Assumptions of the primordial spectrum and cosmological parameter estimation

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The observables of the perturbed universe, CMB anisotropy and large structures, depend on a set of cosmological parameters, as well as, the assumed nature of primordial perturbations. In particular, the shape of the primordial power spectrum (PPS) is, at best, a well motivated assumption. It is known that the assumed functional form of the PPS in cosmological parameter estimation can affect the best fit parameters and their relative confidence limits. In this letter, we demonstrate that a specific assumed form actually drives the best fit parameters into distinct basins of likelihood in the space of cosmological parameters where the likelihood resists improvement via modifications to the PPS. The regions where considerably better likelihoods are obtained allowing free form PPS lie outside these basins. In the absence of a preferred model of inflation, this raises a concern that current cosmological parameters estimates are strongly prejudiced by the assumed form of PPS. Our results strongly motivate approaches toward simultaneous estimation of the cosmological parameters and the shape of the primordial spectrum from upcoming cosmological data. It is equally important for theorists to keep an open mind towards early universe scenarios that produce features in the PPS.

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1. INTRODUCTION

Precision measurements of anisotropy and polarization in the Cosmic Microwave Background (CMB), in conjunction with observations of the large scale structure, suggest that the primordial density perturbation is dominantly adiabatic and has a nearly scale invariant spectrum [1, 2]. This is in good agreement with most simple inflationary scenarios which predict nearly power law or scale invariant forms of the primordial perturbation [3, 4, 5]. However, despite the strong theoretical appeal and simplicity of a featureless primordial spectrum, our results highlight that the determination of the shape of the primordial power spectrum directly from observations with minimal theoretical bias would be a critical requirement in cosmology.

The observables of the perturbed universe, such as, CMB anisotropy galaxy surveys and weak lensing etc., all depend on a set of cosmological parameters describing the current universe, as well as, the parameters characterizing the presumed nature of the initial perturbations. While certain characteristics of the initial perturbations, such as, the adiabatic nature, tensor contribution, can, and are, being tested independently, the shape of the primordial power spectrum remains, at best, a well motivated assumption.

It is important to distinguish between the cosmological parameters that describe the present universe, from that characterizing the initial conditions, specifically, the PPS, $P(k)$. However, it is prevalent in cosmological parameter estimation to treat the two sets identically. Based on sampling on a coarse grid in the cosmological parameter space, we have already shown that the CMB data is sensitive to the PPS [6]. The best fit cosmological parameters with free form PPS have much enhanced likelihoods and the preferred regions significantly separated from the best fit parameters obtained with assumed power-law PPS. It is also known that specific features in the PPS can dramatically improve the fit to data (eg. ref. [7], and references therein).

In this letter, we bring forth another important issue introduced by our prior ignorance about the PPS. While, known correlations between cosmological parameters is always folded into parameter estimation, the analogous situation for $P(k)$ is not as widely appreciated. An assumed functional form for the PPS, is equivalent to an analysis with a free form PPS, where, say, $P(k)$, is estimated in separate k bins, but, then one imposes strong correlation between the power in different bins. As we show in this work, the assumed form (equivalently, the implied correlations in $P(k)$ at different k,) drive the significant number of degrees of freedom available in the cosmological parameters to adjust into suitable specific combinations. Hence, the assumed form of PPS could be dominant in selecting the best fit regions (eg., ref. [8] where features in the PPS lead to very different best fit cosmological parameters). For specific functional forms, the corresponding best-fit models lie entrenched in distinct basins in the parameter space. Our results show that in these basins, the likelihood is remarkably robust to variations in the PPS. We conclude that there are sufficient degrees of freedom in the cosmological parameters to mould the fit around the constraints imposed by the assumed form of the PPS.

In this letter, we elucidate this issue in the context of CMB data from WMAP for three different well known assumed forms of the primordial spectrum, $P(k)$, (i.) scale-invariant Harrison-Zeldovich (HZ), (ii.) scale-free Power-law (PL) and, (iii.) Power law with running(RN). The methodology and analysis is described in §2. The results are given in §3 and we give the conclusion of our work in §4.

2. METHOD AND ANALYSIS

The angular power spectrum of CMB anisotropy, C_l, is a convolution of the PPS, $P(k)$ generated in the early universe with a radiative transport kernel, $G(l,k)$, determined by the current values of the cosmological parameters. The precision measurements of C_l, and the concordance of cosmological parameters measured from other cosmological observations allow the possibility of direct recovery of $P(k)$ from the observations. In our analysis we use an improved (error-sensitive) Richardson-Lucy (RL) method of deconvolution to reconstruct the optimized primordial power spectrum at each point in the parameter space [6, 9, 10, 11, 12]. The RL based method has been demonstrated to be an effective method to recover $P(k)$ from C_l measurements [6, 11, 12] (look at ref. [13] for some other reconstruction methods).

In this letter, we study the improvement in likelihood allowed by an 'optimal' free-form PPS at points in the cosmological parameter space around the best-fit region for the three different assumed form of PPS, viz., Harrison-Zeldovich (HZ), power-law (PL) and power-law with running (RN). We apply our deconvolution method to reconstruct an 'optimal' form of the PPS at each point [6].

Markov-chain Monte-Carlo (MCMC) samples of parameters provide a fair sampling of the parameter space around the best-fit point. We use the MCMC chains generated based on 3 year data by the WMAP team for parameter estimation with HZ, PL and RN forms of the PPS. We reconstruct the optimized PPS for each point of these chains and obtain the 'optimal' PPS likelihood based on the reconstructed spectrum.

We limit our attention to the flat ΛCDM cosmological model and consider the four dimensional parameter space, H_0, τ, Ω_b and Ω_{0m}. This corresponds to a minimalistic "Vanilla Model", a flat ΛCDM parametrized by six parameters (n_s, A_s, H_0, τ, Ω_b, Ω_{dm}). In case of Harrison-Zeldovich (HZ) PPS assumption, $n_s = 1$, leaving only 5 parameters. The case of assuming a constant running in the spectral index (RN), n_{run}, leads to 7 parameters. The dimensionality in the three models is different solely due to the parameters of the assumed PPS. Hence, in our analysis we always have a four-dimensional space of cosmological parameters (since we recover the optimal PPS).

In order to represent the likelihood in a four dimensional parameter space we find it convenient to define a normalized distance, ρ, between two points,

$$\rho(a,b) = \sqrt{\Sigma_i (P_i^a - P_i^b)^2/(\sigma_i^b)^2}, \tag{2.1}$$

where P_i^a and P_i^b are the value of i^{th} cosmological parameter at point 'a' and point 'b', respectively. To ensure that equal separations along different parameters have a similar meaning, we divide $P_i^a - P_i^b$ by standard deviation σ_i^b at point 'b'. We assign point 'b' to be a best fit point where σ_i^b are the 1σ confidence limits derived by WMAP team from the corresponding MCMC chains. Since we are primarily interested in studying the region around the best-fit point, ρ provides a convenient definition of distances to other points with respect to it. (Note, the 'distance' ρ is 'asymmetric' in 'a' and 'b' when $\sigma_i^b \neq \sigma_i^a$ and should interpreted accordingly).

3. RESULTS

The simplest characterization of the likelihood landscape, $\mathcal{L}(P_i)$, around the best-fit point is to study its behavior as ρ increases with separation from the best-fit point. The trend in the likelihood can then be compared for two cases – assuming a form of primordial spectrum, or allowing a optimal free form. (We use the effective chi-square, $\chi^2 \equiv -2\ln\mathcal{L}$ instead of $\mathcal{L}$.)

We now define ρ_c as the distance between each point in the given MCMC samples to the best-fit point. For each point, we compute the effective χ^2 difference, $\Delta\chi^2$ (i.e., twice the relative log-likelihood) with respect to this best fit point, both, for the likelihood obtained under the assumed PPS, and, with a free form PPS (the optimal PPS recover in our deconvolution). Fig. 1 shows scatter-plots of $\Delta\chi^2$ vs ρ_c for the case of power-law (PL) and running power-law (RN) assumptions of the primordial spectrum. Green crosses show the expected behavior that locally the likelihoods worsens with ρ_c as points depart from the best-fit parameters. In the other hand, the red pluses mark the same points in the cosmological parameter space, but for the $\Delta\chi^2$ obtained under free-form optimal PPS. It is clear from Fig. 1, that a free-form optimal PPS can very markedly improve the likelihood relative to that in assumed form PPS. What is more remarkable is that the improvement through optimal free-form PPS is suppressed in a basin around $\rho_c \lesssim 1$. This is apparent in the absence of red plus marks near the lower left corners of the plots.

It is also interesting to mention that the basins for the three assumed forms of the PPS are very distinct and non-over-lapping. The parameter distances between the best fit points assuming HZ, power law and power-law with running forms of the primordial spectrum are quite large. We have $\rho_{(PL,HZ)} = 14.56$, $\rho_{(RN,HZ)} = 43.79$, $\rho_{(HZ,PL)} = 6.89$, $\rho_{(RN,PL)} = 4.85$, $\rho_{(HZ,RN)} = 11.09$ and $\rho_{(PL,RN)} = 2.44$. It is important also to note that the best fit point obtained under one assumed form of PPS may be disfavored with a high confidence by another assumption.

As mentioned above, in the basins around the best-fit points it is very difficult to get a significantly better likelihood allowing for a free form PPS. It is instructive to explore the nature of these basins and the trends of likelihood assuming the free form PPS for each of parameters $\Omega_b h^2$, $\Omega_{0m} h^2$, h, and τ. To do so, for each parameter,

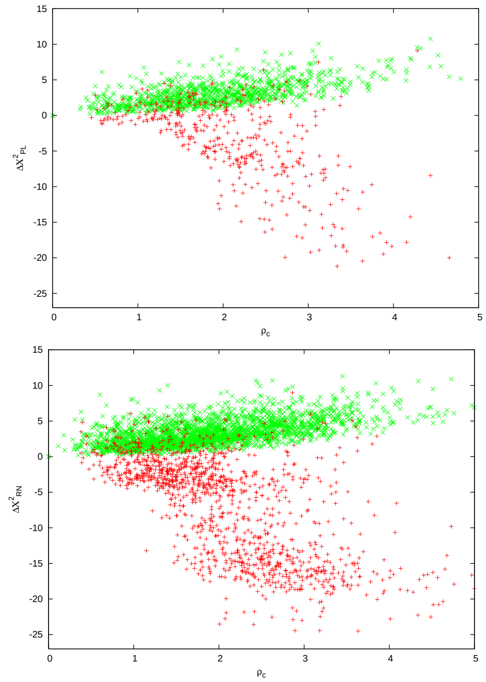

FIG. 1: The panels show the comparative scatter-plots of relative χ^2, with, and without, optimal $P(k)$, versus normalized distance, ρ_c, (eqn. 2.1) in parameter space of the sample points for sub-samples of the MCMC chains generated by the WMAP team. Green crosses show the $\Delta\chi^2$ relative to the best fit value. Red pluses mark the same points in the parameter space but with χ^2 derived after 'optimization' of the primordial power spectrum. The *top* and *bottom* panels corresponds to MCMC chains assuming 'Power Law' (PL) form and 'running power law'(RN) forms PPS, respectively. The obvious absence of red points with significantly negative $\Delta\chi^2$ for $\rho_c \lesssim 1$ mark the basins for each assumed PPS where, no (or, minor) improvement in likelihood is seen even invoking a free-form 'optimal' PPS. The basins for three assumed PPS are non-overlapping. For comparison, in the PL case (Upper panel), the distances to the best-fit HZ model $\rho_{(HZ,PL)} = 6.89$ and RN Model $\rho_{(RN,PL)} = 4.85$, respectively. In the RN case (Lower panel), distance to best-fit HZ and PL models are $\rho_{(HZ,RN)} = 11.09$ and $\rho_{(PL,RN)} = 2.44$, respectively.

i we split the separation, ρ, between points, a and b in the parameters into a separation $\Delta P_i = (P_i^a - P_i^b)/\sigma_i^b$, along the parameter and the 'perpendicular' distance $\rho_{i_\perp} = \sqrt{\sum_{j\neq i}(P_j^a - P_j^b)^2/(\sigma_j^b)^2}$ measuring the separation in the other three parameters.

Fig. 2 shows for the PL PPS case, a $2D$ surface representation of the optimized $\Delta\chi^2$ around the best fit point plotted against ΔP_i and $\rho_{i_\perp}$ for each of the parameters. We have weighted the neighbouring sample points by their Euclidean distance in the parameter space to assign an average likelihood at each point. The color palette is chosen such that red (blue) regions have poorer (better) likelihood than the reference value of best-fit model. The white regions have likelihood comparable to the best fit value. In this representation, the figures clearly show that in all cases, there is a plateau in the parameter space (the red regions) enclosing the best fit point where a free form PPS does not improve the likelihood. The location of the best-fit points are marked by red arrows. Outside these basins, there are blue regions where optimal free form PPS leads to very significant improvement in the likelihood. (However, note that these are far from being the global minima for optimal PPS cosmological parameter estimates – as shown in ref. [6], there are models with much higher likelihood.)

The plots in Fig.2, also supplement the PL plot in Fig. 1 (top), by indicating the direction in the parameter space from the best fit models where likelihood resists improvement against modifications to the PPS.

4. CONCLUSIONS

In this paper we have shown that the assumed form of the primordial power spectrum (PPS) plays a key role in the determination of cosmological parameters. In fact, the functional form of the PPS forces the best-fit cosmological parameters to specific preferred basins of high likelihood to the data. These estimated cosmological parameters are then significantly biased. It is similar to a case where in an N dimensional parameter space of a model, we fix the values of m parameters ($m < N$) and vary the other $N - m$ parameters to fit an observation. The resultant best fit values of these $N - m$ parameters can be very different, depending on the values assigned to the fixed m parameters. If there is no good reason to select a particular set of fixed values, the determination of the rest of the parameters remains under question. Assumption of assumed (say, power-law) form of the primordial spectrum can also be interpreted as a very strong, specific correlation between $P(k)$ at different k. This assumption is similar to setting values for the m parameters with specific correlations. We surmise that the assumed form of the PPS could be the dominant reason that in the basins for each assumed form it was not possible to achieve a marked improvement in χ^2 by allowing optimal free-form PPS (see Fig.2).

In summary, we show that the apparently 'robust' determination of cosmological parameters under an assumed form of $P(k)$ may be misleading and could well largely reflect the inherent correlations in the power at different k implied by the assumed form of the PPS. We conclude that is very important to allow for deviations from scale invariant, scale free or, simple phenomenological extensions of the same, in the PPS while estimating cosmological parameters. This provides strong motivation to pursue approaches that simultaneously determine both, the cosmological parameters, as well as, the primordial power spectrum from observations. The rapid improvement in cosmological observations, such as, the CMB polarization spectra, holds much promise towards this goal. It is not unlikely that early universe scenarios that produce features in PPS could in fact be favored by

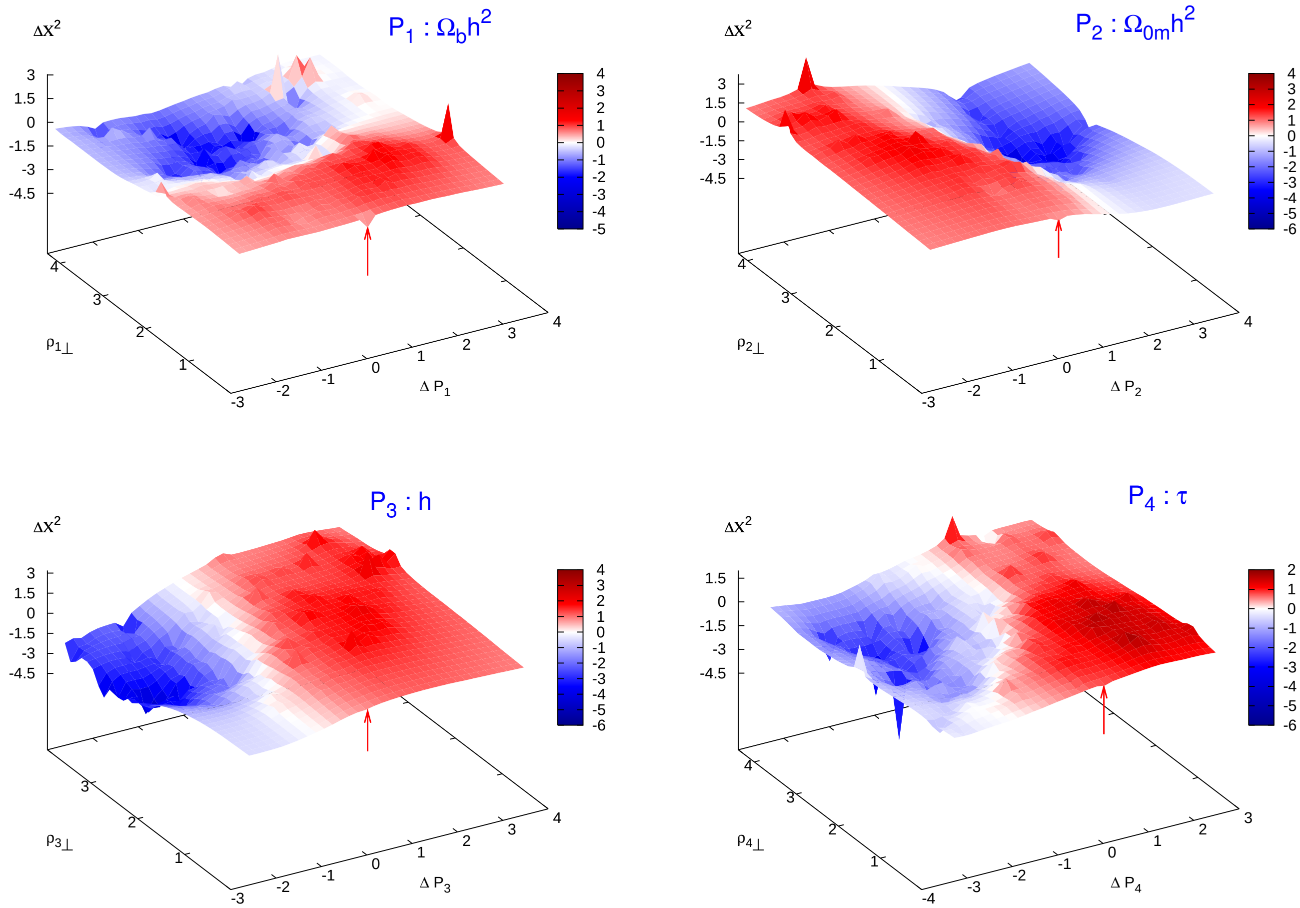

FIG. 2: A $2D$ surface representation of the optimized $\Delta\chi^2$ around the best-fit point for Power law PPS case for the four parameters. For each parameter, i, ΔP_i measures separation along the parameter, and $\rho_{i_\perp}$ measures the separation in the three other parameters. Regions in the parameter space with $\Delta\chi^2 > 0$ are shown by red color and are separated by a white band (representing $\Delta\chi^2 \approx 0$) from the regions with $\Delta\chi^2 < 0$ shown by blue color. Red plateaus represent the regions where allowing the free form primordial spectrum does not improve the likelihood. Red arrows show the position of best fit point assuming PL form of PPS.

data.

Acknowledgments

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