nuclear reactions.

* Sometimes the suffix Z is suppressed.

The unknown chemical potentials can be eliminated between (44) and similar relations for N_p and N_n. The result is expressed in this form

$$X_Q = \tfrac{1}{2} g_Q A^{5/2} X_p^Z X_n^{A-Z} \xi^{A-1} \exp\left(\frac{B_Q}{kT}\right), \tag{46}$$

where

$$\xi = \tfrac{1}{2} N_N \left(\frac{mkT}{2\pi\hbar^2}\right)^{-3/2}. \tag{47}$$

For an appreciable buildup of complex nuclei, T must drop to a low enough value to make $\exp(B/kT)$ large enough to compensate for the smallness of ξ^{A-1}. This happens for nucleus Q when T has dropped down to

$$T_Q \sim \frac{B_Q}{k(A-1)|\ln \xi|}. \tag{48}$$

Let us consider what happens when we apply the above formula to the nucleus of ^{4}He. The binding energy of this nucleus is $\cong 4.3 \times 10^{-5}$ erg. If we substitute this value in (48) and estimate N_N from the presently observed value of nucleon density of around 10^{-6} cm^{-3}, we find that T_Q is as low as $\sim 3 \times 10^9$ K. However, at this low temperature the number densities of participating nucleons are so low that four-body encounters leading to the formation of ^{4}He are extremely rare. Thus, the underlying assumption of thermodynamic equilibrium (which requires frequent collisions) leading to (48) becomes invalid. We therefore need to proceed in a less ambitious fashion in order to describe the buildup of complex nuclei.

Hence, we try using two-body collisions (which are not so rare) to describe the buildup of heavier nuclei. Thus deuterium (d), tritium (^{3}H), and helium (^{3}He, ^{4}He) are formed via reactions like

$$\begin{aligned} &p + n \longleftrightarrow d + \gamma, \\ &d + d \longleftrightarrow {}^3\text{He} + n \longleftrightarrow {}^3\text{H} + p, \\ &{}^3\text{H} + d \longleftrightarrow {}^4\text{He} + n. \end{aligned} \tag{49}$$

Since formation of deuterium involves only two-body collisions, it quickly reaches its equilibrium abundance as given by

$$X_d = \frac{3}{\sqrt{2}} X_p X_n \xi \exp\left(\frac{B_d}{kT}\right). \tag{50}$$

However, the binding energy B_d of deuterium is low so that unless T drops to less than 10^9 K, X_d is not high enough to start further reactions leading to ^{3}H, ^{3}He, and ^{4}He. In fact, the reactions given in (49) with the exception of the first one do not proceed fast enough until the temperature has dropped to $\sim 8 \times 10^8$ K.

Although at such temperatures, nucleosynthesis does proceed rapidly enough, it cannot go beyond ^{4}He. This is because there are no stable nuclei with $A = 5$ or

8, and this means we cannot go on adding neutrons and protons to build nuclei heavier than ^{4}He. So the process terminates there. Detailed calculations by several authors have now established this result quite firmly.

So starting with primordial neutrons and protons, we end up finally with ^{4}He nuclei and free protons. All neutrons have been gobbled up by helium nuclei. Thus, if we consider the fraction by weight of primordial helium, it is very simply related to the quantity X_n – the neutron concentration before nucleosynthesis began. Denoting this fraction by weight by the symbol Y, we get

$$Y = 2X_n. \tag{51}$$

In Figure 1 the cosmic weight fractions of ^{4}He, ^{3}He, and ^{2}H and so on are plotted against a parameter η defined by

$$\eta = \left(\frac{\rho_0}{2.7 \times 10^{-26}\ \mathrm{g\,cm^{-3}}}\right)\left(\frac{3}{T_0}\right)^3. \tag{52}$$

Thus, η essentially measures the nucleon density in the early universe through the formula

$$\rho = \eta T_9^3, \qquad T_9 < 3. \tag{53}$$

Note that the ^{4}He weight fraction is insensitive to the parameter η. This is because, as we saw just now, it only depends on X_n; which in turn depends more critically on the epoch when the weak interactions rate fell below the expansion rate. If we go back to (38), we see that in the very early stages the neutron/proton ratio depends on temperature T_*. A faster expansion rate implies that the ratio becomes frozen at a higher temperature and so is higher, thus leading to a higher ^{4}He abundance. However, the expansion rate in the early stage does not depend sensitively on the parameter η. This is why the curve for ^{4}He in Figure 1 is nearly flat, with Y in the neighborhood of 0.25.

In contrast to the behavior of Y, the abundances of other nuclei critically depend on η. These abundances are very small compared to Y. Only deuterium and ^{3}He eventually survive; ^{3}H (tritium) decays to ^{3}He. Of nuclei heavier than ^{4}He, only ^{7}Li (lithium) appears with any appreciable quantities, although smaller than ^{3}He. The most interesting situation exists for deuterium, whose abundance sharply drops as η rises above 10^{-4}. For $T_0 = 3$ K, this corresponds to

$$\rho_0 \sim 2.7 \times 10^{-30}\ \mathrm{g\,cm^{-3}}. \tag{54}$$

Comparing this with the densities of Friedmann models, we see that for $h_0 = 1$, $\Omega_0 \lesssim 0.12$ and, hence, $q_0 \lesssim 0.06$. Here we have used the present Hubble constant H_0 as 100 h_0 km s^{-1} Mpc^{-1} and $\Omega_0 = \rho_0$ /closure density. q_0 is the deceleration parameter. Therefore, if even a small amount of deuterium believed to be primordial in origin were found, Friedmann models of the closed variety would be ruled out. There is, however, a loophole in this argument to which we will return later.

We can sum up by saying that Gamow's expectation that the early hot universe

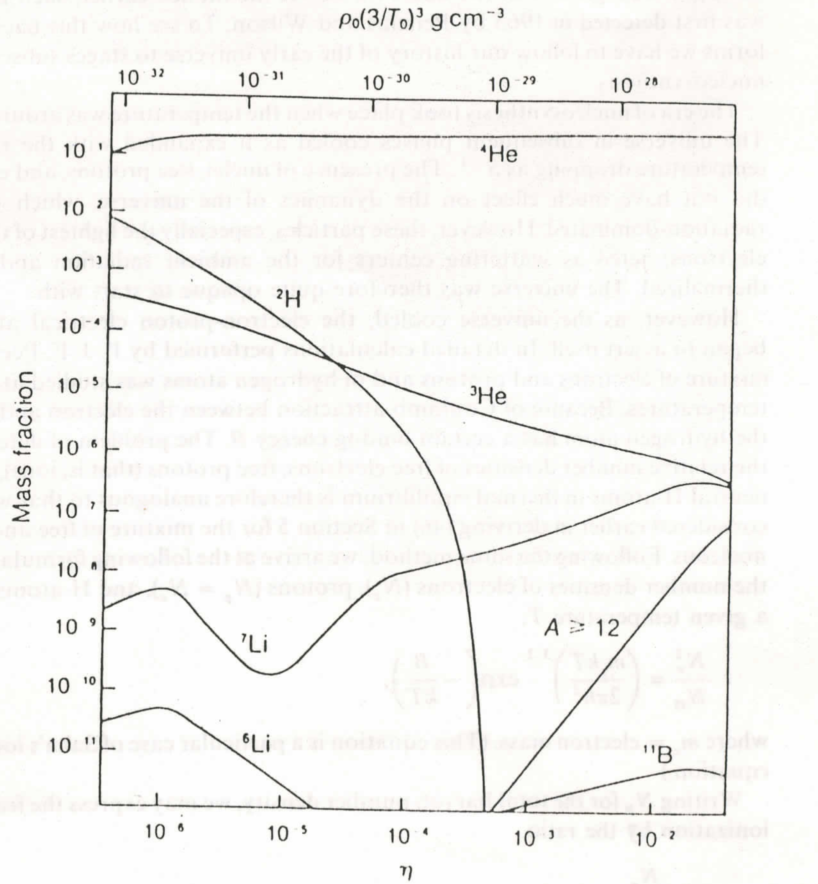

Fig. 1.

would synthesize all types of nuclei has been only partially fulfilled. To obtain complex nuclei heavier than ^{4}He (and possible ^{7}Li), astrophysicists have to look to other sources: the stars. (In Figure 1, the primordial production of nuclei with atomic weights exceeding 11 is shown by the curve $A \geqslant 12$.)

6. The Microwave Background

Gamow and his colleagues Alpher and Herman made another prediction, however, that appears to have received confirmation. This is the prediction that the photons of the early hot era would have cooled down to provide a thermal radiation background in the microwaves. As mentioned earlier, such radiation was first detected in 1965 by Penzias and Wilson. To see how this background forms we have to follow our history of the early universe to stages subsequent to nucleosynthesis.

The era of nucleosynthesis took place when the temperature was around 10^9 K. The universe in subsequent phases cooled as it expanded with the radiation temperature dropping as S^{-1}. The presence of nuclei, free protons, and electrons did not have much effect on the dynamics of the universe, which was still radiation-dominated. However, these particles, especially the lightest of them, the electrons, acted as scattering centers for the ambient radiation and kept it thermalized. The universe was therefore quite opaque to start with.

However, as the universe cooled, the electron-proton electrical attraction began to assert itself. In detailed calculations performed by P. J. E. Peebles, the mixture of electrons and protons and of hydrogen atoms was studied at varying temperatures. Because of Coulomb attraction between the electron and proton, the hydrogen atom has a certain bindng energy B. The problem of determining the relative number densities of free electrons, free protons (that is, ions), and the neutral H-atoms in thermal equilibrium is therefore analogous to that which we considered earlier in deriving (46) in Section 5 for the mixture of free and bound nucleons. Following the same method, we arrive at the following formula relating the number densities of electrons (N_e), protons ($N_p = N_e$), and H-atoms (N_H) at a given temperature T:

$$\frac{N_e^2}{N_H} = \left(\frac{m_e kT}{2\pi h^2}\right)^{3/2} \exp\left(-\frac{B}{kT}\right), \tag{55}$$

where m_e = electron mass. (This equation is a particular case of Saha's ionization equation.)

Writing N_B for the total baryon number density, we may express the fraction of ionization by the ratio

$$x = \frac{N_e}{N_B}.$$

Then, since $N_H = N_B - N_e$, we get from (55)

$$\frac{x^2}{1-x} = \frac{1}{N_B}\left(\frac{m_e kT}{2\pi h^2}\right)^{3/2} \exp\left(-\frac{B}{kT}\right) \tag{56}$$

For the H-atom, $B = 13.59$ eV. Substituting for various quantities on the right-hand side of (56), we can solve for x as a function of T. The results show that x drops sharply from 1 to near zero in the temperature range of ~ 5000 to 2500 K, depending on the value of N_B, that is, on the parameter $\Omega_0 h_0^2$. For example, for $\Omega_0 h_0^2 = 0.1$, $x = 0.003$ at $T = 3000$ K.

Thus, by this time most of the free electrons have been removed from the cosmological brew and, as a result, the main agent responsible for the scattering

of radiation disappears from the scene. The universe becomes effectively transparent to radiation. This is called the 'recombination epoch'.

The transparency of the universe means a light photon can go a long way ($\sim c/H$) without being absorbed or scattered. Therefore, this epoch signifies the beginning of the new phase when matter and radiation become decoupled. This phase has lasted up to the present epoch. During this phase, the frequency of each photon is redshifted according to the rule

$$\nu \propto \frac{1}{S},$$

while the number density of photons falls as

$$N_\gamma \propto \frac{1}{S^3}.$$

It is easy to see that under these conditions the photon distribution function preserves the Planckian form with the temperature dropping as

$$T \propto \frac{1}{S}.$$

A present background temperature of ~ 3 K therefore means that the epoch when matter decoupled from radiation corresponds to a redshift of $\sim 10^3$. However, we also find that the universe also changed over from being radiation-dominated to matter-dominated around the same epoch. Why the transition from opaqueness to transparency and from radiation domination to matter domination should take place around the same time is at present unexplained and must be considered a coincidence.

Another result, as yet unexplained by early universe physics, is the observed ratio of photons to baryons

$$\frac{N_\gamma}{N_B} = 4.57 \times 10^7 (\Omega_0 h_0^2)^{-1} \left(\frac{T_0}{3}\right)^3 \tag{57}$$

This ratio has been conserved since the time the universe became essentially transparent, although both N_γ and N_B can be studied theoretically at even earlier epochs. Why the above ratio and no other? Later we will discuss some ideas from particle physics that are intended to throw light on this mystery.

The important signature of the relic radiation is, however, its spectrum. In addition, we will consider a few effects that may cause small perturbations of the radiation background. But these apart, we should find the background spectrum to be very close to the Planckian form. Observations to date confirm this prediction with $T_0 \cong 2.7$ K.

7. Anisotropies of the Microwave Background

Assuming that the microwave background is the relic of a hot big bang, there are two types of anisotropies that, if found, should give us clues to the early history of the universe, especially about the recombination era. They are discussed below.

7.1. *Small-Angle Anisotropy*

Theories of galaxy formation place lower limits on the fluctuations of $\delta\rho/\rho$, the density contrast at the recombination epoch. Assuming that the fluctuations are adiabatic, the particle number density will vary as the cube of the radiation temperature, therefore

$$\left(\frac{\delta T}{T}\right)_R = \frac{1}{3}\left(\frac{\delta\rho}{\rho}\right)_R, \tag{58}$$

where the subscript R denotes the recombination epoch.

Since the universe is optically thin after this epoch, these fluctuations will be imprinted on the radiation background and would be observed to this day. That is, if we sweep across the sky we should see ups and downs in the background temperature. What should be the order of magnitude of this fluctuation in temperature at the present epoch? Over what characteristic angular size should we observe these fluctuations?

Simple theories of galaxy formation suggest that we should have present-day fluctuations of $(\delta T/T)$ in the range $\sim 3 \times 10^{-3}$ to 10^{-4}. This is of course true on the assumption of optical thinness mentioned before.

The typical angular size of the fluctuation is

$$(\Delta\theta) \cong 23\left(\frac{M}{10^{11} M_{\odot}}\right)^{1/3} (h_0 q_0^2)^{1/3} \text{ arcsec}, \tag{59}$$

where M is the typical galaxy mass. Thus, galaxy formation should leave a characteristic patchiness of the angular size ~ 20 arcsec. Actual observations, however, reveal no fluctuations down to $\Delta T/T < 5 \times 10^{-5}$. These null observations severely constrain the theories of galaxy formation.

7.2. *Large-Angle Anisotropy*

Particle horizons restrict the distances over which physical signals can travel. It could be argued that the universe may not have been homogeneous to start with (as presumed by the cosmological principle), but that it achieved homogeneity by physical transport of energy and momentum. In that case, at any given epoch the size of a homogeneous region cannot exceed the diameter of the particle horizon at that epoch.

Since the last time that there occurred a thorough mixing of the radiation background was at the epoch of redshift z_R, it would be relevant to ask what the size of the particle horizon was at that epoch. Calculations show that the diameter of the horizon was

$$d_H = \frac{4c}{H_0(1+z_R)\sqrt{2q_0 - 1}} \sin^{-1}\sqrt{\frac{2q_0 - 1}{2q_0(1+z_R)}} \tag{60}$$

for the model with $q_0 > 1/2$. Similar results can be obtained for $q_0 \leqslant 1/2$. The angle θ_H subtended by the horizon at us is given by

$$\sin\frac{\theta_H}{2} = \frac{q_0\sqrt{1 + 2q_0 z_R}}{q_0 z_R + (q_0 - 1)\{\sqrt{1 + 2q_0 z_R} - 1\}}. \tag{61}$$

For large redshift, $z_R \sim 1000$, (61) can be approximated to give

$$\theta_H \sim 2\sqrt{\frac{2q_0}{z_R}} \sim 5^{\circ}\sqrt{q_0}. \tag{62}$$

The small angle coming out of this analysis implies that if the radiation background was thermalized at an early epoch such as $z_R = 1000$, the smallness of particle horizons should lead to a patchiness in the present intensity distribution across the sky. Is such patchiness observed? Again, observations show a null result. Thus, the observed smoothness of the microwave background is at present a great embarrassment to theorists.

8. Cosmology and Particle Physics

We have so far discussed the properties of the big bang universe starting from the epoch in which it was $\sim 10^{-4}$ sec old, when a mixture of baryons, mesons, leptons, and photons was in thermodynamic equilibrium with a temperature of $\sim 10^{12}$ K. We discussed how this hot primordial gas evolved as the universe expanded and cooled down. We ended our story with the formation of the helium nucleus, by which time the universe was ~ 3 minutes old.

In the 1960s the above range of epochs would have been considered as describing the early universe. Today the implication of this phrase has changed. The 'early universe' now implies the era preceding the above phase, when matter was in an even more elementary form than that considered above. The reason for this shift lies less in any development in cosmology than in particle physics. The remarkable developments in particle physics, which signify progress towards a unification of the basic interactions of physics, have found their echoes in cosmology.

So far, particle physicists have relied on the use of powerful accelerators to study the interaction of particles at high energy. From elementary quantum theory, it follows that to be able to probe smaller and smaller distances, higher and higher momenta must be achieved. Thus, high-energy accelerators are required in order to probe the structure of particles like the proton or the pion. The present accelerators achieve energies of the oder of a few tens or hundreds of GeV(1 GeV ≡ 10^9 eV).

In theories of unification, however, interesting phenomena are predicted at energies ~ 10^{15} GeV. Energy ranging as high as this value is far beyond what could be achieved by present technology.

It is against this background that particle physicists have turned to cosmology in the realization that the early hot universe is the poor man's high-energy accelerator. This is not the first time physicists have turned to astronomy in order to study the behavior of basic physical processes under conditions unattainable in a terrestrial laboratory. Even before thermal fusion could be achieved on the Earth, physicists were studying the process inside stars.

Naturally, the interplay of cosmology and particle physics that we plan to discuss in this section in highly speculative on both fronts. It depends on the validity of the cosmological model and on the viability of (as yet fluid) ideas of particle physics. This should be borne in mind throughout the various calculations given here.

Let us first consider what particles might exist in the early universe, out of which the baryons and mesons are formed. This information is supplied by particle physics.

The masses of particles like quarks, leptons etc. are expressed in the unit of MeV (1 MeV ≡ 10^6 eV). We have so far not introduced this unit. It is convenient to do so now, since we shall be using many ideas from particle physics where this unit is commonly used. Thus, for each mass m expressed in grams, mc^2 is energy expressed in ergs. We then use the following conversion scale:

$$1 \text{ MeV} = 1.6021917 \times 10^6 \text{ erg}.$$

Further, since we are going to describe the hot universe, it is also convenient to express the temperature in the same unit. Thus, for T expressed in Kelvin, kT is energy expressed in ergs, which can be written in units of MeV. We therefore have

$$1 \text{ gram} = 5.618 \times 10^{26} \text{ MeV},$$

$$1 \text{ Kelvin} = 8.617 \times 10^{11} \text{ MeV}.$$

Although these conversion factors involve many powers of 10, they show why the MeV is a good unit for the early universe. For example, a temperature of the order of 10^{12} K is a few MeV.

We end this section by recalling from earlier work the result that relates the temperature of the universe to its age and is given by the Einstein equation

$$\frac{\dot{S}^2}{S^2} = \frac{8\pi G}{3}\rho. \tag{63}$$

If there are bosons with a total g_b of g-factors and fermions with a total g_f of

g-factors, then

$$\rho c^2 = \tfrac{1}{2} g a T^4 \tag{64}$$

with

$$g = g_b + \tfrac{7}{8} g_f. \tag{65}$$

Thus, we have for g = constant

$$S \propto t^{1/2} \tag{66}$$

with

$$t = \left(\frac{3c^2}{16\pi G a}\right)^{1/2} g^{-1/2} T^{-2}. \tag{67}$$

This relation can be expressed in MeV as

$$t_{\text{second}} = 2.4\, g^{-1/2}\, T_{\text{MeV}}^{-2}. \tag{68}$$

9. Survival of Massive Particles

We will begin with a simple extrapolation of the approach adopted earlier. We will assume in this section that quarks have combined to form particles (and antiparticles) and investigate the criteria that determine the survival of a particular species of particles. In the ideal gas approximation, we will assume the distribution functions to be those given by (9). In the relativistic (high-temperature) approximation of Section 2, we have the following formula for a number density of particles of species A:

$$N_A = \eta g_A N_\gamma = \eta g_A \frac{2.4}{\pi^2}\left(\frac{kT}{ch}\right)^3, \tag{69}$$

where N_γ is the number density of photons and $\eta = 1/2$ for bosons and $\frac{3}{8}$ for fermions. In the nonrelativistic approximation, we get

$$N_A = \frac{g_A}{h^3}\left(\frac{m_A kT}{2\pi}\right)^{3/2} \exp\left(-\frac{m_A c^2}{kT}\right). \tag{70}$$

The assumption leading to (69) or (70) is that the species is in thermodynamic equilibrium with the rest of the particles. For (69) to hold we need $T \gg T_A \equiv m_A c^2/k$, while for (70) to hold we should have $T \ll T_A$. Exactly similar results must hold if species A has antiparticles $\bar{A}$. To fix ideas (since we are eventually going to use these formulae for baryons-protons and neutrons), we will assume A to be a fermion. Thus, $\eta = 3/8$.

In general, A and $\bar{A}$ may annihilate if they are brought together. In a typical reaction, two photons will be produced

$$A + \bar{A} \to \gamma + \gamma. \tag{71}$$

A comparison of the reaction rate with the rate of expansion of the universe finally yields the surviving baryon to photon ratio as

$$\frac{N_A}{N_\gamma} \simeq 2 \times 10^{-18}. \tag{72}$$

Compare this with the observed range:

$$\frac{N_A}{N_\gamma} \simeq 2 \times 10^{-8} (\Omega_0 h_0^2)\left(\frac{T_0}{3}\right)^{-3}. \tag{73}$$

Since $T_0 \sim 3$ and $\Omega_0 h_0^2$ is not expected to be lower than $\sim 10^{-3}$ under the most extreme case, we have a large discrepancy to account for. There is one further point of criticism. If we are sure that the universe is made up predominantly of matter, then $N_A \gg N_{\bar{A}}$ and the formula (73) applies $N_A (\cong N_A - N_{\bar{A}} \cong$ baryon number density). However, our analysis so far is symmetric between matter and antimatter and so leads to $N_A = N_{\bar{A}}$. Clearly new inputs are necessary in the discussion given above if we are to understand why $N_A \gg N_{\bar{A}}$ and why N_A/N_γ is as high as indicated by (73).

10. Problems of the Very Early Universe

The baryon to photon ratio described above poses a problem to be solved by the scenarios purporting to describe the very early universe. It is clear that new inputs are needed to understand the observed ratio N_A/N_γ.

On the observational side, our understanding of the material composition of the universe is largely based on the data provided by the electromagnetic radiation. Since this radiation treats matter and antimatter in a symmetrical manner, the distinction between the two cannot be made by this method. Cosmic rays in the Galaxy and the Local Group of galaxies may be used to argue that we are living in a matter dominated area. But for remote parts of the universe, the assumption of a net baryon number corresponding to (73) is largely a conjecture.

Assuming that (73) is characteristic of the whole universe, we see that new inputs are needed, such as (i) the possibility of baryon nonconserving processes in particle physics, (ii) the probable lack of thermodynamic equilibrium in the processes of the very early universe, and (iii) a likely asymmetry between matter and antimatter at very high energies.

The Grand Unified Theories (GUTs) and other versions of unification attempts are therefore likely to play an important role in explaining N_A/N_γ. For example, the simple SU(5) model has been used to bring this ratio in the range $10^{-12} - 10^{-4}$ which includes the observed range (73). However, since the GUTs themselves are in a state of flux so far as their final form is concerned, it is too early to say that the problem of explaining N_A/N_γ has been solved.

The other important problems of the very early universe are briefly discussed below.

10.1. *The Horizon Problem*

In the very early universe, the size of the particle horizon was exteremely small. At time $t > 0$ the particle horizon restricts the range of communication to $\sim ct$. At $T = 10^{15}$ GeV, $g \simeq 100$ (values characteristic of GUTs-epoch) we get from (68),

$$t \sim 10^{-37}\ \text{s}.$$

Correspondingly, the horizon range is $R_H \sim 3 \times 10^{-27}$ cm. As the temperature dropped from 10^{15} GeV to the present value of 3 K $\sim 2.6 \times 10^{-13}$ GeV, the size R_H would have expanded to

$$R_H \times \frac{10^{15}}{2.6 \times 10^{-13}} \sim 10\ \text{cm}.$$

Compared to the present size, $\sim 10^{28}$ cm, this is extremely small; thus highlighting the fact that unless the initial composition of the universe was homogeneous, the horizon cannot allow homogeneity to be established later. Clearly a departure from the dynamics of standard model is needed to eliminate the horizon effect. Later articles will offer solutions through inflation (Panchapakesan, Chapter 17) and quantum cosmology (Padmanabhan, Chapter 18).

10.2. *The Flatness Problem*

Consider the field equation

$$\frac{\dot{S}^2}{S^2} + \frac{kc^2}{S^2} = \frac{8\pi G\rho}{3}. \tag{74}$$

In the early universe scenario, we neglected the 'curvature term' kc^2/S^2 in comparison with the dynamical term $\dot{S}^2/S^2$. The ratio of the two terms

$$F = \frac{kc^2/S^2}{\dot{S}^2/S^2} = \frac{kc^2}{\dot{S}^2} \to 0, \tag{75}$$

as $\dot{S} \to \infty$ in standard big bang models: so the approximation was justified.

However, as first pointed out by Dicke and Peebles in 1979, the ratio F is comparable to unity at present and this corresponds to an extremely small value at $t = 1$ s. For $t = 10^{-37}$ s, F may be as small as 10^{-50}. Thus, the present state of the universe seems to have arisen from an extreme fine tuning of the curvature term to the value zero. In other words, all observationally permitted Friedmann models seem to be bunched round the flat $k = 0$ model to within a very narrow range. Why should this be so? Again inflation and quantum cosmology will offer alternative scenarios to explain this effect.

10.3. *The Monopole Problem*

GUTs seem to imply the existence of a magnetic monopole type solution. Monopoles appear to be inevitable consequences of GUTs and once created they are hard to get rid of. Their present density can be estimated as follows.

Since the particle horizon restricts a monopole solution to a region $\lesssim R_H$, the monopole number density at $t = 10^{-37}$ s was $\sim R_H^{-3}$. By expansion, this should fall to $(10\,\text{cm})^{-3}$, as we saw earlier. But the mass of the monopole is $\sim 10^{16}$ GeV, i.e., 0.2×10^{-7} gm. This gives us the monopole mass density at the present epoch as

$$\rho_M \simeq 2 \times 10^{-11}\,\text{gm cm}^{-3}.$$

This is far greater than the closure density of $\sim 10^{-29}$ gm cm^{-3}. Clearly there is something wrong with the monopole survival scenario! The inflationary universe offers a way out of this difficulty.

10.4. *Galaxy Formation*

Despite numerous attempts the detailed scenario of galaxy formation still eludes the theoreticians. Considerable complexity and arbitrariness has now entered the picture because (a) there is some astronomical evidence that there is dark matter in the universe, probably in considerable excess over the visible matter and (b) the GUTs and SUSY offer several candidates for dark matter, e.g. massive neutrinos, gravitinos, photinos, etc., as well as other massive particles that may decay in short timescales. In addition, the inflationary scenarios of various kinds bring fresh inputs to the galaxy-formation problem.

The observed smoothness of the microwave background is still the biggest stumbling block to such theories. In addition, there appears to be problem from massive neutrinos also in that their background induces galaxy formation on a much more clumpy state than observed.

10.5. *Massive Neutrinos*

Recent experiments in the U.S.S.R. suggest that neutrinos may indeed have a small rest mass. This possibility opens up a number of interesting astrophysical consequences. As early as 1972, R. Cowsik and J. McClelland had conjectured

that the 'missing mass' in the universe (that is, the difference between ρ_0 and ρ_N) may be accounted for by relic neutrinos. What can we say today about such a possibility? (ρ_N = mass density of nucleons.)

Let us do the calculations taking $g = 1$ even for massive neutrinos. If the rest mass of the neutrino is larger than $\sim 2 \times 10^4$ eV, they will have small random velocities today. Since experiments suggest m of the order of a few electron volts, we will write

$$m_\nu = M_\nu(\text{eV}).$$

From Table I we know that the number density of neutrinos is 3/8 of the number density of photons of the same temperature. We also know that the number density of photons goes as the cube of the photon temperature. Since in the post $(e^+ - e^-)$ annihilation phase

$$\left(\frac{T_\nu}{T_\gamma}\right)^3 = \frac{4}{11}, \tag{76}$$

we get the present number density of neutrinos as

$$\left(\frac{N_\nu}{N_\gamma}\right)_0 = \frac{3}{22}. \tag{77}$$

Putting everything together, the mass density of neutrinos at present may be expressed as

$$\rho_\nu = \sum \Omega_\nu \rho_c, \tag{78}$$

where Σ denotes sum over all types of neutrinos and

$$\Omega_\nu = \frac{M_\nu}{150}\left(\frac{T_0}{3}\right)^3 h_0^{-2}. \tag{79}$$

Comparing (54) with (79), we see that $2\Omega_\nu$ exceeds the upper limit on Ω_ν even for such a modest value of M_ν as 1.5. If we include all neutrino species, we get the above result as

$$\sum_{\substack{\text{all}\\ \text{species}}} m_\nu \gtrsim 1.5\,\text{eV}.$$

Very massive neutrinos will prove embarrassing for big-bang cosmology. If all neutrinos (which means $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$), have on average a mass of ~ 25 eV, then $\Sigma\Omega_\nu$ is close to 1. Larger masses than this value and/or increase in the number of relic neutrino species would increase $\Sigma\Omega_\nu$ and the overall Ω beyond the closure value $\Omega = 1$. As seen in general, closed universes have shorter ages, and an overall age $< \sim 6 \times 10^9$ years may be embarrassingly small in reality. It has been suggested that under such circumstances, λ-cosmologies might have to be invoked.

These calculations illustrate how astrophysics may provide valuable constraints on properties of elementary particles. We end with another such constraint coming from neutrinos.

In Sections 4 and 5 we found that the primordial helium production depends on the terminal value of N_n/N_p, and that this value goes up if the universe were expanding faster at that epoch. Thus, it could happen that if there were more neutrino flavours (say ≥ 3) then the value of the g-factor would go up and the universe would expand faster so that the primordial value of Y would exceed all reasonable bounds from observations. Thus, cosmologists impose a restriction on particle theories if their picture of the early universe is to be valid.

References

The early work on primordial nucleosynthesis

G. Gamow, Expanding universe and the origin of elements, *Phys. Rev.* 70, 572 (1946).

R. A. Alpher and R. C. Hermann, Evolution of the universe, *Nature* 162, 774 (1948).

R. A. Alpher, H. A. Bethe, and G. Gamow, The origin of chemical elements, *Phys. Rev.* 73, 80 (1948). [This paper, with the sequence of authors Alpher/Bethe/Gamow, led to the name '$\alpha/\beta/\gamma$ theory'.]

Stellar nucleosynthesis

G. R. Burbidge, E. M. Burbidge, W. A. Fowler, and F. Hoyle, Synthesis of the elements in stars, *Rev. Mod. Phys.* 29, 547 (1957).

Later work on primordial nucleosynthesis

C. Hayashi, Proton-neutron concentration ratio in the expanding universe at the stages preceding the formation of the elements, *Progr. Theoret. Phys. (Japan)* 5, 224 (1950).

G. R. Burbidge, Nuclear energy generation and dissipation in galaxies, *PASP* 70, 83 (1958).

F. Hoyle and R. J. Tayler, The mystery of cosmic helium abundance, *Nature* 203, 1108 (1964).

P. J. E. Peebles, Primordial helium abundance and the primordial fireball, *Ap. J.* 146, 542 (1966).

Ya B. Zeldovich, The 'hot' model of the universe, *Usp. Fiz. Nauk* 89, 647 (1966).

R. V. Wagoner, W. A. Fowler, and F. Hoyle, On the synthesis of elements at very high temperatures, *Ap. J.* 148, 3 (1967).

D. Schramm and R. V. Wagoner, Element production in the early universe, *Ann. Rev. Nucl. Sci.* 27, 37 (1977).

Discovery of the microwave background

A. A. Penzias and R. W. Wilson, Measurement of excess antenna temperature at 4080 Mc/s, *Ap. J.* 142, 419 (1965) (see also the textbooks at the end).

Growth of fluctuations

E. Lifshitz, On the gravitational instability of the expanding universe, *J. Phys. (USSR)* 10, 116 (1946).

Analytical approaches to galaxy formation

D. Lynden-Bell, Statistical mechanics of violent relaxation in stellar systems, *Monthly Notices Roy. Astron. Soc.* 136, 101 (1967).

P. J. E. Peebles, Origin of the angular momentum of galaxies, *Ap. J.* 155, 393 (1969).

J. E. Gunn and J. R. Gott, On the infall of matter into clusters of galaxies, *Ap. J.* 176, 1 (1972).

R. A. Sunyaev and Ya. B. Zeldovich, Formation of clusters of galaxies; protocluster fragmentation and intergalactic gas heating, *Astron. Astrophys.* 20, 189 (1972).

S. D. M. White and M. J. Rees, Core condensation in heavy halos: a two stage theory of galaxy formation and clustering, *Monthly Notices Roy. Astron. Soc.* 184, 643 (1978).

A. G. Doroshkevich, E. M. Saar, and S. F. Shandarin, Spatial structure of protoclusters and the formation of galaxies, *Monthly Notices Roy. Astron. Soc.* 184, 643 (1978).

N-body approaches to galaxy formation

A. Toomre, Mergers and some consequences, in R. B. Larson and B. Tinsley (eds.), *Evolution of Galaxies and Stellar Populations*, Yale University Observatory, New Haven, p. 401.

S. J. Aarseth, J. R. Gott, and E. L. Turner, N-body simulations of galaxy clustering, *Ap. J.* 228, 664 (1979).

G. Efstathiou and B. J. T. Jones, The rotation of galaxies: numerical investigations of the tidal torque theory, *Monthly Notices Roy. Astron. Soc.* 186, 133 (1979).

Clustering of galaxies

H. Totsuji and T. Kihara, The correlation function for the distribution of galaxies, *Publ. Astron. Soc. (Japan)* 21 221 (1969).

P. J. E. Peebles, The gravitational instability picture and the nature of distribution of galaxies, *Ap. J.* 189, L51 (1974).

S. J. Aarseth, J. R. Gott, and E. L. Turner, N-body simulations of galaxy clustering, *Ap. J.* 228, 664 (1979).

Nonrelic microwave background

F. Hoyle, N. C. Wickramasinghe, and V. C. Reddish, Solid hydrogen and the microwave background, *Nature* 218, 1124 (1968).

J. V. Narlikar, M. G. Edmunds, and N. C. Wickramasinghe, Limits on a microwave background without the big bang, in M. Rowan-Robinson, (ed.), *Far Infrared Astronomy*, Pergamon Press, New York, p. 131.

M. J. Rees, Origin of pregalactic microwave background, *Nature* 275, 35 (1978).

M. Rowan-Robinson, J. Negroponte, and J. Silk, Distortions of the cosmic microwave background spectrum by dust, *Nature* 281, 635 (1979).

N. C. Rana, Cosmic thermalization and the microwave background radiation, *Monthly Notices Roy. Astron. Soc.* 197, 1125 (1981).

Baryon excess in the early universe

G. Steigman, Observational tests of antimatter cosmologies, *Ann. Rev. Astron. Astrophys.* 14, 339 (1976).

M. Yoshimura, Unified gauge theories and the baryon number of the universe, *Phys. Rev. Lett.* 41, 281 (1978).

S. Weinberg, Baryon-lepton non-conserving processes, *Phys. Rev. Lett.* 43, 1566.

F. Wilczek and A. Zee, Operator analysis of nucleon decay, *Phys. Rev. Lett.* 43, 1571 (1979).

Helium abundance and neutrino types

J. Yang, D. Schramm, G. Steigman, and R. T. Rood, Constraints on cosmology and neutrino physics from big bang nucleosynthesis, *Ap. J.* 227, 697 (1979).

Experimental data on massive neutrinos

V. A. Lyubimov, E. G. Novikov, V. Z. Nozik, E. F. Tretyakov, and V. S. Kozik, Brighton conference on High Energy Physics (1983).

Massive neutrinos and cosmology

R. Cowsik and J. McClelland, An upper limit on the neutrino rest mass, *Phys. Rev. Lett.* 29, 669 (1972).

R. Cowsik and J. McClelland, Gravity of neutrinos of nonzero mass in astrophysics, *Ap. J.* 180, 7 (1973).

S. Tremaine and J. E. Gunn, Dynamical role of light neutral leptons in cosmology, *Phys. Rev. Lett.* 42, 407 (1979).

D. Schramm and G. Steigman, Relic neutrinos and the density of the universe, *Ap. J.* 243, 1 (1981).

Problems of the very early universe

A. D. Linde, The inflationary universe, *Rep. Progr. Phys.* 47, 925 (1984).

Textbooks on nucleosynthesis and big bang cosmology

D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis*, McGraw-Hill, New York (1968).

P. J. E. Peebles, *Physical Cosmology*, Princeton University Press, Princeton (1971).
S. Weinberg, *Gravitation and Cosmology*, Wiley, New York (1972).
P. J. E. Peebles, *The Large-Scale Structure of the Universe*, Princeton University Press, Princeton (1980).
J. V. Narlikar, *Introduction to Cosmology*, Jones and Bartlett, Boston (1983).

6. An Approach to Anisotropic Cosmologies

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1. Motivation

It is perhaps not wrong to consider Einstein's 1917 paper on cosmology as the beginning of modern cosmology. It was a peculiar beginning for a branch of physics. Normally, one has some observational data and the theoretician goes on to build up a structure with a minimum of *ad-hoc* assumptions which help to systematize and 'explain' the data. In 1917, Einstein had hardly any observational data to explain and the little he had, he chose to ignore. Even at that time, it was clear that processes are occurring on a large scale leading to a flow of radiation from celestial bodies and, thanks to the ideas of the special theory of relativity, this could be understood as a continuous change in the distribution of sources of gravitation. Besides, it was already known that matter and radiation, even of the same energy density, differ in their energy-stress tensor. All this was enough to indicate that the universe, even if in equilibrium at the same stage, would not continue in that state. Yet Einstein adopted the assumption of a static universe which was also uniform in space and isotropic.

Later, the assumption of static nature had to be given up to accommodate the Hubble shift of spectral lines from distant galaxies, but the assumptions of spatial homogeneity and isotropy still form the cornerstone of standard cosmology. Indeed, while for a long time there was little empirical evidence in favour of these assumptions, the microwave radiation background is commonly thought to have brought powerful support for the isotropic models.

But if standard cosmology were completely successful, there would hardly be any need to explore other models of the universe, except perhaps for mathematical recreation. Here and there, however, doubts and difficulties remain and anisotropic models have been investigated at different stages in the hope that they may smooth out these difficulties. We just make mention of the problems that plague standard cosmology and give the motivation for the study of anisotropic models:

(a) *The big bang singularity*
Standard cosmology gives, for the metric of the universe*,

* In the following equation and in some of the subsequent ones, the upper and the lower signs correspond, respectively, to the metric signature $(+ - - -)$ and $(- + + +)$.